History of Mathematics, Mathematics Education, School Practice: Case Studies in Linking Different Domains

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In this paper we consider how the domains of mathematics education and history of mathematics may interact in the process of mathematics teaching. We try to tackle the problem by focussing on the teachers' role, which is central in this process (see Figure 1): on the one hand they act as filters of the stimuli and the suggestions coming from curriculum developers and historians of mathematics, on the other hand they are a basic source of outputs from classrooms which allow us to evaluate the experiments carried out. In the diagram of Figure 1 a dotted line represents the problematic relationship between the domains of research in mathematics education and in the history of mathematics, which will be the main concern of the present paper.

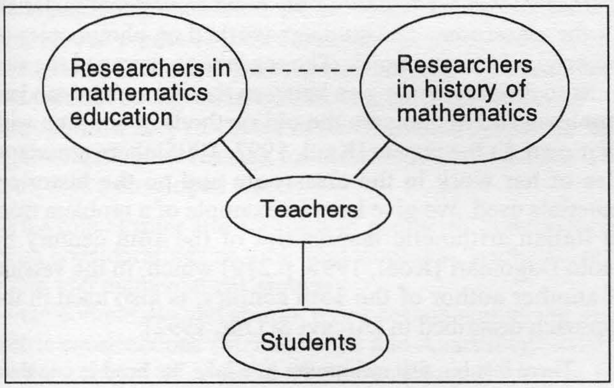


Figure 1

The literature shows that there are historians of mathematics particularly sensitive to pedagogical problems who not only interpret and mediate sources, but also select and discuss specific situations suitable for didactic transposition. Thus teachers have a very wide choice of materials suitable for classroom work [1]. On the other hand some curriculum developers encourage the use of history; in [Barbin *et al.*, 1988; Fauvel, 1990; Furinghetti, 1993] passages from official curricula in which the historical perspective is encouraged are reported.

On the basis of these facts a lot of experience has been accumulated about the uses of history in mathematics teaching. Looking at this experience we can observe some general issues:

— behind the work in classrooms there is the great enthusiasm of the teachers involved and their reliance on history;
— teachers' education and training in the history of mathematics is not homogeneous: a few have consulted original sources and research papers, others use manuals of history of mathematics, popularization books, readers, etc.;
— having introduced history in the classroom the teacher tends to consider the experience positive. This opinion, which is common to the majority of the cases, usually comes from subjective impressions and not from regular and systemic studies of the outputs;
— each experience is a "microworld", that is to say the various experiences are quite scattered and there is no organized network of classes and teachers carrying out analogous experiments. This does not allow us to compare the different results and to establish some trends in the research.

The last two points deserve reflection since the development of the use of history in mathematics teaching — on the efficacy of which we trust — needs a systemic approach based on a strict collaboration between the world of education and that of historians, to make the relationship represented by a dotted line in Figure 1 efficient. Looking for paradigms of work in this field can ensure the experiences are not confined to a few specific (happy) cases, but are transferable to other situations and contexts.

In this connection our contribution here is to offer insights on different kinds of experiences through the analysis of the work in classrooms developed by four teachers. Our study is based on the materials they produced, on their written and oral reports, and on students' protocols.

The case studies

• Teacher A: History and the image of mathematics held by students

At the Second European Summer University held in Braga (July, 1996) an exhibition of «images made by students about their image of mathematics» was presented. It was the result of a project developed with Italian students aged 18 at a *Liceo Artistico*, a type of school oriented to give an education in the visual arts. In this kind of school, mathematics is a compulsory subject, but is quite neglected and disliked by the students. The mathematics teacher was frustrated by this situation and felt the need to investigate the relationship his students had with mathematics [2]. It was not an easy project, taking into account the students' tendency to hide their feelings in order not to displease the teacher. To achieve the goal the teacher planned an activity lasting for two years. Initially students were faced with historical and epistemological problems in mathematics, took part in debates guided by the teacher, and engaged in personal research on the history of mathematics. The sources were classical texts in the history of mathematics and encyclopedias. After this phase of preparation the students were asked to write their impressions and their ideas about the development of mathematics through the centuries. At this point the teacher decided to exploit the abilities acquired by the students thanks to the particular type of school program they followed — their graphical and pictorial abilities. Students were asked to choose one of the following tasks: to make a poster for a hypothetical conference on mathematics; to draw some figure concerned with the history of mathematics; to design the cover of a mathematics book. Each graphical product had to be accompanied by notes explaining which ideas they wished to express, which artistic techniques were used, and, where appropriate, whether some artist's style had inspired them. The students' graphical works were photographed, digitalized through a scanner, and elaborated on the computer by means of specific graphics programs. The materials are very attractive from an aesthetical point of view and are very effective in giving a feeling for how students perceive mathematics, and which issues in its history impressed them. In Figure 2 we give an example: according to the author's explanation it represents mathematics (the world of numbers) and man, who is the link between the two aspects (pure and applied) of mathematics and the world which is the domain of mathematics.

Figure 2

We report here the teacher's description of the mathematical targets which were aimed at in this work:

— to set mathematics in a complete cultural scene;
— to motivate a class which did not achieve brilliant results in scientific subjects;
— to give a general picture of the discipline, to organize systematically the contents encountered in various school years, and to frame their mathematical evolution historically;
— to hint at the interactions between philosophy and mathematics;
— to recognize the contribution made by mathematics to the development of the experimental sciences;
— to develop the students' capacity to work autonomously;
— to give them practice in reading and composing texts and documents (the last two are general educational targets concerning other subjects than mathematics).

In addition, from the point of view of the classroom, this project had the important characteristic of combining different fields of study and promoting different abilities: the use of the computer, the history of art (which was an important source of inspiration for the pictorial styles adopted by the students), the graphic disciplines, history, mathematics, and its history.

This project answered the initiating teacher's purpose of collecting ideas on students' relationships with mathematics: the history of mathematics proved a good means for eliciting hidden beliefs and conceptions.

• Teacher B: History as a source of problems

This case, as with the previous one, stemmed from the teacher's need to motivate students who are not interested in mathematics. Teacher B used problems taken from arithmetic texts dating from the Middle Ages to the 16th century. The importance of these texts for studying the passage from arithmetic to algebra is widely recognized and some mathematics educators have based their work on students' difficulties on them — [Radford, 1995] for one. In our case the teacher mainly exploited the appeal that this kind of presentation of problems may have for introducing students to problem solving. She is doing research on 16th century arithmetic books and her deep knowledge of the original sources allows her to choose the most convenient materials for the classroom. The students worked on photocopies of the original Dutch copies. The teacher encouraged the students to discover analogies between the old and the modern problems and to compare the old methods of solution with their own. In the papers [Kool, 1992, 1995] there are examples of her work in the classroom and on the historical materials used. We give here an example of a problem from an Italian arithmetic manuscript of the 14th century by Paolo Dagomari [Kool, 1995, p.219] which, in the version of another author of the 15th century, is also used in the approach described in [Arcavi & Ofir, 1992]:

> There is a big fish swimming in a lake. Its head is one-third of the whole. Its trunk is one-fourth of the whole, and its tail is 9 feet long. How long is the whole fish?

Here are some of the sentences she writes in support of the choices she made in working with history. "When a colleague asks me if and how to use history I answer: Do not *talk* about the history of mathematics in your classroom, but do it, use it!! Use historical problems in your teaching for reasons of variety and to give your pupils something extra! The extras that historical problems bring to your pupils are historical insights and mathematical insights. Historical problems may intervene at the end of the learning process as an extra exercise or the application of a new learned mathematical topic, or at the beginning to stimulate pupils to develop their own individual strategies".

• Teacher C: History as an optional activity

This teacher teaches at the secondary level (ages 16-19) in a type of school, called *Liceo Scientifico*, where mathematics has a fundamental role. This does not mean that all the students like and/or understand this discipline, nevertheless the problem of motivating them is less pressing than in the previous cases. On the other hand the mathematical program is so full of topics that it is difficult to find space for new activities; for this reason the historical activity we are reporting was carried out outside scheduled school time and with students who volunteered. Even if these two conditions (the extra-curricular and non-compulsory natures of the activity) make consideration of the case of teacher C less transferable to "normal" situations, nevertheless the experiment is an interesting source for studying the introduction of history. In particular, we point out, it can be carefully evaluated, since the small number of students attending the lessons (16) allowed the teacher to observe their ways of thinking and working. At the end of each lesson students filled in a questionnaire intended to collect information on the quality of the course, on their appreciation of the topics and of the exposition by the teacher. A very detailed final questionnaire was given for the overall evaluation of the course. It was also possible to give the students a voice: they exposed the results of their research and accepted the criticisms of their colleagues. The students, being volunteers, were not necessarily gifted, but were certainly curious, thus the classes were very alive and the students worked with a will.

The teacher is very fond of the history of mathematics, a collector of old books in this field: the idea for the activity we are referring to comes from an item in his collection. The subject was conics [3]. The course fell into two parts, of 8 hours each. In the first part students were given a historical and cultural background concerning the birth and development of the theory of conics in Greek antiquity with the aim of illuminating the reasons for the names of the conics. The main topics developed were:

— the role of ruler and compass in geometric constructions (with considerations of their philosophical, aesthetic, religious, and mathematical aspects);
— the complexity and simplicity of certain significant geometric constructions (Menaechmus and Anaritius);
— the concept of a curve (construction and criteria for classifying);
— transformations: of a problem into another (Hippocrates), and of geometrical figures (Book II of Euclid's *Elements*: parallelograms and triangles);
— the symptoma of conics.

In developing these lessons it was also a precise concern of the teacher to set mathematics in a wide cultural context through frequent readings concerning mathematics in different fields, both literary and philosophical, e.g., texts by Proclus, Vitruvius, Eratosthenes, Aristotle, J. Donne, T.S. Eliot and J. Joyce.

The second part of the course was centred on studying conics from the French text *Traité des sections coniques, et autres courbes anciennes* by de la Chapelle [second edition, Debure, Paris, 1765], see [de la Chapelle, 1750]. This treatise was chosen for its clarity of presentation of the topic. As is recounted in [Itard, 1952], this author was very concerned with the problems of mathematics teaching, a matter for lively discussion in the 18th century; one need only think of the didactic work of Clairaut. In the preface the didactic purposes and the strategy of leading the students to each discovery step by step are described [de la Chapelle, 1750, p.V].

Along the same line that we find in Clairaut's *Éléments de géométrie*, de la Chapelle pays attention not only to the "pure" theory (as «Messieurs de la Hire, Guisnée, and L'Hôpital» do when treating conics sections), but also to applications «aux Arts» [*ibidem*, p.IV].

The approach to this text was planned so that the students' role would be very active. They used worksheets of increasing difficulty prepared by the teacher (4 for the parabola, 3 for the ellipse, 1 for the hyperbola, and finally 1 for the conchoid and its use in the duplication of the cube).

At first glance this experiment seems mainly a way to implement a short course in the history of mathematics. But in studying the students' reactions we see the educational potentialities of such a course and that some of the educational outputs could be transferred by the teacher to normal curriculum activities. The work of teacher C shows that really, as [Heiede, 1992] puts it, doing the history of mathematics is doing mathematics. Interpreting the unfamiliar (and old) language (French) of the text revealed itself as a good way of grasping the mathematical concepts and the procedures behind them. It obliged students to work step by step organizing their work and their reasoning carefully. The same happened too with the interpretation of the figures in the de la Chapelle text. For example, students were presented with Figure 3 (Figure 10 in the edition of 1765) where, according to the usage at that time, the graphical elements referring to propositions VI, VII, VIII, IX, XIV are superimposed. The students learned to disentangle the different graphical components by relating them to the appropriate propositions, and thus acquired an ability (operating with figures, historical or not) often neglected in classroom practice.

According to the words of the teacher himself the aims of the historical work carried out in classroom are: «to train the students — within the limits of the context — to search for historical sources, to become students of the ancient authors.» He adds that his choice of aims relies on the pedagogical conviction that «to create an environment and a way of thinking may give rise to an enthusiasm for discovery and thus foster the process of understanding. In addition, the habit of right quotation is good training in intellectual honesty, in discussion».

The particular historical source favours the pedagogical fall-out desired by the teacher since, as we said above, de la Chapelle's text has a strong didactic flavour. It is interesting that one student (at least) grasped (obviously unconsciously) the pedagogical method behind the book by de La Chapelle, as appears in her answer to the question in the final questionnaire evaluating the course [Testa, 1996, p.453]:

> «I ask myself more questions, I no longer keep to what I am told to study but I try to reach a more thorough understanding. In tackling problems, I now proceed methodically, step-by-step, point by point. Things must be seen in depth, we have to investigate their nature and not accept them as they are. Now I very often organize my study by following schemes or building them; moreover, I try to reason in a

more systematic way. I no longer focus my attention on a detail, but consider a problem in general, by transforming it, if possible, into a more simple one».

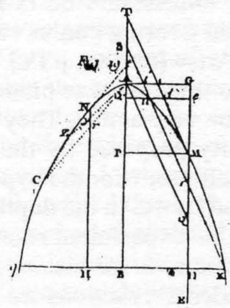


Figure 3

Teacher D: History as a different approach to concepts

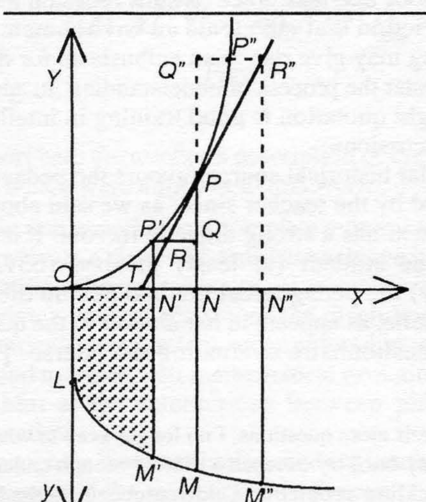
This teacher teaches at the secondary level (ages 16-19) in a Liceo Scientifico. She is also doing research in the history of mathematics; knowledge of the field has led her to the conviction that she would find there the answer to some teaching problems. Here we give some examples of her way of working concerning calculus and proof.

In calculus she felt that one of the more difficult concepts to understand is the concept of area and the related concept of integral; in particular, it is difficult to grasp the link between the derivative and the integral. In this connection she offers students the re-elaboration below based on lesson X from *Lectiones opticae et geometricae* of Isaac Barrow (1674), London; second edition, pts.I and II) [4].

About proof, she considers that in the past the method of analysis and synthesis was efficiently used both in research and in teaching. We find a clear presentation of this method already in the *Collectiones mathematicae* [Pappi …, 1660]. In his book on mathematical reasoning [*Des méthodes dans la science de raisonnement*, Gauthier-Villars, Paris, 1865] J.-M. C. Duhamel states that the double method of analysis and synthesis, which is good for making discoveries, with the addition of more reasons can be employed to explain the discovery. She presents this method in classroom through the work of Marin Ghetaldi [5]. An example of the problems proposed to students is shown on the following, page taken from the book *De resolutione et compositione mathematica libri quinque* [Ghetaldi, 1630] by Marin Ghetaldi. The solution is presented according to the schematization («conspectus») also used by Ghetaldi in other problems of the book [6]; we consider this schematization very efficient from the didactic point of view.

The method of analysis is also illustrated for the students through the study of passages of Descartes's *Discours sur la méthode* [Leiden, 1637]. With this presentation the teacher sheds light on the important links between Euclidean geometry, algebra, and analytical geometry, given by the analytical method, in line with the ideas on the development of algebra in relation to the evolution of this method discussed in [Radford, 1996].

The teacher thinks it is "politically correct" to present students with examples of proofs developed by analysis and synthesis, since this is the way most proofs of theorems studied in school were constructed. Usually textbooks present only one way; this makes certain procedures in the proofs of the calculus theorems seem unjustified *a priori* and very cunning (see, for example, Rolle's Theorem and the proofs of the theorems on limits). In this case one of the values of a historical presentation is its capacity to make the rules explicit. Teacher D observed that students presented with the method of analysis and synthesis become free and easy in applying it to different situations, even when they are not required by the teacher to do this. We stress that the use of history in this case allowed the teacher to satisfy the students' need for explicitation, which is frustrated in the usual teaching.



Let $y = f(x)$ an increasing function, $MN = f(ON)$. The Oy axis is turned down. Let OY another axis turned up. MN is extended with a segment NP equal to the area of $OLMN$. When the point M varies P describe a new curve. Take on the x axis a segment $NT = NP/MN$, the right line PT is tangent in P to the new curve. Proof:

Let $N' \neq N \quad ON' < ON$

$PQ = PN - QN = PN - P'N' = \text{Area}(OLMN) - \text{Area}(OLM'N') = \text{Area}(N'M'MN')$

The triangles PQR and PNT *are* similar

$PQ : QR = PN : NT = PN : NP/MN = MN$

$QR = PQ/MN = \text{Area}(N'M'MN')/MN < MN \cdot MN'/MN = P'Q$

$QR < P'Q$, P' is on the left side of the right line.

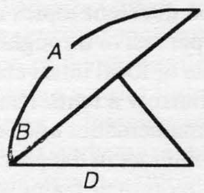
Let $N'' \neq N \quad ON'' > ON$

$Q''R'' > Q''P''$, P'' on the left side of the right line.

Then in a neighbour of P the curve is on the same side in respect to the right line PT and, consequently, this last one is the tangent right line (note that Barrow uses the ancient definition of tangent).

The passage from the given curve to the new one means to make an integration; the inverse passage is made by constructing the tangent PT whose angular coefficient (our derivative of the function NP) is $NP/TN = MN$ (see the initial function).

Problema VIII (Ghetaldi, 1630, p.92-93): «Data base trianguli, angulum rectum subtendente, & differentia crurum. Invenire triangulum» [7].



90°, basis D, sides' difference B

Resolutio

Suppose the problem solved

Let A = sum of the sides

$A/2 + B/2 = l_M$

$A/2 - B/2 = l_m$

$l_M{}^2 + l_m{}^2 = D^2$

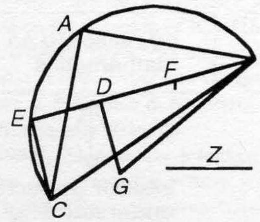
$A^2/2 + B^2/2 = D^2$

$A^2 + B^2 = 2D^2$

$A^2 = 2D^2 - B^2$

(sides' sum)^2 = 2basis^2 - (sides' difference)^2

[this, as observed by the author, is the «porisme», which allows to find the sum of the sides]



90°, basis AB, sides' difference Z

Compositio

$EB^2/2 + FB^2/2 = D^2$

$EB^2 + FB^2 = 2AB^2$

$EB^2 = 2AB^2 - FB^2$

$EB^2 = 2AB^2 - Z^2$

$EB^2 = CB^2 - EC^2$

Construction:

Let $AB \perp AC$, $AB = AC$, construct the semicircle with diameter BC and take $EC = Z$.

EB satisfies the porisme (is the sum of the sides), that is to say

Take $FB = EC$ and $FE/2 = DF$; construct $DG \perp DE$, $DG = DE$

BDG is the triangle we search for; this is proved by the *compositio* (see below), then

$DG^2 + DB^2 = GB^2$

$GB^2 = AB^2$

$GB = AB$,

the angle GDB is right for the costruction and the difference of the sides Z is equal to $FB = EC$ for the costruction.

The analytical process gives a relation among the segments which can be translated into a geometric construction, thus providing the synthetic solution of the problem.

From the use of history in mathematics teaching to its integration

All the teachers considered here had a common aim in their teaching, which may be expressed (with extreme compression) by the statement of one of them. To pursue this aim they have spontaneously chosen to work with history (in general this is not required by the mathematics curricula). We can infer that all of them have a particular leaning towards history: it is obvious that teachers bring to the classroom their tastes, their abilities, and their knowledge. In particular, a good knowledge of the history of mathematics (including a sufficient familiarity with original sources) fosters pedagogical creativity and mastering the uses of history in mathematical activities.

The teachers who decide to work with the history of mathematics have to answer (consciously or not) the questions that teacher B poses to herself after having found suitable historical materials [Kool, 1995, p.215]:

> «Where can I use it, with which pupils, at which moment? Which aids can I give my pupils to solve the problem? Do I want my pupils to solve it in a specific way? Can I connect other questions to the given problem? Do I confront my pupils only with the historical problem or with the historical solution too? Shall I ask my pupils to study the historical solution method to see why it works and to compare it with their own modern solution method?».

The answers a teacher gives to these questions shape the style of his or her teaching. But the key point in shaping this style is that the history has to satisfy the teachers' needs arising out of their mathematics teaching; the different ways of introducing history are strongly influenced by the different needs they may feel.

To explain the role played by the factors at issue in the process of using history we outline two main streams of intervention of history into mathematics teaching arising from our case studies (Figure 4). One stream is aimed at promoting mathematics, the other at reflecting on mathematics; the first is linked with the "social" role of the discipline and its image, the second mainly concerns aspects interior to the discipline, such as development and the understanding of it.

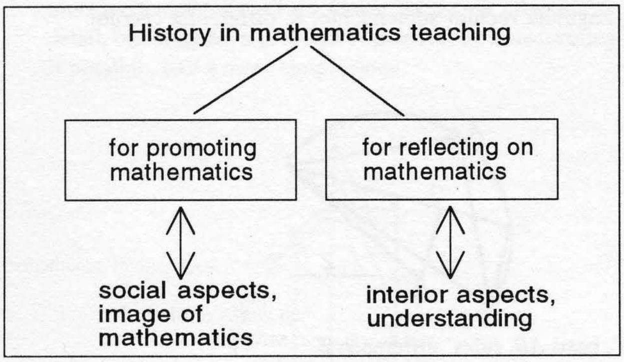


Figure 4

Cases A and B fall into the first stream; cases C and D into the second; obviously a "promotional" intervention may be a first step to a "reflection" phase, as we can see in B's case where the appealing old problems are also used for discussing solution methods and the problems of notation. In our opinion the fundamental value of following the first stream of Figure 4 is its acknowledgement of the need for the teacher to create the atmosphere he/she feels suitable to his/her work. Both teachers A and B work in a context unfavourable to mathematics and feel their main problem is to give students a reaaon for doing mathematics. The pedagogical elements on which these experiences base their success are variations in the classroom routine, surprise, and links with other aspects of the students' cultural life. In case A the influence of the context (the artistic orientation of their curriculum) brings the teacher to the need for emphasizing the cultural value of mathematics and the place of this discipline in the development of human culture. Teacher B works in the Dutch context, where important projects on mathematics teaching are centred on dealing with problems taken from real life; she says that her first inspiration for motivating students took this direction, and afterwards she arrived at old problems which gave the same feeling. Ultimately, with historical problems she succeeded in raising interest in present-day problems.

The second stream of Figure 4 ("reflection") is more complex. Here the focus is not on the appeal of history (which of course can be present, but not in a determining way), but on the nature of mathematics learning/understanding. We focus on case D to see the variety of situations it touches. If we consider the examples given here and the others reported in [Furinghetti & Somaglia, in press], we observe that Figure 4 can be enriched by further specification of the kind of reflection we can introduce with history, as shown in Figure 5.

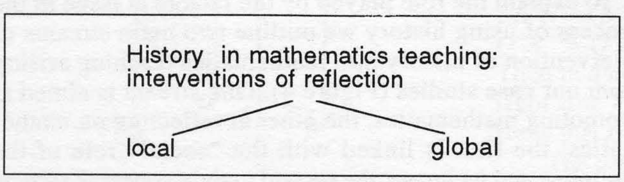


Figure 5

By *local interventions* we mean those uses of history which concern the introduction of a concept or a procedure, though confined to a specific case; by *global interventions* we mean cases in which the use of history touches on a whole method of working on different topics and situations, rendering the intervention pervasive throughout the learning process.

An example of local intervention may be the didactic unit inspired by Barrow's tenth lesson. We use this example to analyse how mathematics education and the history of mathematics may interact in the process of teaching/learning. We refer to the theoretical framework given in [Sfard, 1995]. This author stresses the duality that exists in mathematics between computational and abstract objects which are linked to operational (in the first case) or structural (in the second) conceptions of them. The two kinds of objects are related and the author explains the relation with a model consisting of three steps: interiorization → condensation → reification. In the first step a certain object is presented, in the second the object undergoes some computational processes that provoke the condensation of it, in the third step the object achieves a structural status. In our example the focus on a non-traditional computational process provided by the Barrow procedure offers favourable soil for the condensation to be nurtured (see Figure 6).

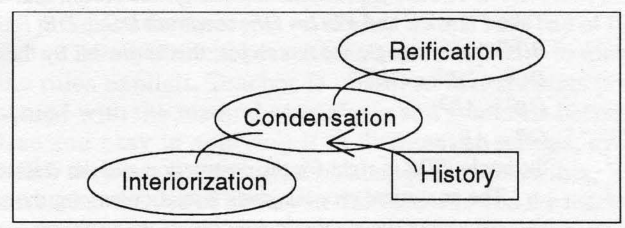


Figure 6

In this case the student may arrive at the structural stage in the reification of a concept in a new domain (calculus) through a process performed in a familiar domain (elementary geometry). The historical contest activates a different frame for tackling the same problem, thus promoting students' flexibility, identified by mathematics educators as a basic ability for solving problems, see [Gray & Tall, 1993].

An example of global intervention may be the introduction of the analysis/synthesis method through Ghetaldi's work and the extension of its use to other situations. The relation existing between educational and historical research is even more evident if we look at the literature on proof. In [Alibert & Thomas, 1991] the authors analyse the problem of proof and propose a four-level method previously studied by Uri Leron. Their aim is to give students a way of following the construction of the paths in a proof. The pattern of this schematization of proof fits quite well with the analysis/synthesis pattern, even though the authors do not mention this historical method and probably constructed their schematization by themselves by exploiting their competences in the field of mathematics education. More recently, in the Topic Group on proof at ICME 8 in Sevilla (July 1996), the classification and the examples of proving in secondary schools examined there included the analysis/synthesis method [Ibañes & Ortega, 1996].

This unconscious convergence of methodology is the most convincing confirmation of the orientation advocated by some authors towards forging a strict link between the study of the cognitive processes of learning and the results of an epistemological analysis to study the problem of incorporating history in mathematics teaching. The naïve approach which consists of transposing directly historical passages directly into the classroom, bypassing all methodological considerations, remains at a rather superficial level and does not lead to didactic situations that are significantly helpful to learning.

When the history of mathematics is introduced in some form into mathematics teaching many authors, instead of using expressions such as «the use [introduction] of history», prefer to use the expression «the integration of history» which really better fits the idea of efficient teaching as well as efficient analysis of the cognitive processes of learning and understanding. The word "integration" underscores a use of history in mathematics teaching developed according to the following steps: — to single out the common objectives of learning mathematics and learning the history of mathematics; — to develop them, exploiting the specificities of the two fields involved; — to analyse the cognitive results in the light of patterns of educational and epistemological research.

Notes

[1] For a survey on of historically-related articles see: the special issue of the journal *For the learning of mathematics*, [v.11. n.2, 1991], [Calinger, 1996], [Swetz, Fauvel, Bekken, Johansson & Katz, 1995)] and the Proceedings of the European summer universities [Montpellier, 1993 and Braga, 1996].

[2] For a brief presentation of the exhibition see p.51 in the booklet of the program of the *Second Summer University* (Braga).

[3] For a detailed description of the experience see [Testa, 1996].

[4] Recently other authors have recognized the explanatory character of this lesson [Flashman, 1996].

[5] The mathematician Marin Ghetaldi was ambassador of the Republic of Venice, travelled all over Europe and met important mathematicians such as Clavius and Viète. For information on the work of this author see [Dadić , 1984, 1996]

[6] The letters and the notations of the figures are taken from Ghetaldi's text.

[7] Given the hypothenuse of a right triangle and the sides' difference find the sides.

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