The Stereotyped Nature of School Word Problems

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Where is the failure in the teaching of word problems?

The motivation to teach word problems in arithmetic in schools depends on a strong assumption which says that arithmetic is a powerful tool which enables one to gain additional new numerical information by means of arithmetical manipulations on the basis of that which is already known. Arithmetic is, then, a privileged language for dealing with almost every area and issue which requires a numerical answer, and word problems serve as the area of teaching "applied arithmetic", par excellence. Mastering applied arithmetic seems to be one of the most significant realms of knowledge for future living and it is frequently defined as one of the basic skills for the average citizen. No wonder, then, that schools have struggled for years to teach word problems in school, and hundreds of documents are piling up on this issue.

The continuous volume of documentation and research in this area is due to a consensus that most students fail in solving word problems, beyond the simple ones [NAEP, 1979, Lindquist, 1979]. The failure, however, is on two grounds:

(a) Many children fail to understand the essence of applied arithmetic at school and develop defensive strategies and a kind of phobia which usually end in failure in performance, and,

(b) Word problems at school do not resemble problems in real life situations, and they are not considered by the children themselves to be related to the real world.

To illustrate the above two points we will present some typical reactions of children from the primary level.

In the first case, after reading a simple word problem, Rachel (age 8:6) said to her teacher: "I understand the words and the numbers well, just tell me whether to add or to subtract" [Nesher, 1976, unpublished, see also Brown & Küchemann, 1977]. The teacher, for alleged educational considerations, refused to answer this specific question and politely explained to Rachel: "But this is what you are supposed to find out for yourself." Rachel, however, did not benefit from such an answer.

A similar misunderstanding happened to Jacob (age 7:2) who in an answer to the following question: "There are 4 buttons on the front of the shirt and one button on the pocket of the shirt, how many buttons are there altogether?" answered immediately: "five." Then when asked to write down a mathematical sentence about the same question, he wrote: "7+2= 9." Jacob could not understand what was wrong with his sentence. He, of course, did not understand that such a simple problem, about a shirt and five buttons, hides behind it the whole philosophy of teaching him how to use the privileged language of arithmetic in real life situations. He did not need the privileged language in this case. He knew how to answer the simple question, he knew how to solve more complicated symbolic sentences, but yet he failed the task given to him: to connect the two.

Steffe and Thompson [1979] in a study of 47 first graders in a clinical setting wrote (about one subset of them):

> *It seems clear to us that these children were not engaging in problem solving, at least as problem solving is conventionally thought of. They did not make constructions to serve the satisfaction of a primary goal "how many" — setting and satisfying subgoals in doing so (p.38)*

There is also additional evidence about the fact that children are engaged in the special activity of "solving word problems" without relating them to any real life experiences but rather accepting them as part of the school ritual. Note for example the following "problems" composed by first and second graders when asked to tell a "story" which would correspond to the mathematical sentence 1 + 6 = 7 [Nesher, 1976; unpublished]:

> *Johnny (second grade). "Mummy bought an iron and then she bought six more irons. Now she has seven irons"*

Clearly, this story does not represent an elaboration of a real life situation, but rather Johnny's school experience concerning task requirements. He was given some numbers and was asked to do something with them, and it really did not matter what he did as long as he responded promptly.

Similarly, Ruth who was completing first grade when asked to compose a story for the numerical expression "3 + 4", said "I ate three cups and four plates ..." (ibid). She, too, was more concerned with fulfilling her respondent role than monitoring the plausibility of her story. These kinds of answers are a result of school learning. They could not have been learned through the children's real life experience. It is part of the faulty learning that takes place at school that we present our well-defined structure of knowledge, but do not know how to match it to the child's understanding of the same task in the general framework of school. While to us the area of word problems in arithmetic is one which presents the child with real life situations, to the child it is one more irrelevant, somewhat capricious school task where one has to do some computations from given verbal data.

In what follows we will try:

a) To demonstrate that school's arithmetic word problems (hereafter: SCH-PROB) *do not* resemble problems in real life that call for numerical answers and computations (hereafter: REAL-PROB).

Then, we will try to answer the question:

b) What *can* SCH-PROB teach, or, what is the educational goal that such a component in our curriculum should aim at?

The above argument will lead to the question:
c) Can we suggest another approach to the teaching of SCH-PROB?

Comparing school arithmetic word problems with quantitative problems in real life

Let us start with an illustration. Imagine your neighbor Mr Smith would like to build a fence around his swimming pool He faces a real problem, which calls for some numerical computations The formulation of his problem in real life will be:

(I) "How much will it cost to construct a fence around Mr. Smith's swimming pool?"

The presentation of the problem does not simultaneously involve the specification of all the information required for the solution of the problem. In fact, no numerical data is given at all. One of the major tasks would then be that of finding the relevant information for the solution of the problem.

In looking for the relevant information it would appear that some *qualitative* decisions have to be made, which may include numerical manipulations and testing alternatives to the numerical answer (for example, decisions concerning the type of material to be used for the fence, its cost per unit, the cost of transportation, the question of whether Mr. Smith wants a strip of lawn along the sides of the pool and how wide it will be, etc).

The verbal formulation of the problem does not provide any clues concerning the required mathematical operations and at times there may not even be a unique answer to the problem, but a set of options, one of which is to be selected.

The complexity of the example about Mr. Smith's pool is due to the *qualitative* considerations which have to be made in choosing the *relevant dimensions to be quantified*. Since it is not clear to what extent it is the task of the arithmetic lesson to develop qualitative reasoning of a practical nature and practice decision-making concerning personal preferences, the traditional approach of our arithmetic instruction has been to develop a *simplified* form of word problems in which all the qualitative decisions have already been made.

Mr. Smith's real problem is likely to be formulated in our schools as follows:

(II) "Mr. Smith's swimming pool is 12 m long and 8 m. wide. Mr. Smith wants to build a wooden fence around it, leaving a 2 m. wide strip of lawn all around the pool. How much will the fence cost if a meter of fencing costs $10 (cost inclusive of all materials and labor)?"

It seems clear that the two different formulations of Mr Smith's problem (I and II) make intrinsically different cognitive demands One should not, then, be surprised that children who know how to solve simple quantitative problems of addition and subtraction out of school [Gelman, 1979; Carpenter and Moser, 1979; Ginsburg, 1979] fail to solve similar problems when they are given at school. The requirements in these "similar" problems are quite different.

It could be argued that the instructional value of school word problems lies in their simplicity relative to "real problems", and they serve, therefore, as a tool at the beginning stages, since the young child is given a set of data, formulated with great conciseness, which enables him to concentrate on the main task of choosing the mathematical sentence that would lead him to the solution.

This would indeed be the case if it were not for the problems created in the form of learned responses which may be detrimental to future problem solving activities. We maintain that school arithmetic word problems as they are formulated, induce in the young learner an unwanted rigidity, so that before he even grasps the idea of the possibility of applying mathematics to the solution of everyday problems, his responses have already become overly structured. Thus, when he reads a word problem (SCH-PROB) he tends to disregard the actual situation described and to engage in exploring the possible combinations of numbers occurring in the text (see the case of Rachel on page 2). The example of Jacob (on page 2) raises also another problem If the simplification of the problem is carried too far, a child may not see any purpose in "saying a mathematical sentence" at all. He cannot, therefore, appreciate the language of arithmetic as a privileged one

Let us now turn now to examine more carefully the difference between school problems and real problems.

On the stereotyped nature of arithmetic school problems

One of the assumptions underlying the rationale for presenting the 'simplified' text of word problems in school, instead of real problems, is the assumption that this text is in fact a description of real life situations. The language employed is supposed to be a natural language, easily understood by the children, so that instead of probing into all kinds of complex qualitative decisions the child is presented directly with a more advanced stage, where all the qualitative decisions have already been made. A linguistic analysis of word problems in arithmetic, however, points out [Nesher-Katriel, 1977] that they must be recognized as a very special type of text, whose interpretation is shaped by the language game of arithmetic instruction, and cannot be considered as a text describing real life situations. Some points which may illuminate the stereotyped nature of SCH-PROBs will be mentioned here. They will refer to the semantic, referential and stylistic aspects of the texts

The semantic aspect

Understanding the textual statement of a word problem requires a recognition of the existence of semantic dependencies between the strings underlying a word problem text. These dependencies dictate in most cases a special interpretation of the lexical items given in the text. For example, the following problem [taken from Nesher-Katriel, 1977]:

(III) "Two boys were running to the classroom and three boys were walking to the classroom How many boys altogether arrived at the classroom?"

The verbs "walk" and "run" have largely overlapping meanings and differ only in the specification of the manner of motion. When two such verbs occur in a narrative discourse in close proximity as they do here — "two boys were running and two boys were walking. . ." — they are interpreted with an emphasis on their differential semantic features and not on their common ones. The interpretation of the verbs seems to be based on the general pragmatic assumption that the juxtaposition of lexical items belonging to the same semantic field is instrumental for the purpose of singling out their differential meanings (in this case the manner of motion.) If one is not interested in emphasizing the manner of motion, one would simply say, "Four boys went to . " (or "Four boys were walking . ") instead of "Two boys were running . and two boys were walking . ". The reading of these verbs (run, walk) in the word problem above is quite different.

(a) If the children ran to the classroom then:
  (1) they moved toward the classroom
  (2) their motion was fast

(b) If the children walked to the classroom then:
  (3) they moved toward the classroom
  (4) their motion was not fast.

In a word problem text, the interpretation of the above semantic features of "run" and "walk" is shaped by the underlying logical (additive) structure of the text. Thus, one can add the two sets of boys, since they share features (1) and (3). The union will be of those who arrived at the classroom (which share the same interpretation). The manner of their motion, which was the main reading in a narrative discourse, is of minor significance. It is mentioned in the text only to establish the fact that the two sets of boys mentioned, are disjoint sets (another logical constraint), and the child is supposed to disregard it. This last condition could be achieved, however, in many other ways (with adjectives, for example, "two big boys and two small boys ...").

Our claim is that interpreting the same linguistic entries in the context of word problems of arithmetic is *not the same* as in the context of another type of discourse, and arithmetic word problems do not constitute a part of natural language use. Therefore, the understanding a child has of given lexical items in ordinary language might not always facilitate his understanding of the text of the word problem.

The referential (ontological) aspect

The universe of "objects" mentioned in a SCH-PROB is a special domain. The actual *identity* of the objects in these texts is far less relevant than the relationships between their semantic classes. In fact, the question of strict identification rarely arises, since the reading of numerically quantified arguments usually involves non-specific reference. Thus in a SCH-PROB one will typically find the formulation:

(IV) "Three boys went to the beach . . "

and not:

(V) "Jacob, Joseph and Dan went to the beach . ."

which one may find in narrative. The latter formulation if given in a SCH-PROB is considered to be a complication, since it requires a further transformation to change it to (IV).

In a SCH-PROB text none but those "objects" mentioned in the text exist. Furthermore, the objects mentioned in the text persist for the duration of the problem, and nothing changes in them. Consider the example:

(VI) "Ruth had eight candies in the morning. At noon she gave two of them to her sister Susan. How many candies does she have now?"

How about the possibility of eating some candies during the day? How does such a description correspond to the child's experience?

Our claim is that the "objects" described in SCH-PROBs are not *real* for the child. These are different objects, which are subject to different rules, rules which the child is going to learn in school.

The stylistic aspect

A SCH-PROB in contrast to a unit of narrative discourse is numerically overloaded in an artificial manner, disregarding all other attributes of the mentioned objects. All the characteristics which do not help directly to understand the logical structure of the problem are considered to be "superfluous" or "extraneous" [Nesher, 1977; Lindquist, 1979]. The notion of "superfluous information" and "missing information" are relative to the underlying *logical* or arithmetical structure the author of the text had in mind.

The need to give all the required information on the one hand, and the tendency to create as abbreviated a text as possible on the other hand, result in a text which is laconic, very concise and in many cases employs very difficult syntactic structures (see problem II).

Our claim in this section is that SCH-PROBs are not stated in ordinary language and they do not relate to the child's experience. They are stereotyped in their style and in their semantic interpretation and describe objects and events which have no reality, and thus bear no resemblance to any "real world". In what follows we will try to show that it is the lack of a real context on the one hand, and the stereotyped formulation of the SCH-PROBs texts on the other hand, which encourage the children's mis-learning.

Aspects of research on arithmetic word problems

Research on solving word problems has many facets and has shifted focus in recent years. We do not intend to summarize the enormous amount of research existing in this field, but rather to point out the progress achieved during the last decade in understanding that word problem solving involves several levels of interpretation.

The initial preoccupation with the *logical component* of the word problem [Jerman and Rees, 1972; Resnick et al, 1973; Nesher, 1976; Weaver, 1979], soon gave way to the realization that the verbal formulation of the text should also be taken into consideration.

Research on the *linguistic component* of word problems first dealt with surface structure variables, namely, *syntactical* variables and isolated *lexical items* [Paige and Simon, 1966; Jerman, 1973, 1974; Searle, 1974; Wheat and Kulm, 1976; Linville, 1976; Nesher and Teubal, 1974]. In recent years there has been growth in research dealing with the *semantic* analysis of the texts of word problems [Vergnaud, 1976, 1979; Nesher and Katriel, 1977, 1978; Nesher, 1979; Greeno, 1979; Carpenter and Moser, 1979; Moser, 1979].

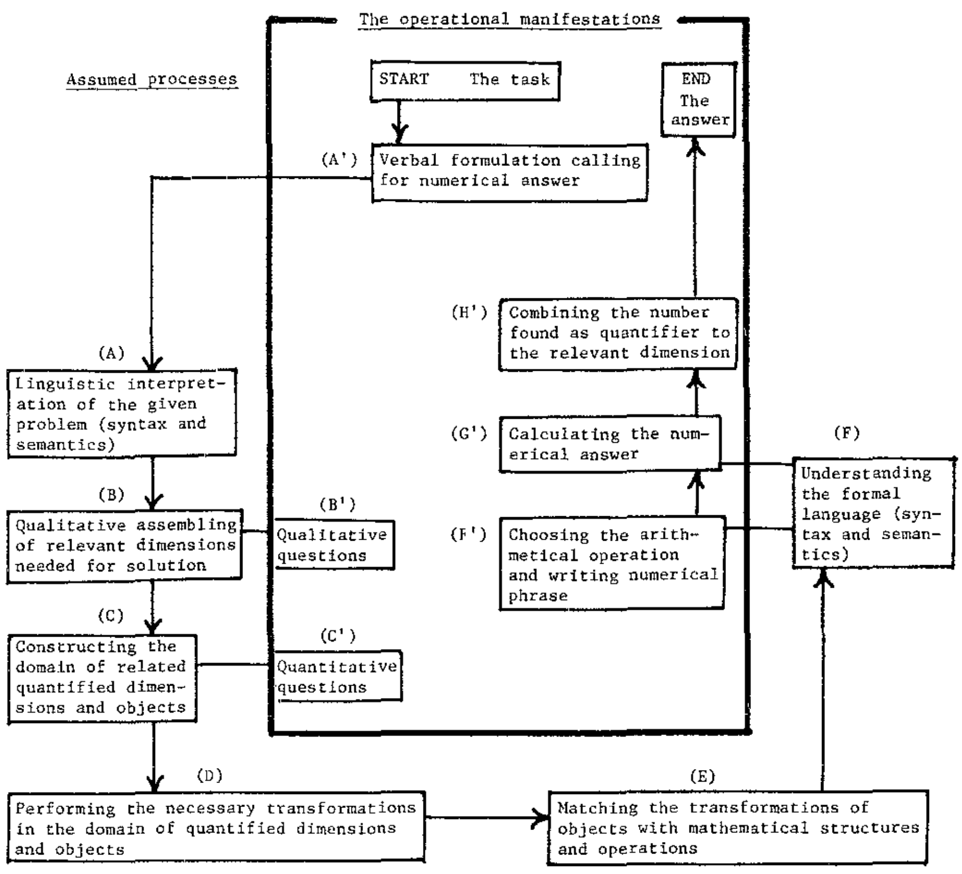
Unfortunately, the analysis of all the above variables left us with unexplained difficulties These require additional variables, that seem to us to be *pragmatic* variables The first indication of the role of such contextual variables appeared with the need to make a distinction between dynamic and static texts

In a *dynamic* word problem one can distinguish between the *initial* state of affairs (at time t_1), a change in that event that occurs later, (at time t_2), and *a final* state (at t_3) [Vergnaud, 1976, 1979; Nesher and Katriel, 1978]. Greeno [1979] describes it as a cause/change word problem.

A static word problem refers to a single state of affairs. Its underlying strings refer to sub-parts of the entirety Moser [1979] calls it a part-part whole problem*.

* The above distinctions are taken from the sample of addition and subtraction word problems It is assumed, however, that one can find similar distinctions in other word problems

If one examines this distinction more closely one realizes that it relates to the *pragmatic level*. This is a distinction between a description of *an action* and a description of a static *configuration* Empirical studies [Nesher, 1978, 1979; Carpenter and Moser, 1979] have consistently showed that the static text is more difficult for the child than the dynamic one These findings are surprising. The nature of the formal operations which the child has to perform is extra-temporal [Piaget, 1960]. Thus it seems at first that the transition from a *static* text to its *formal* representation would be easy because of their resemblance in that respect. (In fact, most text books start with such static word problems.) Since this is not the case one must assume that cognitive processes beyond the processing of language are involved. We suggest, then, that in solving word problems the child has to penetrate to a deeper level, the *pragmatic* level, at which he



A SCHEME FOR SOLVING QUANTITATIVE REAL PROBLEMS

(When reading this scheme try to elaborate the example of Mr. Smith's swimming pool)

Figure 1

performs manipulations in the rather abstract domain of events and configurations. We would like to suggest in the next section a wider theoretical scheme which will also account for this pragmatic component

A theoretical scheme for solving quantitative real problems

Since we feel that the attempts to understand fully the source of the difficulties in solving SCH-PROB on the basis of logical, syntactic and semantic analyses have failed, we suggest a different strategy to attack this issue. We suggest the elaboration of a schematic theory about the way in which REAL-PROBs are understood and solved outside of the school. Equipped with this theoretical scheme we will then try to examine what the teacher does in school in posing SCH-PROB and then, with the use of the same theoretical scheme, we will summarize the strategies used by the students to solve SCH-PROB

From our understanding of human action, of linguistic behavior, and of formal arithmetic, we would like to suggest a tentative theory formulated with the above concepts

Our scheme (see Figure 1) will first distinguish between *overt behaviors* (operational manifestations which are presented in the center of the scheme, steps A′, B′, C′, F′, G′ and H′), and the *underlying processes,* which according to our approach should be assumed as a basis for the overt behaviors (they are marked in the periphery of the scheme, steps A,B,C,D,E,F). The arrows indicate the sequence by which the solution of the problem takes place. Note that D and E are both assumed steps which do not have overt counterparts, but yet we consider them to be the main steps in the process of applying arithmetic to real life situations

The theoretical scheme assumes that both ordinary language and mathematical language have their own semantic and syntactic interpretations (see steps A and F, respectively). The scheme also suggests a process by which the solver has to construct for himself the qualitative and quantitative dimensions which bear on the solving of a problem (steps B-B′ and C-C′)

The scheme suggested in Figure 1, although it seems complicated, is, according to our analysis, the minimal route that should be taken in order to solve a real problem which calls for a numerical answer. The scheme does not describe SCH-PROB posing or solving. This will be dealt with separately in the following section. We assume that step D is the *pragmatic* essence of the task, in which one, by imaginary transformations of the quantified objects and dimensions, envisages the goal state towards which he is directed; therefore it is a necessary condition for solving the problem. Step E is the parallel underlying mathematical structure [see Nesher et al, 1974]. We will refer to step D as the *pragmatic deep structure*, which consists of the abstract pragmatic domain of objects and the abstract transformations performed in this domain

Our assumption is that without real transformations, in one's *mind* in the abstract pragmatic domain of relevant objects, and without explicit awareness about its dimensions and their functional relations, it is impossible to *apply* mathematics in a meaningful manner, even in cases in which the mathematical language is fully comprehended. This is in accord with what Wittgenstein wrote about understanding and anticipating reality in the act of ordering (*Philosophical Investigations* #188):

> *That when you meant it your mind as it were flew ahead and took all the steps before you physically arrived at this or that one*

Our theoretical scheme also suggests an explanation for the fact that dynamic texts are easier than static texts. If step D is crucial in the process of solving word problems, it is also the step in which the distinction between a dynamic and a static text is the most apparent.

In a dynamic text the linguistic formulation explicitly states what transformations are to be performed with the objects in the abstract pragmatic domain (step D). In a static text, however, the relationship between the subsets is usually qualified by static attributes (adjectives, location, possessions, etc.) and the operation is to be performed mentally in the abstract pragmatic domain without any previous linguistic suggestion for the operation. Thus, the transformations in the pragmatic abstract domain D require more efforts in order to reach the union operation explicitly required in E (in the case of additive structures). The abstract transformation is of switching the focus from a set to its subset and vice-versa. Vergnaud [1979] takes a similar stand in his further distinction between actions, transformations and operations, and the various stages in a dynamic text. Vergnaud's distinction between transformations and operations is suggested in our scheme as the distinction between steps D and E. The above theoretical scheme is only a hypothesis that should be further elaborated and articulated, as well as empirically examined. We regard it as a first approximation, based on the research now available. At the moment, we will try with the aid of the above scheme to characterize more specifically some sources of mislearning in schools

Steps missing in the learning of school problems

As already mentioned, for the sake of simplicity, in SCH-PROB all the qualitative and quantitative considerations for a given REAL-PROB have already been made by the author of the text.

Thus, steps B′ (B) and C′ (C) (of Figure 1) are already embedded in the formulation of the SCH-PROB. In fact, the author of the SCH-PROB usually starts with step F′ or E. He has in mind a mathematical operation, or a mathematical structure with whose applications in real life he would like the students to become acquainted. The author then chooses one of the real life contexts and imagines a situation (step D) which will call for the application of the given mathematical structure (step E). In order to simplify it for the student he then adds, in the most concise manner, all the qualitative and quantitative information needed for solving the problem, and arrives at a kind of SCH-PROB which has all the stereotyped characteristics already described

To compare it to our theoretical scheme (Figure 1), the

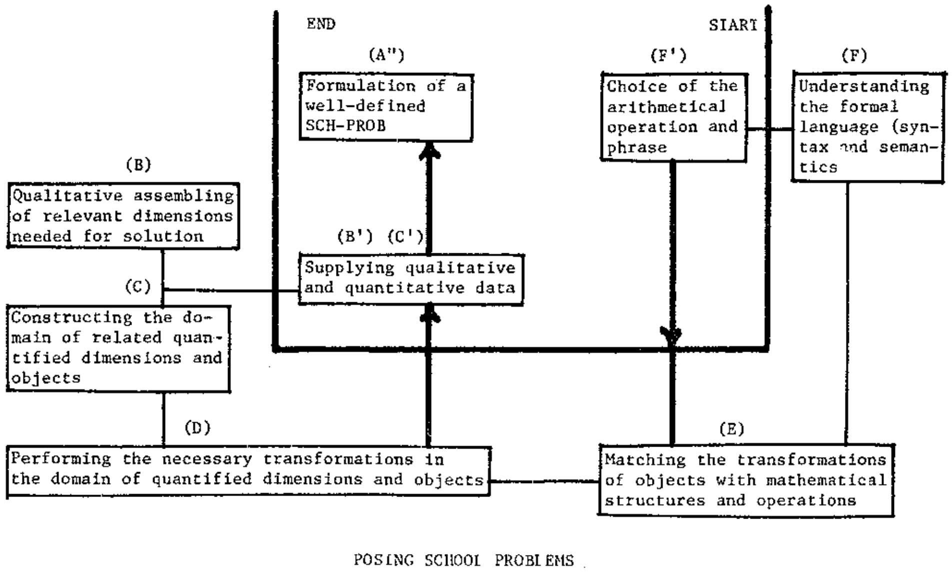


Figure 2

author is performing the following steps in *posing* SCH-PROB (Figure 2).

The student who receives the well-defined SCH-PROB does not have to be engaged in qualitative and quantitative decisions (steps B and C) In most cases, however, he is not able, because of the condensed style, to reconstruct the context from which the data was taken. In short, he is not able to imagine the domain of objects and transformations that the author had in mind. Instead he develops another strategy. He tries from the verbal formulation of the SCH-PROB text to infer directly the needed mathematical operation (Figure 3).

Can one succeed with such a strategy? In fact, this strategy is continuously fostered at school. It starts with the fact that the goal is stated for the sutdent in the framework of a mathematics class and is mainly formulated in terms of steps F′ and G′: to find the corresponding mathematical operation and to calculate the numerical answer.

With this change in goal, from solving a real problem which calls for examining alternatives, to a mere calculation with numerical data, other "economical" methods are encouraged; among them are the learning of artificial lexical clues, and use of analogy with previous successful and seemingly similar problems. In fact, there are many teachers who recommend such methods as major means to the "understanding" of word problems.

Our claim is that without reaching the point at which the student is penetrating to the pragmatic deep structure of the problem given to him, his method of solution will be a guess, a matter of chance, or trial and error We claim further, that the history of formulating stereotyped word problems in school makes it easy to develop mis-guiding strategies for traditionally known texts.

One should note that Figure 2 (posing SCH-PROB) and Figure 3 (solving SCH-PROB) are both parts of the entire process of solving quantitative REAL-PROB (Figure 1). Unfortunately most students in school, in the absence of steps B′ and C′, take the short cut plotted by the solid-line arrows (steps A′ — F′ — G′) and do not have access to the most important parts of this process, namely, comprehending and operating in the pragmatic deep structure and matching it with the relevant mathematical structure (steps D and E) In our view, the deletion of steps B′ and C′ at school encourages such short cuts and therefore does not train for solving REAL-PROB.

There is widespread evidence within the literature describing students' difficulties in reconstructing step D [Carpenter and Moser, 1979; Fuson, 1979]. What is most striking is the students' inability to employ their own knowledge about the context in question when it is presented to them in the style of SCH-PROB and in the context of school An illustration will best illuminate the last point

The following word problem was presented to fifth graders

(SCH-PROB VII) "What will be the temperature of water in a container if you pour 1 jug of water at 80° F and 1 jug of water ar 40° F into it?" The answer given by many children was "120° F"! If, however, you ask the same children REAL-PROB (VII′), "What will the water be like if you pour hot water and cold water into one container?" their answer will always be, "You get lukewarm water "

As seen, the children have had experience with the context presented to them in REAL-PROB problem VII′, yet it was not evident in their response to SCH-PROB VII. The rule they have learned at school is that when you put things *together*, you add their numbers (Similar well known examples concern two faucets filling one swimming-pool, or two workers completing a certain job, etc.)

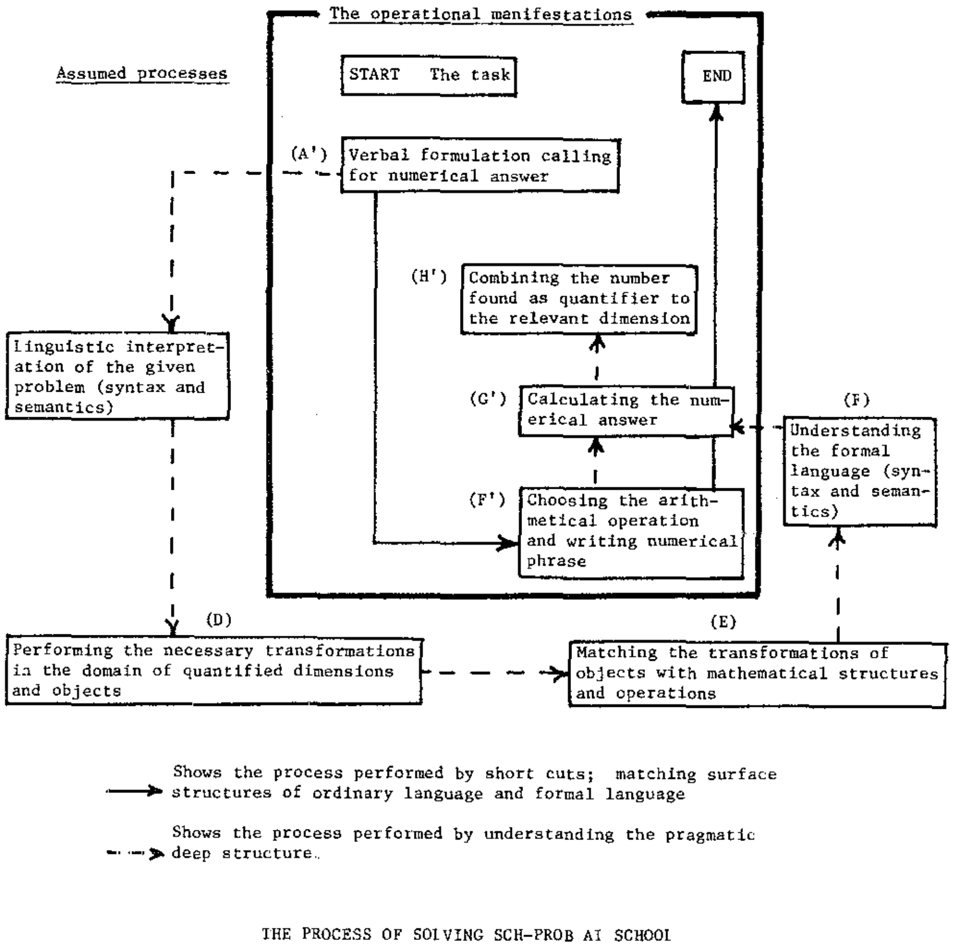


Figure 3

In all these cases it is clear that even a qualitative analysis of the problem (step B) could avoid such absurd answers. The absurdity, however, was not noticed by the fifth graders since, for them, their experience with mixing water in the real world does not have any connection to their experience with SCH-PROB. SCH-PROB are school tasks for which they simply have to supply a numerical answer according to very superficial rules of addition, subtraction, etc

To overcome the superficial rules, one must follow the entire route described by Figure 1 and avoid short cuts as described in Figure 3. Only if steps B,C, D and E are part of the solution of the problem can one assume that the child's knowledge of the real world will affect the solution of SCH-PROB.

The essential part of teaching "applied arithmetic" lies in the ability to perform mentally anticipated operations in D and in the transition from step D to step E, that is, the transition from events and transformations in the pragmatic deep structure to its corresponding mathematical model. This cannot be replaced by direct translation from surface structure expressions in ordinary language to surface structure formulations in mathematical language (e.g. "together" or "more" means "+", etc.). In fact, if one is looking, for example, for a subset in an additive structure, the operations "+" or "−" are equally suitable depending on the specific surface structure you choose to express the given relationship. (This refers to the knowledge of F.)

With the aid of the above theoretical scheme (Figure 1) it is possible now to review the rich literature existing in the field and to show specific variables which relate to specific steps. This enterprise, however, is beyond the scope of the present paper.

Is there a way out?

The theoretical scheme can serve as a hypothesis for further research. In reviewing the research literature we could not find research which illuminate the variables involved in steps B′ and C′, which we assume to be significant for step D. There are, however, some studies in which the child was asked to manipulate materials or to make drawings, according to the descriptions of the text, which shed some light on step D [Carpenter and Moser, 1979; A. Quintero, M.I.T., personal communication].

There is also another interesting experiment, run by Project TORQUE [1979], in which arithmetic calculations are embedded in a full-fledged context, which has merit of its own based upon the quality of its content. In these texts, quantitative questions are fully integrated and comprise a minor part of it. The texts run from one to three full pages and contain a variety of information. Thus, these texts try to build a richer description of the context, in which steps B and C are naturally embedded, and it is easier for the student to advance to step D. Unfortunately, these are only beginnings and no systematic follow-up has yet been done and reported.

One can also think about a methodology, in which only the main problem will be posed, and the child will have to reach all the needed information (steps B′ and C′) and reconstruct the relevant context (steps B, C, D and E). For example, the following simple REAL-PROB (VIII): "Do I have enough money to return home?" What kinds of questions may one ask in order to solve the above problem? Where are you? Where is your home? Do you take the subway, the bus, or a cab? How much will it cost you to buy a ticket in each case? How much money do you have? etc. This series of questions may not be the most "efficient" approach, yet many adults, in fact, ask such questions before solving the main problem (VIII). Note that the parallel SCH-PROB will be:

(SCH-PROB VIII′) "I now have $X and a ticket to my home cost $Y. Do I have enough money to return home?"

It seems that the SCH-PROB VIII′ is simpler than REAL-PROB VIII. Yet almost every adult who was asked to solve REAL-PROB VIII asked all kinds of "superfluous" questions which probably helped him to construct the pragmatic deep structure of the problem.

We have not yet experimented with enough children to be able to substantiate our hypothetical scheme, but we regard it as a research program which will assist us in articulating variables related to B and C and to evaluate more precisely the degree of mismatch of our SCH-PROB with the child's competence to reconstruct the pragmatic context in which he is able to operate. It is our hope that when such a research program is completed some better routes for teaching real problems at school will emerge.

I am indebted to Tamar Katriel of Haifa University for collaborating on the first part of this paper and to Dr. D. Nesher for his assistance on the second part. I wish to thank David Wheeler, John Richards and Marion Walter for their comments on the earlier draft of this paper.

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