The Influence of Teachers on Children's Image of Mathematics

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I have found myself returning to certain of the questions raised by Stephen I. Brown in his communication "The right to be wrong" in FLM 10, 3:

- *What does it say about the kind of question teachers and students explore if essentially all errors are eventually detected by the end of a period?* [p. 37]
- *What is there of intellectual and personal worth that we ought to be doing in mathematics classrooms for which the concept of error (regardless of how imaginatively we deal with it) makes little sense?* [p. 38]

The first question brings to mind for me some words, heard on a television programme, of the late Richard Feynman which have been quoted by me many times to students:

I have spent all my life looking for the problem which I will spend the rest of my life trying to solve

When I was first a head of a mathematics department, the teachers of the first year classes met together for lunch each week to share what they were doing and going to do. Over time we came to share a culture and the meetings became a pleasure. One year a teacher new to the school joined the group and I can remember sharing a particular idea with which I was comfortable. I would ask the children to imagine a spider's web they had seen — really picture it — so clearly that they could draw it on the board or describe it in detail. the images would then be shared and we would work on the question of what was the same, what different about the images. Invariably the children's images were enlargements and we would go on to do some work related to this idea. The new teacher said: *But how does a spider make its web?* and thus started a process of discovery through which I found out that, in fact, spiders' webs are spirals and I have a current question: *how does the spider fix the first thread of its web?* This does not alter the fact that children will still produce enlargements in response to the original invitation, but suggests a difference in approach and quality of this teacher's perceptions.

This same teacher was adept at using the concerns of his pupils. Taking a mixed ability group of children away for a weekend doing mathematics he had had to give way to an oncoming car whilst trying to overtake, much to the amusement of the children. On return, the children commented on this incident as they were rehearsing what had happened for those in their class who had not gone away. There was some discussion which resulted in work lasting for three weeks on the question: *how long does it take to overtake?*

Talk of detecting most errors by the end of the lesson does not make much sense in this environment, perhaps because the teacher was not working to a pre-conceived structure. The children and the teacher were all engaged in coming to understand better. Also, the concept of error itself makes little sense in this case where the children, aged 11, are grappling with velocity, acceleration, distance, time and modelling and measuring from their own experiences and perceptions over an appreciable amount of time

The second question brings to mind a more personal story: When I left the classroom, some six years ago, I was employed in posts where I worked in other people's classrooms These experiences were a continual source of surprise, recognition, disturbance (resonance and dissonance) and consequently, through reflection, personal learning. I found myself asking: *Is this a classroom in which it's all right to be wrong?* as a way of analysing teacher and pupil behaviours. By this mechanism I came to identify behaviours which

(a) felt comfortable, e.g. children being open about sharing their ideas — justifying them, yet letting go of or refining them if they proved less than satisfying and

(b) felt uncomfortable, e.g. children putting their hands up to ask: *Is this right?* after each example

I have used the "function game" as described in *Starting points* [Banwell, Saunders, Tahta, 1972] a lot and wrote up the idea as a possible lesson for *An addendum to Cockcroft* [L. Brown, Waddingham, 1982] An example of a disturbance created by the use of the questions: *Is this a classroom in which it's all right to be wrong?* occurred when the first student teacher with whom I worked as school practice supervisor, Gaye Stanley, used the game on my first visit to her. She wrote a starting number, followed by an arrow and then offered the chalk to the class looking for a volunteer to come and write a possible next number. I broke out into a cold sweat. Difference! I noticed that in my own beginning to the function game I would always provide a starting number, arrow, related number... repeat slowly, working on working out my rule, provide another starting number, slowly, arrow, and then offer the chalk. This felt so different to Gaye's invitation to try what must amount to no more than guesses and I realised that my understanding of the infinite number of possibilities for the first related number had been instrumental in my closing of a possible avenue of exploration for a class. At the time I wrote:

For these students who have never played the game before it encourages trial and error. When I play the game with a class I am encouraging them to expect one "right" answer, perhaps encouraging the fear of "getting it wrong". The second rule was more difficult and after 6 or so guesses, the teacher asked them if they wanted to know the first one. The general impression was "no". The end of the lesson came as we were working on

3 $\longrightarrow$ 12
4 $\longrightarrow$ 20
11 $\longrightarrow$ 132

and some students stay behind to talk. They have more than one way of producing a rule and are involved and energised

I am now aware that there are always an infinite number of possibilities and I can choose to give more information or not dependent on circumstances since I am not locked into one behaviour pattern which I have ceased to question.

Experiences such as these led me to an interest in the influence teachers have over their pupils' images of mathematics. My question: *is this a classroom in which it's all right to be wrong?* is related to just one aspect of a larger picture.

In 1988, for the dissertation part of a taught M. Ed., I worked at devising a set of instruments which might allow me to explore whether a particular teacher did, in fact, influence the image of mathematics of their pupils in the same way. I found research studies on:

- children's attitude to mathematics and on their perception of it
- teacher's views of mathematics and of mathematics teaching
- identifying characteristics of good practice
- working in the complex space which encompasses the children, their teacher and the mathematics

and I read reports [e.g. Buerk, 1982, Vertes, 1981] of teachers with a strong philosophy who apparently influenced their pupils in some way. This reading all fed into the research design.

Research design

I give details here of the instruments used in the research, quoting without attribution from my final dissertation [L. Brown, 1988], referencing others who contributed to my thinking at this stage.

Definitions of image and influence

The personal theory [Kelly, 1955; Claxton, 1984] which an individual holds about mathematics at the present time which will include feelings, expectations, experiences and confidences was called that individual's *image* of mathematics.

An influence of the teacher on children's image of mathematics will therefore be defined as how the children's personal theories of mathematics have undergone a common change or adaption through working with the teacher.

The total of influences of the teacher on a particular child's image of mathematics would be expected to be greater than that of any common changes, but the identification of common changes would help the teacher to identify those of their personal beliefs which are most apparent to their pupils.

In looking for common changes in the set of pupils taught by a particular teacher, I will not, therefore, be considering the difference in adaption of personal theories which might be apparent, say, in the subset of boys and the subset of girls within the pupils.

Choice of teachers

I imposed a number of constraints of which the most important for me were:

- each teacher would have a strong personal philosophy and be considered to be effective either by advisory teachers or their head of department
- a range of philosophies would be represented such as use of an individualised learning scheme, School Mathematics Project (SMP 11-16) published by Cambridge University Press, skill in exposition, "not using a text-book"
- each teacher would have taught a group of pupils for at least a full year before the process started.

In the end I worked with four teachers, the fourth one being chosen because his own philosophy of teaching mathematics was undergoing change.

Choice of pupils

With each teacher I worked with one class and asked each teacher to choose six pupils, two of whom did respond to whatever they did with them, two of whom did not respond and the other two to make up imbalance, e.g. gender. They were not to inform me of the reasons for their choices. Given the definition of influence above I would be looking for similarity of responses between the teacher and *all* six of their students even though one-third of them did not respond! Given this constraint, at this stage of the process, I was sure that I would find no evidence of such influence!

Pre-visit

I felt that it was important to experience the teacher working with the group of pupils and this visit started the fieldwork. My record of this visit was in the form of notes, written at the time, of events that happened in the lesson using reportive statements only, e.g. "Teacher says "Now make 22"" [A.M. Brown, 1987]. At this early stage of the process I wanted to avoid other categories quoted by A.M. Brown such as interpretative and prescriptive. I had to work quite hard prior to this first visit to get into the habit of doing this. Each teacher was asked not to prepare a special lesson for me to observe, but just to do what he or she had planned to do.

Interview with six pupils

Each interview was semi-structured [Walsh, 1985] with a basic script for me to follow from which I could ask contingent questions as in a clinical interview [Ginsburg, 1981]. I tape-recorded each interview and later transcribed the tapes. The interviews began with us engaging in some mathematics which was chosen by the pupil from five alternatives. This was partly as a vehicle for us to get to know each other before the "questions" and partly as a vehicle to allow a choice of mathematical activity to be made which then led to a discussion of why that choice was made and the possibility of learning something about the pupils' view of mathematics indirectly. The five activities were chosen to offer a spread of categories of mathematical experience: practical work/geometry, numerical/algebraic, an investigation, applied/bookwork, a problem. The work of Hoyles [1985] in *asking pupils to recall particular episodes* and Thomas [1987] in *the formulation of the questions in the affective domain* were influential in the design of the following script for the interviews with the pupils.

Opening statement:
On the table in front of you there are, in fact, five different activities, Although we will not have time, probably to finish the activity in any sense, could you choose one for us to get involved in?
After approximately 15 minutes:
Your answer to this question might be the same as your first answer. If instead of asking you to choose an activity for us to get involved in, I'd asked you to choose the one you thought was most mathematical, would you have chosen differently?
Stories:
For this part of the interview I am going to make some statements and, for each one, see what is brought to mind by what I way. Try to remember the event so clearly that you can tell me a story about what happened.
a) Tell me about an activity you have done recently in a maths lesson, and although you probably did not think so at the time, it is brought to mind now when I say, there you are, sitting a maths lesson and what you are doing does feel like mathematics
b) Tell me about an activity you have done recently in a maths lesson, and although you probably did not think so at the time, it is brought to mind now when I say, there you are, sitting in a maths lesson and what you are doing does not feel like mathematics
c) Imagine a time when you felt good in a maths lesson.
d) Imagine a time when you felt bad in a maths lesson
e) What I am interested in is your image of mathematics. So far you have indicated in your responses to the various statements and activities that maths is.. Is there anything else you'd like to add that has not been covered so far to the question: What is mathematics to you?

Some of these questions might seem long-winded but experience showed that they precipitated direct responses from the pupils without any need for them to clarify what I meant. The precise wording developed over time. The last question proved very useful in that I could get feed-back from them about what I thought I had heard.

Interview with the teacher

I kept the interviews with the teachers as close as possible to those with the children and also taped them. They engaged with the mathematical activity and then I asked them to describe to me their criteria for choosing the children for interview. The story questions were the same, followed by:

- *Can you say in one word how you feel about mathematics?*
- *Can you say in one word how you feel about teaching mathematics?*
- *And finally a chance to mention anything you would like to that you do not think we have covered naturally so far*

The techniques of asking for stories through which to probe the underlying tenets [Joy Davis, John Mason at a seminar, *Changing ways*, Association of Teachers of Mathematics Easter Course 1988] of the teachers worked well and I felt comfortable in the interviews which lasted in some cases over an hour. As the interviews progressed I reflected on the techniques I was using to encourage the teachers to talk more. The most effective technique I called "summing up". Clearly, on the later tapes, there are comments from me, in response to a particular story of statement, which are little more than a simple reiteration, e.g. *So you think that . (repetition)....* These comments seem to provoke either agreement or disagreement on the part of the teacher followed by clarification and further examples.

Post-visit

The final visit was to observe the teacher and, as with the pre-visit, to simply note reportive observations [A.M. Brown, 1987] unless anything from the previous experience in the interviews was brought to mind in which case I would change to interpretative mode [A.M. Brown, 1987] continuing in reportive mode when sufficient notes had been taken to ensure that the link could be remembered. In practice this was the least satisfactory part of the whole process since those pupils who had been through the interviews wanted to engage with me and I became an active part of each of the lessons and "observation" was difficult. Having worked hard to observe in the pre-visit in reportive mode I also found it difficult to move into a more interpretative style

A large amount of evidence was collected which made fascinating reading. The four diagrams (Figures 2 to 5) present what I found to be strong strands linking the images of the teachers with those of their pupils:

- Teacher A through challenging the pupils leaves with them an image of mathematics as initially hard but easy when it's sorted out
- Teacher B through using the structure of the SMP 11-16 individualised learning booklets leaves with the pupils an image of mathematics as a set of titles from their booklets.
- Teacher C sees mathematics as a framework of ideas which all link with each other and leaves with the pupils an image of mathematics based on using and applying it.
- Teacher D and the pupils have a common image of mathematics as enjoyable.

These links really did, as I have said before, come as a surprise to me. I had expected that my conditions on the pupils, namely some who did and others who did not respond to their teachers, would ensure that influence, defined as being an image common to all the six pupils and their teacher, could not be present.

One other surprise, and here I come back to the initial stories of my reactions to Stephen's questions, was that the 24 children I interviewed all seemed, on the whole, genuinely to be engaging with their mathematics. Where were all these pupils who hate mathematics and cannot see the relevance of it? Where are the children lacking in confidence, scared of making a mistake? Certainly not all children I interviewed would have said maths was wonderful; some thought it hard at times, others boring at times, but my overwhelming impression was of children working in classrooms where it was all right to be wrong. They were children who were learning something in mathematics lessons and had a feeling of progress

So, these four teachers are seemingly capable on this evidence of working with pupils so that they end up seeing mathematics in a particular way. This seems to have positive effects on the pupils' performance in mathematics. Many questions were raised for me by these statements which reading Stephen's letter reminded me of. The most pressing question is: *what do these teacher do?* They have a strong philosophy, yes, but if, say, I am starting to teach and know that I want to create an environment in my classroom where children learn from their mistakes rather than feel that they have failed, are there specific strategies I can try?

It would take a longer term research study to identify particular strategies which a particular teacher uses to achieve a particular classroom ethos, but the following example might give some flavour of possibilities:

Whilst observing a lesson given by Teacher C, I noticed him use a particular teaching technique. A pupil offered an explanation of how they had begun to tackle a problem. The other pupils were invited to close their eyes and put up their hand if they had started in the same way. In fact, in the lesson observed, only two pupils did so. An alternative start was requested and the pupils again closed their eyes and put up their hands if this was their way of starting. The process continued with more information being collected and these different starts were then used for further exploration:

— *What was the aim of the people who drew the radius?*

In the interview with Teacher C, without prompting from me, the technique was referred to:

— *This idea of closing eyes, I got that from a junior school I went to and I thought it works so well, because people have the chance to make a decision anonymously. I can put up my hand for something I really do believe and nobody will laugh at me.*

Teacher C's reasons for using this technique fit with his philosophy of allowing each pupil to develop their own framework.

My attention was arrested by the following comment from pupil C6:

— *There are several solutions to one problem. If you go round the class asking, you'll come up with six or seven ...You can experiment with the maths. That's a good thing about this GCSE*, like a real mathematician*

Here pupil C6 attributes to GCSE something which would seem to be directly related to Teacher C's teaching technique of eye-closing. For a longer term study, it would have been informative to go back to the other pupils in set C and explore their reactions to the technique. Perhaps this technique is transferable and others could use it to establish the richness of frameworks in their classrooms?

I was not looking for such techniques when observing the other teachers since the idea for looking for such strategies developed out of this incident. It was so powerful that I knew that there would be occasions when I would use it in my own teaching in the future. The question arises, would it be possible to observe teachers who *do* influence their pupils to be effective mathematicians in particular ways and identify transferable strategies which might allow another teacher access to the same influence?

As Teacher C himself, states, this strategy does not work with every class that he teaches, but, when it does work, it makes his task easier. He himself observed a primary teacher using the technique and liked it; it fitted with what he was trying to do, and so he has adapted it to his own style. This technique is transferable in that it is easy to explain and it is clear why you might want to use it. It establishes your classroom to be one where everyone's ideas are respected and thought through.

So, the beginnings of another area for exploration which overlaps with those questions posed by Stephen I. Brown. For me the issue is one of identifying the links between the influence of the teachers on their pupils' image of mathematics and the teacher behaviours; is any of this "transferable"? In particular, what strategies might be effective in establishing classrooms where "it's all right to be wrong"?

*GCSE = General Certificate of Secondary Education, the current 16^+ examination in England and Wales, with up to 40% of the final marks given for more process-oriented work during the course

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Pupil A2

Mathematics is about organising things so that they are easier to think about. When things are already easy they are not worth thinking about and not mathematics. It's more demanding than just number.

Pupil A3

When you look at all those numbers you don't read it, it looks difficult. I think maths is more about problems as well as figures. Sorting out things Finding easier ways to do things.

Teacher A : influence through challenge

Control, insight and challenge. In one word mathematics is compelling. When kids report 'It's all so easy and you can make it as complicated as you like and it's just as easy' about a piece of GCSE coursework about algebra, that's at the heart of what I'm trying to do with my teaching.

Pupil A1

I don't understand it sometimes. I get a bit lost, but in the end I can do it.

Pupil A4

I think maths is a hard subject.

Pupil A6

I think I felt good it was building those classrooms. Feeling good when I'd finished it, achieving, actually working it out. We'd not really known what to do - all our own work really.

Pupil A5

That making a scale-model was really hard, interesting but hard. I got to understand it a bit better, it took about a week to understand.

Pupil B2

Reflections 2 felt like maths and *Dice-games* (a booklet) didn't. I just did it. It was just a game.

Pupil B3

We had to find out what the temperature was in the booklet. It just seemed interesting. If a booklet is hard it's not interesting. I asked myself 'Why are we doing this (reflection booklet) because we don't need it for our exams.'

Teacher B : influence through structure

Although Teacher B did make some comments about what mathematics was to her these did not appear in the pupils' comments. Teacher B does use the SMP 11-16 individualised learning scheme and reports pleasure in looking up and seeing her pupils all working hard.

Pupil B1

I've done two booklets on fractions. I thought fractions was quite hard, but then I found I was doing them. *Dice-games* (a booklet) was no good - it was a game.

Pupil B4

Most of the booklets I do I can understand what I'm supposed to do so I just do it. I don't like it in one booklet where we have to make up a story - it's more like English.

Pupil B6

I feel bad in maths when I'm doing the wrong booklet and I get put back on the one I'm supposed to be doing.

Pupil B5

I thought *Algebra dice-games* (a booklet) felt like maths. Lines and circles (an activity not a booklet) doesn't because it's got nothing to do with numbers

Pupil C2

I prefer having to solve it myself. It gives you that satisfaction of not having to take it from a book. I enjoy mathematics. I find it more of a challenge than a chore. The problem-solving exercises would help me because I could imagine how I felt and go logically through the steps.

Pupil C3

Mathematics is problem-solving. In Connect-4 I'd start by experimenting on a smaller grid to see if there's any pattern and be able to predict; maybe changing the number of counters which you have to make a row

Teacher C : influence through philosophy

I think the whole idea of a problem is that you model it and make it solvable

Mathematics is a framework and mathematics teaching is fun. Fun when you see the children building their own frameworks which are not necessarily your frameworks.

Pupil C1

You've got to actually solve things for yourself which aren't in a book. That's not really what I thought maths was going to be in the earlier years because that was just numeral sort of maths. You can relate it more to things outside, it's not just like a picture on the board, you can imagine it.

Pupil C4

I think maths is just applying stuff that you have learned in the lesson in reality.

Pupil C6

There was a real problem there. I understood what was happening and there were so many different types of maths used to find the final answer. Maths was numbers to me. I felt that in maths everyone knew the answer but as time's gone on I've discovered that even Teacher C doesn't know all the answers - so maths has changed - you can experiment.

Pupil C5

Maths is using what I already know like trigonometry and measurement.

Pupil D2

Maths is good. Feeling bad in a maths lesson doesn't happen. We don't do cards in this school.

Pupil D3

The maths we do at this school now is a lot different because it's more interesting because you go further into things and you do different things to help you learn. In our primary school we just worked from cards.

Teacher D : influence through enjoyment

Challenge - not being forced by directed questions. Being able to capture your own thought processes, explain them and generalise. In one word searching. They enjoy their maths lessons more; they talk about maths and share their ideas with each other

Pupil D1

It's interesting, the things that we're doing. Sometimes it gets boring. I prefer the way they do it here. They just do different things. All we did in the primary school was do cards.

Pupil D4

When I was in junior school I thought it was just sums. It's not. Everything's got maths in it really. Puzzles, shapes, finding out formulas, numbers, patterns, symmetry, equations.

Pupil D6

At junior school you only used to play about with numbers, but at this school you play around with shapes and sizes and things like that. I find maths fun and very enjoyable, because we cover a lot of things that are round our life now.

Pupil D5

Maths is interesting and one of my best subjects. I've always felt quite happy here.