AUDIENCE, STYLE AND CRITICISM

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> We make out of the quarrel with others, rhetoric, but of the quarrel with ourselves, poetry (Yeats, 1918, p. 21)

Throughout its near thirty-year existence, this journal has published articles on a very wide range of topics deemed pertinent 'for the learning of mathematics'. Among them, the theme of the practices of adult mathematicians themselves has recurred, whether insider accounts by professionals (e.g. Henderson, 1981; Leron, 1985; Mazur, 2004; Thurston, 1995) or pieces by others about the views of mathematicians (e.g. Burton, 1999a, 1999b) and their spoken and written cultural practices (e.g. Agassi, 1981; Morgan, 1996; Smith and Hungwe, 1998). And this is quite aside from significant attention to explorations concerned with the history of mathematics and ethnomathematics.

In this article, we first attend to questions of style in written mathematics, particularly from a discursive point of view, expanding on Morgan's (1998) linguistic feature analysis to include attention to rhetorical style and the potential influence of audience. Next we turn to some aesthetic considerations of mathematical style, as well as examining published guides to 'good' mathematical writing. We then address questions of taste and criticism as well as the varied public settings in which mathematical writing might be undertaken, before finally returning to explore the above quotation.

Discursive aspects of mathematical style

Morgan (1998) has made an attempt to characterise certain discourse features of a modern research article in mathematics. Her list includes:

- widespread use of nominalizations rather than verbal forms, which transforms processes into objects and also serves to obscure agency;
- use of non-active verb forms;
- absence of reference to human activity;
- distant authorial voice and lack of direct address;
- continuous present tense throughout;
- preference for imperatives over pronoun use (*let, define, consider, suppose, …*);
- prevalent use of connectives, marking the relation of each sentence to antecedent and subsequent sentences (e.g. but, hence, so, then, therefore, …).

In addition, these various features interact to support the discursive effects of others. Subsequent work (e.g. Burton and Morgan, 2000; Nardi and Iannone, 2005) has explored the views of practicing mathematicians about facets of written mathematical style, not least in relation to novice university student attempts to acquire a greater command of it. These are some of the surface features that identify a mathematics journal article as a paradigmatic text (in the sense of Bruner, 1986 - see [1]), one that speaks its own truth, a truth created in part by the very textual means by which it is asserted.

Part of Foucault's (1980) work reflects centrally the way disciplinary regimes become installed in institutions, including the creation of 'régimes of truth':

> The important thing here, I believe, is that truth isn't outside power, or lacking in power [...] truth isn't the reward of free spirits, the child of protracted solitude, nor the privilege of those who have succeeded in liberating themselves. Truth is a thing of this world: it is produced only by virtue of multiple forms of constraint. And it induces regular effects of power. (p. 131)
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> 'Truth' is to be understood as a system of ordered procedures for the production, regulation, distribution, circulation and operation of statements. 'Truth' is linked in a circular relation with systems of power which produce and sustain it, and to effects of power which it induces and which extend it. A 'régime' of truth. (p. 133)

However, it is not simply a question of *how* it should be done; there are always issues of why *this* rather than *that* form. (See Csiszar, 2003, for an extensive exploration of this question.) What is achieved by this particular style? One such achievement of the writing, Foucault would suggest, is its truth. What would be lost were it altered or hybridized, especially in relation to a more narrative rather than a paradigmatic mode? Part of the declared intent is to render utterly transparent the 'logical structure' of the text. But part of a possibly more covert agenda has to do with creating the very sense of decontextualised authority and certainty that is then claimed as the hallmark of mathematics.

Solomon and O'Neill (1998) firmly contrast the mathematical from the narrative (illustrating their argument on a variety of texts authored by the nineteenth-century Irish mathematician William Rowan Hamilton), arguing the difference lies precisely in this 'glue' of logical versus chronological structuring (and their surface manifestations in terms of verb tense, personal pronoun use, connectives between sentences and other lexical choices). Interestingly, in Hamilton's range of mathematical writing - the focus of their work - the syntactic glue changes depending on whether he was writing diary notes to himself, letters to friends or when he was writing his journal articles or monographs (ostensibly addressed to his colleagues).

> An examination of the letter and the notebook reveals a more complex structure than a simple narrative. The

texts contain two distinct component texts: a *mathematical* text is embedded within a *personal narrative* The difference between the texts is indicated by the tense system, the choice of deictic reference and the forms of textual cohesion employed (p. 216; *italics in original*)

What might a more narrative style in mathematics look like? How would it differ from what is currently offered in professional mathematical accounts and are such differences significant? In particular, might it provide greater scope for writing about images of mathematics rather than solely its body (Corry, 2001, 2006, also discussed below)? These questions have led us to examine the nature of the semantic/syntactic 'glue' that holds mathematical texts together, the glue that seemed problematically absent in novices' prose But also, for us, there is an adjacent locus of interest, perhaps of greater interest, and that is the question of audience and its role (both presumptive and actual) in shaping mathematical writing.

A question of audience, a matter of address

> Euclid's attitude [towards the reader] is perfectly straightforward: there is no sign that he notices the existence of readers at all. [...] The reader is never addressed. (Fauvel, 1988, p. 25)

One of the more taxing questions implicated in the complex interrelationship between language and mathematics has to do with the shaping of form by content and of content by form. One of the less considered aspects of this mutual influence has to do with the nature and influence of the *audience* for the language, especially written mathematical language where the empirical reader (one possible but by no means exclusive audience) is likely not be co-present with the author, either temporally or spatially Yet as Bakhtin (1952/1986, p. 95) was insistent in claiming, every human utterance is addressed *to* someone, a phenomenon he termed *addressivity*, namely an orientation towards an other. One such question of audience signalled by this section's title, then, concerns the addressivity of a mathematical text (see also Pimm, Beatty and Moss, 2007). This is not always a straightforward matter, as the earlier quotation from John Fauvel indicated, especially where a stylistic convention exists that values its denial.

Mathematicians are not unaware of these issues. Norman Steenrod (1973), for instance, in the opening paragraphs of his contribution to the Mathematical Association of America monograph *How to Write Mathematics*, remarked:

> A major objection to laying down criteria for the excellence of an exposition is that the effectiveness of an expository effort depends so heavily on the knowledge and experience of the reader. A clean and exquisitely precise demonstration to one reader is a bore to another who has seen the like elsewhere. The same reader can find one part tediously clear and another part mystifying even though the author believed he gave both parts equally detailed treatment. (p. 1)

And Paul Halmos (1973), in his extensive contribution to the same publication, declared:

> I like to specify my audience not only in some vague, large sense (e.g., professional topologists, or second year graduate students), but also in a very specific, personal sense. It helps me to think of a person, perhaps someone I discussed the subject with two years ago, or perhaps a deliberately obtuse, friendly colleague, and then to keep him in mind as I write (p. 22)

While this may ease the writer's challenge, such a virtual audience may nevertheless have little impact on the readability of the end result.

Formal mathematics has not always been written in this way. As there have been a variety of written genres used in sophisticated mathematics over the centuries, one need not go back in time very far to find variations. Richard Dedekind (1879/1924), for instance, declared that his considerations regarding the continuity of the straight line:

> are so familiar and well known to all that many will regard their repetition quite superfluous. Still I regarded this recapitulation as necessary to prepare properly for the main question. For, the way in which the irrational numbers are usually introduced is based directly upon the conception of extensive magnitudes - which itself is nowhere carefully defined - and explains number as the result of measuring such a magnitude by another of the same kind. Instead of this I demand that arithmetic shall be developed out of itself. (pp. 9-10)

In terms of defining continuity, he went on to observe:

> As already said I think I shall not err in assuming that everyone will at once grant the truth of this statement; the majority of my readers will be very much disappointed in learning that by this commonplace remark the secret of continuity is to be revealed To this I may say that I am glad if everyone finds the above principle so obvious and so in harmony with his own ideas of line; for I am utterly unable to adduce a proof of its correctness, nor has anyone the power. (pp 11-12)

He then reveals the hidden assumption of continuity that had been used in both geometry and analysis for centuries, without explicit attention being paid to it.

Dedekind's account has a number of striking characteristics when read in relation to published late-twentieth-century mathematical writing. It has a strong first-person narrative voice and is addressed to a general, undifferentiated 'everyone' or 'all', who are assumed to have certain knowledge, details of presumption which contribute to what Eco (1979) has termed the 'model reader' of a text. (For discussion of this notion in relation to mathematical text, see Love and Pimm, 1995.)

Addressivity and the linguistic features associated with the paradigmatic mode of mathematical writing, constitute one dimension of style. If these features were the only ones, it might be expected that mathematical style could be fully proceduralised. Yet, as Csiszar (2003) emphasizes, mathematical guides emphasise the importance of "fashioning" mathematical writing so as to make it compelling, understood and appreciated. The mathematician Wolfgang Krull (1930/1987) brings the aesthetic dimension of mathematical style to the fore:

Mathematicians are not concerned merely with finding and proving theorems, they also want to arrange and assemble the theorems so that they appear not only correct but evident and compelling. Such a goal, I feel, is aesthetic rather than epistemological. (p. 49)

Aesthetic aspects of mathematical style

Henderson and Taimina (2006) recount how one of geometer David Henderson's published papers evoked a rash of requests from other mathematicians

> [The paper] has a very concise, simple (half-page) proof This proof has provoked more questions from other mathematicians than any other of his research papers and most of the questions were of the sort: "Why is it true?", "Where did it come from?", "How did you see it?" They accepted the proof logically, yet were not satisfied. (p. 66)

These questions relate to what Corry (2001, 2006) describes as the *image* of mathematics, which he distinguishes from its *body*; together they form "two interconnected layers of mathematics knowledge" (2006, p. 135) While the body includes "questions directly related to the subject matter of any given mathematical discipline: theorems, proofs, techniques, open problems", the images of mathematics "refer to, and help elucidating, questions arising from the body of knowledge but which in general are not part of, and cannot be settled within, the body of knowledge itself". The images may thus include views about the internal organisation of mathematics into fields and sub-fields or even the stated importance of one sub-field over another, or the perceived relationship between mathematics and, say, theoretical physics (see Jaffe and Quinn, 1993, and Csiszar, 2003). Note that the body is marked as singular, whereas there is a plurality of possible images.

Mathematicians simply do not customarily write about their images, in Corry's sense, even if they write from them Indeed, Corry contends that the black letters and symbols on white pages that constitute formal mathematical texts - that is, the research journal article itself - cannot bring forth forms and colours that constitute the images of mathematics. The same can be said with respect to the arts: questions of categorisation and importance, for instance, are usually settled by outside commentators, most notably art critics who employ aesthetic notions and devices. Indeed, Halmos, in *How to Write Mathematics,* ascribes this situation in mathematics to the desire for efficiency and cumulativeness

> The discoverer of an idea, who may of course be the same as its expositor, stumbled on it helter-skelter, inefficiently, almost at random. If there were no way to trim, to consolidate, and to rearrange the discovery, every student would have to recapitulate it, there would be no advantage to be gained from standing "on the shoulders of giants" (p. 23)

The theme of omission - like Halmos's trimming and consolidation - pervades discussions of style in mathematics. In the *A Manual for Authors of Mathematical Papers* (AMS, 1990), we find, for example, the following advice: "Omit any computation which is routine (i.e. does not depend on unexpected tricks) Merely indicate the starting point, describe the procedure, and state the outcome" (p. 2) Further, we find this distinction between good and bad practice.

> It is good research practice to analyze an argument by breaking it into a succession of lemmas, each stated with maximum generality It is usually bad practice to try to publish such an analysis, since it is likely to be long and un-interesting. The reader wants to see the path - not examine it with a microscope. (p. 2)

Mathematical writing, according to these guides, should be novel (omitting "routine" parts), interesting (by omitting details and analyses, and providing the central plot of the argument), generative (by exposing maximally generative lemmas), and simple (making those lemmas do all the work). This style privileges Krull's "evident and compelling" over Henderson and Taimina's perspicuous and meaningful

While the Halmos quotation above separates the context of discovery from that of "writing it up", we question the extent to which aesthetic style in mathematical writing can be so summarily disconnected. In other words, do these styles of writing start having an effect on what is considered important and interesting in mathematics? Thurston's (1995) discussion of his early work might well suggest that excessive generalization and simplification (through lemma-writing) may have undesirable effects on the field, and compromise what is taken to be interesting about a specific result. Indeed, Lakatos's (1976) imaginary pupils Alpha and Gamma try to figure out how far they can generalise Euler's formula before it ceases to be *interesting*. Alpha insists that it is a question of taste. So Gamma asks, "Why not have mathematical critics just as you have literary critics, to develop mathematical taste by public criticism?" (p. 98)

Gamma's suggestion bears further consideration, particularly in view of the fact that mathematicians have long seen themselves as artists (rather than scientists), without always following up on the implications of such a designation. For example, speaking of mathematical writing again, Ewing (1984) asserts the aesthetic nature of mathematics production:

> Without taste, mathematics ceases to be Art. Like scientists, we should continue to perform experiments, by any means available But like artists, sculptors, and composers, we must exercise judgment about what should be placed on public display (p. 4)

However, he fails to note that unlike the artist, sculptor and composer, the mathematician is not subject to public criticism

Tymoczko (1993) acknowledges this situation and attempts to provide at least an existence argument for the possibility of engaging in aesthetic criticism Instead of focusing on whether a particular proof is good or bad (or is beautiful or not), he attempts a form of criticism that resembles that of the art critic, who provides "a way of seeing or hearing a work of art [... by means of] drawing attention to certain features of the world and in so doing enhance[s] our appreciation or articulates for us the movement of our own feelings" (p. 71)

As his test case, Tymoczko chooses the standard proof of the Fundamental Theorem of Arithmetic, which states than any natural number greater than 1 is either prime or a composite of powers of primes with a canonical factorisation. He violates the usual uniform pace of mathematical proof by introducing a certain rhythm to his discussion that lingers over some statements while flying over others. He also makes explicit the psychological dimensions of the argument, pointing out that, for example, the ease of the first step (in which one proves existence) only leads greater frustration when the difficulties of uniqueness are encountered. He hints at some of the satisfying and persuasive components of the proof, which have much to do with aesthetics (for example, the proof is graced by the presence of the Euclidean GCD algorithm, which pops up rather unexpectedly - and it is always satisfying to be able to call upon a good technique).

Tymoczko refers to his attempt at criticism as a "performance" through which he seeks to describe a "lived work". Indeed, through his use of foreshadowing (the difficulties yet to come) and flashbacks (the reflection on a particularly useful technique), he re-inscribes the proof in time. In so doing, he also changes the nature of his authorial voice, as well as the manner of address to the reader: switching from 'we' and 'our' to 'me' and 'my' in such asides, as well as directly addressing the reader as 'you'. Tymoczko claims that his performance of the proof will be more memorable.

A problem of audience arises here too. For whom is such criticism written? Knowing that he cannot expect too many people to understand the mathematics in question, Tymoczko marks as his audience professional mathematicians themselves. Thus, the professional mathematician is not only the one to create new proofs, but also the one who is to *appreciate* the proofs of others - and is sometimes also the one who will "perform" the proof for colleagues and students (see Sinclair, 2005, and Pimm, 2007, for more on this implicit theatrical analogy). We note that this sudden shrinking of potential participants in Tymoczko's attempt at mathematical criticism feels slightly nepotistic, keeping things in-house. We further note that criticism, in artistic circles, may also be critical - pointing to ways in which artists have failed to communicate their work, to achieve coherence, or to transform their audiences. Gamma was alluding to this more critical role of criticism.

If criticism is important for mathematics, as people like Corry, Lakatos, and Tymoczko have argued, then we propose that it is even more important for mathematics education, which of necessity works on the borders between the image and body of mathematics. The aesthetic dimension of mathematical style (which is the province of criticism) is centrally involved in the elucidating, explaining, and challenging questions that arise about the body of mathematics. What it means to do mathematics, or to be a mathematician, changes; and such changes may either transpire under the influence of eminent mathematicians or it may undergo public scrutiny.

Between rhetoric and poetry

The opening observation of Yeats's identifies two different antagonists with regard to arguments (or 'quarrels' to use his term), namely others and himself. The question of audience when someone is actually *doing* mathematics may well not be the same as when someone is writing mathematics intended for publication. Both have interesting demands and specificities. Both raise questions with regard to mathematical style.

This article has been focused on the act of writing mathematics for others. However, as Foucault indicated in his comments about truth being produced, the doing of mathematics cannot be cleanly separated from the writing of mathematics (in the same way that the context of discovery cannot be separated from the context of justification). We end this paper with two outline examples of this non-distinction at work.

The first comes from on-going work undertaken by Sinclair to study mathematicians' ways of working on problems. In one interview with a combinatorist, where the interviewer has asked the mathematician to describe the process of collaboration, the following description is given: "If it is somebody who is already familiar with these objects then you talk at what we say a high level. But really what it means is that you can provide broad brushstrokes and the person who has thought about this object would know the aspects that you are talking about and you can think together at a high level." Using "broad brushstrokes" to communicate seems very akin to the style of mathematical writing described above, in which the desire to "see the path" is of utmost concern.

In the interview, the mathematician continued on to describe how mathematical collaboration might look somewhat different amongst mathematicians not at the same level: "But if you bring somebody is who is not familiar with or only has cursory familiarity [...] they have to work [...] I show them a paper that explains and then I leave them. They have to go a couple of hours or overnight and go through the process and try to get up to the point of familiarity." Thus, in collaboration too, mathematicians talk to each other much as they write to each other, many adopting a similar style in which the rearranged, trimmed, recapitulated mathematical idea is not only preferred, but assumed to be the best means through which to gain "familiarity" with the idea.

As a second, somewhat related example, we have briefly mentioned the issue of mathematical addressivity. Is there always an audience for mathematics, even when it is a lone mathematician at work alone? Is there always an 'other' for mathematics, an addressee being addressed?

Jacques Nimier (1976) cites a high school student: *Avec les mathématiques, il n'y a personne, on est seul.* (With mathematics, there is no one, one is alone.) (p. 56). Yet Jacques-Alain Miller (2004), the contested inheritor of the Lacanian mantle (according to Sherry Turkle, 1992), has made a challenging contrary claim in this regard. Writing in a collection of essays about mathematics and psychoanalysis, Miller draws on a line of Appolinaire's from *The Mouldering Enchanter*, to the effect that *'Celui qui mange n'est plus seul'* ("The one who eats is no longer alone"). Miller's claim is - to paraphrase him - that the mathematician never dines alone. In other words, when the mathematician is feeding on the sweet fruit of mathematics, there is always an other present.

We like the temporal suggestion ('no longer') of the original Apollinaire, namely that there is a loss of aloneness when engaged in mathematics and that this may actually be one reason for engaging in it. This leaves us with the question as posed by Pimm (2007, p. 25), "Who is the mathematician's 'whom', the whomother toward whom the mathematics is oriented?", in response to the inquiry 'Whom have I the pleasure of addressing?'

Most of this article has been about issues of criticism, style and audience. The title of this closing section identifies our sense of mathematics situating itself in *between* rhetoric and poetry, in Yeats's sense of the two different audiences for a quarrel. The poetry of mathematics arises from an often-repeated sense of solitariness in communion with an apparently pre-existing entity. The rhetoric of mathematics draws on a more clearly identified sense of 'other' being addressed. The claim we have just made about these two not being clearly distinguishable suggests that mathematics partakes of both poetry and rhetoric and we lose sight of one or the other at our peril.

Notes

[1] Bruner (1986) makes this general distinction between what he terms narrative and paradigmatic modes of thought. Narratives involve the recounting of sequences of events: "The sequence carries the meaning: contrast the stock market collapsed, the government resigned with the government resigned, the stock market collapsed. But not every sequence is worth recounting" (p. 121). In opposition, the paradigmatic mode "seeks to transcend the particular by higher and higher reaching for abstraction" (p. 13).

As Healy and Sinclair (2007) write: "Paradigmatic thinking is an explicit form of reasoning about the world of facts whereas the narrative mode employs tacit knowledge implied in the telling (and often encourages reading between the lines) and while the paradigmatic favours the indicative mode of speech, the narrative mode is often expressed using the subjunctive verbal mood, or at least through linguistic markers that express possibility, wishes, emotion, judgments or statements that may be contrary to facts in hand." (p. 6)

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