PROBLEMATIZING THE METAPHORS OF UNPACKING, DECONSTRUCTING, AND DECOMPRESSING MATHEMATICS

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Over the past two decades, the question of what makes mathematics teacher knowledge specialized has often been framed from the perspective of the practices teachers must engage in to teach effectively. Scholars in mathematics education research have leaned heavily upon the metaphors of unpacking, deconstructing, and decompressing mathematics in discourses about mathematics teacher knowledge to describe core practices of mathematics teacher work. For example, Ma (1999) proposed the metaphor of *unpacking* mathematics to describe the process of taking apart and uncovering complex mathematical ideas to access their constitutive analogies, illustrations, and representations. Similarly, Ball and Bass (2000) proposed the metaphorical language of *deconstructing* and *decompressing* mathematics into less polished and final forms to talk and think about the practice of making elemental components accessible and visible. Suggestive of the pervasiveness of these metaphors, Google Scholar estimates that Ma (1999) and Ball and Bass (2000) have been cumulatively cited upwards of 7,000 times.

According to Ball, Thames and Phelps (2008), this particular work of teaching mathematics—unpacking, deconstructing, and decompressing mathematics—requires "a body of mathematical knowledge specialized to teaching" (p. 401). Thus, the metaphors of unpacking, deconstructing, and decompressing mathematics have been used not only to mark the specificity of a central practice of mathematics teacher work, but also to summon into existence a particular form or type of mathematical knowledge that teachers need to know and be able to use (*i.e.,*'specialized content knowledge'), a category constructed as distinct from other forms or types of mathematical knowledge that are not necessarily needed for mathematics teaching (*i.e.,*'common content knowledge'). In essence, these metaphors function to both categorically describe and formally legitimize particular forms or types of knowledge as more or less distinctive to the practice of teaching mathematics. Some scholars have taken this argument yet further by basing the 'essence' of mathematics teacher knowledge on an *explicit* recognition of unpacking mathematics, whereas doing mathematics requires only an *implicit* recognition of such unpacking (Hodgen, 2011, pp. 34–35).

This article focuses on the epistemic constraints of these metaphors (unpacking, deconstructing, and decompressing mathematics) and opens up the complexity of these metaphors for a deeper analysis of how they explicitly and tacitly shape epistemic assumptions and perspectives. There is, of course, nothing wrong with using metaphors to describe practices of mathematics teacher work. Indeed, many theoretical ideas in academic discourses derive from some relatively familiar metaphors (for a discussion on metaphors in thinking about preparing mathematics for teaching, see Scheiner, Godino, Montes, Pino-Fan & Climent, 2022). However, not all metaphors are equally appropriate, and some may even be problematic as they are based on assumptions or carry connotations that can be misleading.

The main question raised here is whether the well-known metaphors of unpacking, deconstructing, and decompressing mathematics are based on assumptions or contain associations that may be problematic or misleading about fundamental aspects of teaching and learning mathematics and play a role in the ongoing process of colonization. It is not argued that scholars who use these metaphors hold such positions or that the work of these scholars is simplistic, but that the metaphors of unpacking, deconstructing, and decompressing mathematics should not be accepted uncritically, and should be problematized. Problematizing here means not only questioning the appropriateness of the metaphorical language used in describing the practices of teacher work, but also questioning the underlying assumptions that scholars make about these practices in order to arrive at the metaphors they use. In other words, we ask how these metaphors shape our epistemic views, both in direct or overt ways and in ways that result from assumptions that arise from the tacit structures of these metaphors.

Before proceeding in earnest, it is worth noting that there are other perspectives from which we might seek to problematize these metaphors. For example, we might choose to problematize instead the object to which the metaphor is applied, arguing that teachers should be more concerned with unpacking the mathematical knowings of students rather than unpacking the mathematics they themselves have internalized. Indeed, we are inclined to agree with and advance such suggestions, especially given the way they reposition student knowledge from the margins to the center in a manner that provides greater opportunities to promote equity-focused approaches to teaching and learning. We consider our focus on the underlying metaphor as a complementary critique, and we find value in multitudinous approaches to problematizing metaphor. We all swim the rivers of hegemony, and these waters need to be troubled from many perspectives so that we might begin to break free of the current.

On the importance of critically examining metaphors

The use of metaphors can provide profound and imaginative ways of thinking about complex phenomena; however, they can also be misleading by distorting the fundamental issues at play. It is therefore critical to problematize the metaphors used in discourses about mathematics teacher practices, as well as the inferences drawn from those metaphors and the assumptions that underlie them.

Far from being simple rhetorical devices, metaphors are (one of) the essential building blocks by which we make sense of our lived worlds. Much like mathematical structures, they function as cognitive isomorphisms that anchor what we come to know in what we already know. From embodied experiences of collections of objects, we abstract basic arithmetic through metaphorical mappings: equality from collections with the same number of objects, addition from pushing two or more collections together, subtraction from the act of removing a smaller collection from a larger collection (Lakoff & Núñez, 2000). We come to understand what it is to learn through metaphor as well: For example, learning can be viewed through the metaphor of acquisition or the metaphor of participation (Sfard, 1998). Even the word 'literal', the ostensible antonym of 'metaphor', is itself a metaphor, drawn from the verb 'linire', meaning 'to smear', just as one might smear a 'literal' description or image upon a page. In short, metaphors are something to be taken seriously by scholars, insofar as they are responsible for constructing our world just as much as we are responsible for constructing them.

Of particular interest is that when we critically examine our metaphors of choice, we often find how our explicit and tacit knowledge of the metaphor itself can smuggle assumptions into the topic or content we are describing or navigating with that metaphor. That is, a metaphor like 'unpacking' ties into our knowledge of the physical act of unpacking, and our prior knowledge of that physical act shapes our assumptions. Metaphors in and of themselves have structural assumptions that act as affordances and constraints independent of the particular characteristics painted upon them. Modifying the paint changes the surface characteristics but ultimately has no effect on the affordances and constraints imposed by the underlying metaphor.

Problematizing the metaphors of unpacking, deconstructing, and decompressing

Let us first consider what the terms unpacking, deconstructing, and decompressing mean to most scholars writing on this topic. The unifying theme of unpacking, deconstructing, and decompressing mathematics reflects the belief that mathematical knowledge can be broken down into more basic elements. The prefixes 'un-' and 'de-' in the terms unpacking, deconstructing, and decompressing are used to express an inversion of the processes of packing, constructing, and compressing mathematics—processes that are considered crucial to the work of mathematicians. In this sense, unpacking, deconstructing, and decompressing can be understood as a kind of 'undoing' of packing, constructing, and compressing. In short, these prefixes suggest a nontrivial sort of reversibility. Mathematical knowledge can be unpacked and packed, deconstructed and constructed, decompressed and compressed in this way—much like objects in the material world, such as a chair or a table, that can be taken apart and put back together. Take as an example Ball and Bass' (2000) oft-quoted statement that teachers need to 'decompress' their previously compressed knowledge:

> One needs to be able to deconstruct one's own mathematical knowledge into less polished and final form, where elemental components are accessible and visible. [...] Paradoxically, most personal knowledge of subject matter knowledge, which is desirably and usefully compressed, can be ironically inadequate for teaching. [...] Indeed, its polished, compressed form can obscure one's ability to discern how learners are thinking at the roots of that knowledge. Because teachers must be able to work with content for students in its growing, not finished state, they must be able to do something perverse: work backward from mature and compressed understanding of the content to unpack its constituent elements. (p. 98)

One of the claims underlying this quote is that experts such as mathematicians or teachers (note the explicit assumption that knowledge is held individually rather than collectively or in more broadly distributed ways) have highly compressed forms of mathematical knowledge, whereas novices such as students generally develop their mathematical knowledge from more uncompressed understandings. Many scholars may be willing to concede the claim that the development of mathematical knowledge involves processes of knowledge compression. Tall (2013), for example, has pointed out that mathematical learning involves the blending of mathematical ideas, which involves the compression of knowledge and the emergence of new knowledge. The development of mathematical thinking is then seen as "a combination of compression and blending of knowledge structures to produce crystalline concepts that can lead to imaginative new ways of thinking mathematically in new contexts" (p. 28).

The other claim by Ball and Bass (2000)—that the reverse is also true, that teachers can "work backward from mature and compressed understanding of the content to unpack its constituent elements" (p. 98)—is not widely accepted, nor can it be supported by existing research on mathematical knowing and learning. For example, Scheiner (2020) has pointed to a significant problem with the idea of reversibility of compression. The problem lies in the assumed reducibility of knowledge: the assumption that prior knowledges are combined to form new knowledge while maintaining discreteness and separability of those constitutive knowings, and the concomitant assumption that knowledge can be deconstructed back into more basic elements [1].

The quoted statement by Ball and Bass gives the impression that the logic of unpacking mathematics is based on the view that knowledge resembles 'regular things' such as chairs or tables. In such a view, deconstruction is seen as taking knowledge apart, to make it 'less polished and final' and more 'elemental'. Knowledge construction, on the other hand, is then similar to putting together regular things with static structure, such as the parts of a chair, to make or build something more complicated.

However, such a view of knowledge paints a picture of mathematical learning that is at odds with research on mathematical knowing and learning and is therefore contested by many scholars in the field. Towers and Davis (2002), for example, challenged the architectural understanding of knowledge and argued instead for a viewpoint that "points to the complex history of organic forms" (p. 316). Similarly, Brown (2014) challenged the interpretation of knowledge as a regular thing, proposing instead to construe knowledge as "dynamic entities with emergent structure that react often unpredictably to influences and that are more organic, unable to be easily assembled, disassembled, and reassembled" (p. 1472).

Situating the metaphors against the backdrop of cultural hegemonies

Knowledge is never isolated, but is always in a complex relationship with other knowings. Here, we observe how this is a case where the tacit meanings and limitations of these metaphors may contribute to the ongoing process of colonization. It is argued here that these metaphors can be read as part and parcel of the broad collection of material and symbolic assemblages that fuel and propel colonization.

Since our aim is to consider these metaphors against the backdrop of cultural hegemony, we have made the pragmatic decision to center colonization, especially insofar as colonization reflects a conglomerate of social forces. These forces are commonly viewed or focused on in relatively disparate terms in research, a tendency that is itself related to the social pressure towards compartmentalization, which we will discuss below. While our language focuses on colonization *per se,* we add that we consider it important for readers to bear in mind the complex entanglement of colonization with the material and symbolic constitution of oppressive systems of white supremacy, amato-cis-hetero male supremacy, abled supremacy, class supremacy, and so forth—these systems exist in conversation, each recursively (re-)informing the others, and though pragmatism demands we select a focus here one need be conscientious about bearing in mind their interrelationship. In acknowledging their interconnectedness, we make space for richer engagement and conversation about and around the ideas we share, while aiming to embrace a truth that is often intentionally excluded from the space of education research—that these violent social forces "silently embrace and dance a dance of violence, holding each other so close that their boundaries blur and disappear" (Bowers & Lawler, 2021, p. 326).

Compartmentalization

Compartmentalization is one of the central characteristics and forces of colonization, and it is manifest in many forms in Western societies, *e.g.,* ethical, aesthetic, spiritual, legal, physical, and epistemological (Fanon, 1961/2005). This force saturates our cultural norms and assumptions, including in the space of educational practice and research (Patel, 2016). Of particular note for our purposes here is the way compartmentalization manifests epistemologically in our discourse and practice (Bowers, 2022), a reification of epistemic violence which, in Fanon's perspective, can only be confronted and deconstructed through greater violence. Apropos of our current exploration, the violence we meet this force with is purely rhetorical, comprised only of our relatively tame use here of some fairly evocative language. The point of this language is to unsettle privileged eyes for the purpose of enabling and encouraging introspection (Wheatley, 2005), as for many of marginalized background or positionality these discussions are not purely theoretical, but instead a sincere reflection of everyday experiential reality (Bowers, 2022).

As elaborated above, the three metaphors of unpacking, deconstructing, and decompressing mathematics carry with them various assumptions about mathematical knowledge: in particular, that more complex mathematics knowings are comprised of simpler knowings that somehow remain discrete and separable following their synthesis into something new. This basic assumption is typical of the colonizing center of cultural hegemony: the notion that almost any object of knowledge or interest can be broken down into discrete parts that can then be considered individually without losing anything essential in the process of this deconstruction. This epistemic frame stands in stark contrast to, for example, various indigenous knowledge systems that tend to view life as holistic, complex, and interdependent [2]. Further, it serves very real and destructive functions in hegemonic discourse, both in the sense that "compartmentalizing complex wholes into disparate pieces facilitates the naming and ordering of those pieces and parts in order to have dominion over them" (Patel, 2016, p. 19), and in the sense that focusing on individual parts and factors obscures more holistic views and makes it more difficult to challenge overall systems (Patel, 2016).

One example of this compartmentalizing hegemony particularly relevant to education research is Piaget's (1971) structuralism, which remains a dominant force both in and of itself and through various constructivist perspectives informed by it, and whose influence can be read as part of the sociohistorical journey towards the widespread adoption by mathematics education researchers of metaphors such as those being problematized here. In particular, we refer here to Piaget's (1971) perspective on the relationship between dialectical modes of thinking and his own structuralism, encapsulated in excerpts such as this:

> An examination of this debate [propagated contra Piaget by Levi-Strauss and Sartre] seems to us all the more in order because both of the antagonists [Levi-Strauss and Sartre] appear to us to have forgotten the fundamental fact that in the domain of the sciences themselves structuralism has always been linked with a constructivism from which the epithet 'dialectical' can hardly be withheld—the emphasis upon historical development, opposition between contraries, and 'Aufhebungen' is surely just as characteristic of constructivism as of dialectic, and that the idea of wholeness figures centrally in structuralist as in dialectical modes of thought is obvious. (Piaget, 1971, p. 121)

In such observations about "opposition between contraries" Piaget seems to understand Hegel's finding of *meaning in contradiction* as equivalent or analogous to saying that *we will sometimes disagree, but can ultimately arrive at a* *resolution constructible from the true elements of discrepant arguments.* Hegelian synthesis, however, is wholly distinct from constructivist synthesis. For a structuralist or constructivist, new knowings may be viewed as constituted of prior, simpler knowings. For a Hegelian (or a Marxist, Žižekian *etc.*), the thesis (abstract) and antithesis (negative) are not discretely or separably present in the synthesis (concrete). Piaget's structuralism, like compartmentalized approaches to knowledge building in general, bypasses the negative moment of determination. In short, through the works of Piaget and those who employed his work as the basis for their own, we see a prime example of how compartmentalizing metaphors act as a hegemonic cultural force in our work, and of how naturally unpacking, deconstructing, and decompressing follow from and reify structuralist perspectives.

To fully explore how the compartmentalization built into these metaphors creates space for colonizing violence to fester would extend far beyond the scope of what can be covered in this article, and so we must largely limit ourselves to raising the connection rather than fully dismantling it. However, to give some sense of what could be explored or reflected upon, we offer here a specific way these restrictions reify colonization or run the risk of further reifying colonization. For example, the way they structure mathematical knowledge as comprised discretely of simpler knowings epistemically restricts any connections we might draw to culture or axiology, tacitly and probabilistically excluding marginalized perspectives from the perceived realm of relevant conversation.

Individualism

Like compartmentalization, enculturation towards individualism is a well-recorded part of the process of colonization (Lomawaima, 1995). Indeed, we could reasonably frame individualism as one particular mode of compartmentalization, wherein the locus of knowledge is compartmentalized to individuals (and specifically individual minds rather than bodies in many instantiations), but here we surface it separately to draw attention to a distinct form of onto-epistemic violence.

A recurrent characteristic of the metaphors unpacking, deconstructing, and decompressing, and how they are deployed, is a perception of knowledge and being as fundamentally individual. These metaphors are commonly used to describe how individual teachers and mathematicians have mathematical knowledge that they can/should unpack to effectively share that knowledge. Even in the alternate problematization of these metaphors we proposed in the introduction, where the object of knowledge to be unpacked is shifted from the teacher to student knowledge, there is a cultural pull to understand student knowledge in individual terms. There is an assumption that knowledge subject to these metaphors can be primarily understood through a frame of individual ownership and possession, which sees both an explicit representation in the language of "one's own mathematical knowledge" (Ball & Bass, 2000, p. 98) and a tacit representation through the dominant cultural understanding of objects subject to (un-)packing and (de-) construction as being under the dominion of individual ownership.

As with compartmentalization, we cannot fully explore and unpack how this becomes manifest as onto-epistemic violence, and we would encourage readers to explore the topic more deeply through other readings (some of which have been cited here). In lieu of such an exploration, we offer a brief sketch of an alternative for readers who may not have had the opportunity to consider such alternatives before. Rather than understanding knowledge and being in individualistic terms, we can make sense of these knowings in collectivist terms. Through a humanist lens, we can conceptualize knowledge as distributed across people (*e.g.,* classes, communities). Expanding to post-humanist perspectives, knowledge can be more broadly conceptualized as distributed across both people and other nonhuman elements of the environment (*e.g.,* tools, institutions, plants/animals, broader aesthetics). This type of perspective sees representation in situative theory as well as in various indigenous and diasporic approaches, but is largely absent from and broadly misunderstood within the portions of our disciplinary culture constructed as at the center. Any of these approaches towards a more collectivist vision create greater space for *othered* positionalities, perspectives, and experiences in a manner that can contribute to our fields ongoing interconnected growth towards more equitable futures.

Conclusion

In summary, the logic of unpacking, deconstructing, and decompressing mathematics is based on the reversibility of packing, constructing, and compressing mathematics and seems to suggest the view that knowledge is in some ways similar to the physical objects of the material world. Although such a view may be intuitively compelling, it is problematic in the light of existing research on mathematical knowing and learning, and is misleading because it suggests that knowledge is decomposable into more elemental bits and pieces. Furthermore, these metaphors can be read as playing a role in the ongoing process of colonization. Thus, when these metaphors are used, they need to be understood, applied, and analyzed conscientiously and critically.

While our purpose here is not to argue for a specific alternative, in our quest to trouble the implicit and explicit constraints imposed by our metaphors of choice we do want to extend an invitation to look beyond what hegemony deems possible. The challenge is to approach metaphors simultaneously with unrestrained imagination and with a critical eye, and to grow rhizomatically in our awareness of the kinds of knowing they exclude and the ways in which they shape what can be known and who can do the knowing. Here, we echo Patel's (2016) view that in order to move (mathematics) education research forward, we must begin "with an intentional reckoning with the worldviews used to formulate, conduct, and share research" (p. 20), including conscious interrogation of the tacit reproduction of worldview reflected and reified in our choices of metaphors for knowing and learning.

A question still to be addressed in the field is whether alternative metaphors, such as 'elementarization' or 'recontextualization', are more appropriate for speaking about central practices of mathematics teacher work (see Scheiner *et al.,* 2022) and what implications they have for conceptualizing mathematics teacher knowledge. We suspect, however, that alternative metaphors are more likely to be found in the work and thinking of scholars constructed as existing at the margins—work that defies the taken-for-granted assumptions of the center and instead takes as its foundation perspectives ranging from the indigenous or diasporic to the queer or dis/abled and beyond. What was once taken for granted is no longer adequate, and perhaps never was.

Notes

[1] These assumptions are perhaps particularly relevant to the metaphors of unpacking and deconstructing, whose everyday counterparts generally hold these properties quite tangibly.

[2] A word of caution: In our use of the word 'various' here, we are trying to avoid constructing Indigenous people or meaning-making systems in monolithic terms. The variety of contemporary and historical indigenous ways of knowing and being is as prismatically expansive as life itself. For one example of work detailing indigenous perspectives that do center a certain type of compartmentalization, de Castro (1998) describes some Amerindian perspectives that incorporate a version of compartmentalization at once familiar and quite distinct from that which exists in the current cultural center.

References

Ball, D. L. & Bass, H. (2000) Interweaving content and pedagogy in teaching and learning to teach: knowing and using mathematics. In Boaler, J. (Ed.) *Multiple Perspectives on Mathematics Teaching and Learning,* 83–104. Ablex.

Ball, D. L., Thames, M. H. & Phelps, G. (2008) Content knowledge for teaching: what makes it special? *Journal of Teacher Education* **59**(5), 389–407.

Bowers, D. M. (2022) Examples, exceptions, and monster barring: unsettling centrism in mathematics education. *Journal for Theoretical and Marginal Mathematics Education* **1**(1), Article 0103.

Bowers, D. M. & Lawler, B. R. (2021) Anarchism as a methodological foundation in mathematics education: a portrait of resistance. In Kollosche, D. (Ed.) *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference,* Vol. 1, 321–330. Tredition.

Brown, D. E. (2014) Students' conceptions as dynamically emergent structures. *Science & Education* **23**(7), 1463–1483.

de Castro, E. V. (1998) Cosmological deixis and Amerindian perspectivism. *The Journal of the Royal Anthropological Institute* **4**(3), 469–488.

Fanon, F. (2005) *The Wretched of the Earth* (Philcox, R., trans). Grove Press. (Original work published 1961)

Hodgen, J. (2011) Knowing and identity: a situated theory of mathematics knowledge in teaching. In Rowland, T. & Ruthven, K. (Eds.) *Mathematical Knowledge in Teaching,* 27–42. Springer.

Lakoff, G. & Núñez, R. E. (2000) *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being.* Basic Books.

Lomawaima, K. T. (1995) Educating Native Americans. In Banks, J. & Banks, C.A.M. (Eds.) *Handbook of Research on Multicultural Education,* 331–342. MacMillan.

Ma, L. (1999) *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States.* Lawrence Erlbaum.

Patel, L. (2016) *Decolonizing Educational Research: From Ownership to Answerability.* Routledge.

Piaget, J. (1971) *Structuralism*. Routledge.

Scheiner, T. (2020) Dealing with opposing theoretical perspectives: knowledge in structures or knowledge in pieces? *Educational Studies in Mathematics* **104**(1), 127–145.

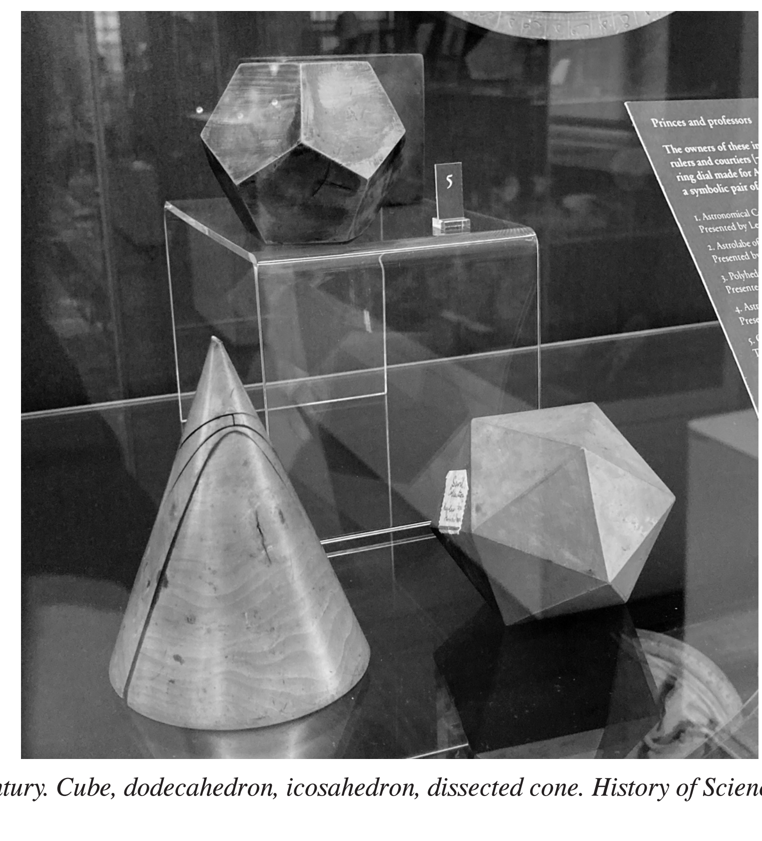
Scheiner, T., Godino, J.D., Montes, M.A., Pino-Fan, L.R. & Climent, N. (2022) On metaphors in thinking about preparing mathematics for teaching. *Educational Studies in Mathematics* **111**(2), 253–270.

Sfard, A. (1998) On two metaphors for learning and the danger of choosing just one. *Educational Researcher* **27**(2), 4–13.

Tall, D. (2013) *How Humans Learn to Think Mathematically. Exploring the Three Worlds of Mathematics.* Cambridge University Press.

Towers, J. & Davis, B. (2002) Structuring occasions. *Educational Studies in Mathematics* **49**(3), 313–340.

Wheatley, M. J. (2005) *Finding our Way: Leadership for an Uncertain Time.* Berrett-Koehler Publishers.



Geometric solids, 17th Century. Cube, dodecahedron, icosahedron, dissected cone. History of Science Museum, Oxford.