Journal Writing and Learning Mathematics

ANDREW WAYWOOD

There has been a keen interest amongst researchers for some time in the ways language and learning mathematics interact. For useful reviews see Austin and Howson [1979], Bell, Costello, and Küchemann [1985], and Ellerton and Clements [1991] Much of the discussion has centered on the relation of language to concept formation Stemming from this theme of language and mathematics there is a growing awareness of the importance of writing in mathematics [Kennedy, 1985; Bell and Bell, 1985; Withers, 1989; Turner, 1989]. The majority of reported work on writing to learn mathematics is focussed at a primary level. Exceptions to this focus are Bell and Bell [1985], Borasi and Rose [1989] Bell and Bell [1985] is a study of the relation of writing to problem solving They conclude their article with the following words:

> By encouraging students to explain themselves in clear coherent prose, exposition allows them to become more aware of their thinking process and more conscious of the choices they are making as they carry out the computation and analysis involved in solving maths problems. [p 220]

The second exception, Borasi and Rose [1989], is a report of a college level calculus course where journals were kept.

This article is a report on the experimental use of writing in secondary mathematics classrooms. The experiment has been running for four years, is integrated in an existing mathematics curriculum, and has included approximately five hundred students in grade levels seven through eleven It is hoped that this report of that work will help to focus some of the complexity of the relation between writing and the learning of mathematics, and at the same time give a rationale for introducing writing as a vehicle for the learning of mathematics.

The experiment took place at Vaucluse College, which is a Catholic girls secondary college in an inner suburb of Melbourne. There are approximately five hundred students from years seven to twelve. The student population is drawn from diverse socio-economic backgrounds and nationalities. There are many students for whom English is a second language.

In so far as this experiment happened in the day to day context of classroom teaching, "experiment" needs to be understood in its broadest sense: a process of refinement through action and reflection The subjects of this experiment have been both the students and the teachers. The report will be organized around two parts:

(1) Developing a pedegogical model for mathematics journal keeping;
(2) The practicalities of implementing journal writing in the curriculum

The ideas expressed in this paper come from reflecting on the experience of the teachers who have used journals as an instrument of mathematical instruction, and in an analysis of the texts found in the journal writing of the many students who form the subject of the study.

Developing a pedagogical model for mathematics journal keeping

In what follows a developmental structure in journal writing is postulated and subsequently elaborated. Samples of students' writing and teachers' experience are used to support and illustrate the key characteristics of this developmental structure. Some support for the essential reality and robustness of the proposed classification of student text can be found in a complementary study by Clarke, Stephens, and Waywood [in press] of journal use and perceptions of mathematics learning

Journal writing is different

The excerpts reproduced as Figures 1 and 2 at the end of the article were both produced by the same year 10 student in respect of the same lesson Figure 1 is an excerpt from her workbook and Figure 2 from her journal. She is clearly a neat student, the legibility of her work was one of the reasons for choosing it, but otherwise not special. Most mathematics teachers would recognize the workbook excerpt as typical of many students workbooks: namely a collection of exercises and notes, where the only connecting prose, if it occurs at all, is either copied from the board or from the teacher On the other hand the journal excerpt in Figure 2 shows a record of ideas which is rich in connecting prose. In fact the page is a cohesive whole where connections, observations ("we can see that ..."), summaries of procedures ("If the hypotenuse is unknown..."), and many other signs of interaction with the material being taught are evident. The key difference between a workbook and a journal is the degree of involvement required of the student. As will be shown later, there are many ways to write a mathematics journal and many elements that go into keeping one. But at the heart of the process is the use of prose to review, reflect on, and integrate concepts. This idea has undergone extensive development first in the "Writing across the curriculum" movement, and then in the "Writing to learn" movement These movements seem to

For the Learning of Mathematics 12, 2 (June 1992)
FLM Publishing Association. White Rock, British Columbia, Canada

have had only a marginal impact on mathematics instruction. Yet it is clear that mathematics is richer than a collection of algorithms, and writing must be a contender for a way of accessing this richness.

As has been pointed out there is a difference between a work book and a journal. Before proceeding, it will be instructive to look at some more examples of journals because there are also differences between journals.

Figures 3 through 8 illustrate three different approaches to journal writing. Each type is marked by what is expressed and how it is expressed. The entry in Figure 3 is from a year 7 student. The entry is about "happenings", and there is no elaboration. Figure 4 is an entry from a year 10 student, again there is the sense of "telling what happened". Figure 5 is an entry from a year 11 Mathematics student's journal. The purpose of her writing seems clear, namely to summarize some content. Figure 6 is an example of a more discursive summary from a year 9 student Figure 7 is again from a year 8 student. It is not hard to see that there is something else going on that was not happening in the previous examples. It is not summary, and it is not a "telling of what happened". The characteristic feature of this entry seems to be the "to and fro" of the discussion. This same quality is more obvious in Figure 8 which is an entry from a year 11 student in a more advanced mathematics class.

It is essential to note that the differences are not purely at the level of competence in writing. Nor is the primary distinction based on grammatical features, though grammatical structure is relevant in so far as different writing tasks are marked by different grammatical structures Rather the primary features that distinguish the examples have to do with how the text is organized into a discourse

These pieces of work were chosen because they represent three broad approaches to writing in journals, and, along with numerous other examples, serve to define the elements of the following classification

A classification for journal writing

The labels "recount", "summary", "dialogue" were created, in the context of the mathematics curriculum at the school, to describe broad groups of students' journal writing. They are technical terms, created in the context of a mathematics curriculum; they do not refer to any element of language as studied by linguists this of course doesn't exclude a description of these categories from a language perspective. The technical nature of this classification has been dealt with elsewhere [Waywood, 1988]. However, just recognizing differences is not the point of the classification. It is how these styles of writing relate to the learning of mathematics that is the real issue From teachers' experience of the students who write journals the following pragmatic hypothesis was formulated: *The mode of journal writing reflects a stance towards learning on the part of the student.* This is a bold assertion: not because different students might have different dispositions towards learning, nor that different students might have different ways of organizing their writing, but because of the assertion that the two are linked.

In particular, Figures 3 and 4 illustrate a *recount mode*. One of the key features of the recount mode is a reporting of what happened It is in the nature of reporting that the stance towards learning underlying the recount mode is discovered. Reporting is about passive observation In terms of the learning stance: Knowledge is objective, out there, a thing to be observed and handed over from the teacher to the student, and all that is required of the student is to receive. Here are further instances of text in a recount mode.

Example 1
"Today was the day that Mr Waywood was absent, and set us work to do that gave me a lot of thinking to do I don't think that it was very hard but you had to think about what to write for the answer to the questions."

Example 2
"I can find what we done today was pretty easy. It wasn't too hard to pick up. And I think I can learn it quite quickly with a bit of practice."

Example 3
"Today we started on parallax This is supposedly how they find the distance from planet to planet etc and size of it.
We had to do an experiment one person had to use an object which you could move further away from the eye a ruler and you. eg a coin"

Figures 5 and 6 illustrate what has been labelled as writing in *summary mode*. The essential quality of summarizing is the codifying of content. It may be for the purpose of examination preparation, or more generally to gain a mastery of a field of content. This essential feature of summary points to a utilitarian stance towards knowledge. Knowledge is to be used and as such it has to be integrated with what is already known Here are further examples of text in what we have called summary mode

Example 1
"Logarithms are an index which are used to simplify calculations. — The whole number part of a logarithm is called the characteristic The decimal part of a logarithm is called the mantissa."

Example 2
(After plotting the function $y = 2x + 1$) "You will notice that the points are in a straight line. A line can be ruled through the points so that the co-ordinates of any point on the line will satisfy the rule $y = 2x + 1$. In fact when any equation of the form $u = ax + b$ is plotted on the Cartesian plane the graph is a straight line."

Example 3
"Equations - the main word here is solve. Equations have an unknown - there is an answer to the problem Linear techniques revolve around inverse operations, and quadratic equations, different from the above require different techniques to solve them, such as factorization. Equations are a branch of Algebra, and they vary in degrees. For example $2x + 3 = 12$ is a degree 1 equation because x is only to the power 1 $2x^2 + 4x + 11 = 99$ is a degree 2 equation because x is to the power 2 The whole notion of an equation is the equal sign - an equation means solve. You can't solve all the equations the same way, because they are all different, and that is why we have to learn different techniques"

Figures 7 and 8 are classified as *dialogue mode*. The basic feature in this mode is a "to and fro" or interaction between a number of ideas. This "toing and froing" signals a creative stance towards knowledge. Knowledge is what is created or recreated. Here are some further examples of text labelled as dialogue:

Example 1
"The sin of 60 = 0.866025403 .firstly is the sin of 60 infinite I wonder. I think it is because you said the points on a circle is infinite. Then how could the square of 0.866025403...be exactly .75 If it is just an approximation then how could it equal exactly 1. Can you please explain "

Example 2
"Q Can the elimination method be used when the unknowns in both equations have the same sign?

eg $2a + b = 9$
$a + b = 7$

A. I think you'd have to multiply one equation by -1 (on both sides of the equation) I'll try it out.

$-1(2a + b) = -1(9)$
$a + b = 7$

=> $-2a - b = -9$.. (i)
$a + b = 7$.. (ii)

=> $-a = -2$. (i) - (ii)
$a = 2$

=> $a + b = 7$ (substitute a)
$b = 7 - 2$
$b = 5$

This seems to be correct, as both no's $a = 2$ and $b = 5$ are the answers that make the equation true "

Example 3
"Another thing, transposition and substitution, really show you the quality of operations, like division, is sort of like a secondary operation, with multiplication being the real basis behind it. This ties in with my learning about reading division properly (in previous pages), that is, fractions are different forms of multiplication So I guess, that's like Rational numbers (Q) are like a front for multiplication, an extension of multiplication. which came first, Multiplication or division? It would have to be multiplication. They are so similar, No that's not what I mean, I mean they are so strongly connected But it's like division does not really exist, multiplication is more real The same with subtraction Addition and Multiplication are the only real operations "

What underlies the production of journals?

We see in journals that different students organize their learning differently. The considerations that lead to this assertion are rather complex. It arises from considering how different students respond to the same lesson in their journals over many lessons The differences in the volume of entries considered lead us to believe that the variation in mode is not accounted for by varying levels of language proficiency or by raw ability, as assessed by the problem solving and testing components of the curriculum. Underlying the variation arising from language proficiency and ability there seems to be revealed the system of choices that students make in trying to come to grips with a lesson's content Three students might record the same fact from a lesson, but one will be happy merely to say that it was stated (*recount*), one will try to capture it by elaborating with an example of how the fact might be used (*summary*), while the third might enter on a discussion in which the fact is related to other facts and its status questioned (*dialogue*). These are all choices that underlie the construction of the text. It is tentatively concluded that different students orchestrate their learning differently. Each student has a pattern of organization for learning that informs the composition of journal texts It would be of great interest to know if there are more or less appropriate "patterns of organization of learning" for the learning of mathematics.

The task of keeping a journal

The journal as a piece of text is shaped by the tasks that are set by the teachers. Teachers at Vaucluse have now settled on a uniform understanding of the tasks that are to be done in keeping a mathematics journal In various ways students from year 7 to year 11 are instructed to use their journals to:

(1) SUMMARIZE
(2) COLLECT EXAMPLES
(3) ASK QUESTIONS
(4) DISCUSS

The following excerpts from the work requirement documentation given to students at different year levels, will give an idea of how these tasks are conveyed to the students. Of course the most effective communication of the tasks happens in the interaction between teacher and class

YEAR 7 AND 8
Work requirements: The journal is to be written every day you have a mathematics lesson, using a page each day, and dating each page When writing you should be revising your lesson by writing a summary of the main ideas, making comments and asking questions. You can also record your own ideas and any extra investigations you do at home. If you need more space glue in an extra sheet Journals are to be submitted at the end of each unit of work. They are to be up to date at all times

In addition to the above instructions students, at year 7 and 8, are given a specially printed book for keeping their journal This clearly indicates the tasks. See Figure 3

YEAR 10
Work requirements extract: The journal is meant to represent a history of your learning in mathematics. It should include: A summary of all material covered. A precise statement of the parts of the course you don't understand. A collection of important examples. An indication that you are extending yourself by reflecting on, and asking questions about the material. In these units the journals will count for 30% of the assessment. The reason for this is that the journal shows clearly how well you are able to work independently

The important point to be taken from this documentation is that journal writing is embedded in the structure of the mathematics curriculum Students can't interpret their journal work as some incidental extra activity The assessment and reporting policies force them to see it as part of doing mathematics. A corollary to this structural emphasis is that other more traditional homework tasks have to be de-emphasized. Experience has shown that journals are more effective when implemented at the level of the school curriculum than at the classroom level. Part of the reason for this is that the development journal writing supports spans more than a single year of schooling. There is evidence that a number of years of journal use changes students' attitudes to mathematics and learning mathematics [Clarke, Stephens, and Waywood, 1990]

How is the journal writing task understood?

A first rationale for the journal tasks was that they seemed to cover what intelligent learning is about. As a task for journal writing "summary" was intended to refer to the use of writing as a means of integrating, as distinct from aggregating, knowledge. "Questioning" has an obvious place in any learning. "Collecting examples" is included because of the special place that examples play in the development of mathematical thought. And "discussing" is an activity that requires involvement. However experience with children trying to carry out these tasks has led to the view that these words are only tokens for the changing interpretations in students' minds. Each of these tasks is interpreted differently, depending on the students' stance towards learning and their experience with journal writing. In the following list we have tried to match interpretations of a task with levels of development. The levels of development (Junior, Middle, Senior) are labelled according to year levels in secondary schooling. this labelling is misleading in that development in journal writing isn't tied to age but rather to experience with journal writing. They have been labelled in the way they have because of the assumption, at Vaucluse, that journals are kept at all levels.

Interpretations of the "WRITE SUMMARIES" task

Junior	Make an effort to record what was taught
Middle	Aim to master the content of the course. In particular, keep good notes so tests can be studied for.
Senior	Aim to form an overview of a topic. Write cumulative summaries.

Interpretations of the "COLLECT EXAMPLES" task

Junior	Examples are collected to show how to get answers
Middle	Examples are collected to show how to apply algorithms and formulae
Senior	Examples are collected because they illustrate ideas and serve as mnemonics for the content of the topic

Interpretations of the "ASK QUESTIONS" task

Junior	Questions are for finding out how to do something. The focus is getting sums right.
Middle	Questions are for probing connections, why one thing follows on from another.
Senior	Questions create lines of inquiry and are tools of analysis

Interpretations of the "DISCUSS" task

Junior	Discuss tends to mean give a description, either of the "external happenings", or, in some cases, the "internal happenings", i.e. a description of thoughts or feelings.
Middle	There is a strong emphasis on summary, and so discussing tends to mean "sort out".
Senior	At this level the focus moves towards trying to communicate. It is implications that have to be discussed.

The entries in the list are idealized in so far as no group of students would verbalize their understanding of the tasks exactly as they have been formulated in the above table. But as the following quote from a year 10 student shows, students do interpret the requirements of the task.

> These are the requirements for my journal.
> I decided to write them right in the beginning so that I can always refer to them
> 1. Summaries. I have to write down everything that he said. What we talked about.
> 2. Collect examples. The examples will show us how to do things if we need help.
> 3. Discuss. Write what I understood and what I didn't, what I learnt and what I still am having difficulties.
> 4. Ask questions. Shows that we are thinking

What seems certain from our experience is that students' interpretations of what they are doing in journal writing changes for most students as they spend more time working with journals. This is supported by the findings of the Vaucluse Study [Clarke *et al*, 1990] which was designed to ascertain students' perceptions and attitudes in relation to journal use and mathematics learning. The entries in the list are derived from recording what the majority of students, at any one level, are doing in response to the tasks. The heuristic being appealed to is that students *intend* to do what is required of them, so what they do reflects their understanding of what they were asked to do. It is not surprising that as one moves through the list in each category there is a shift in emphasis from happenings to relations. This movement is paralleled in the text by a movement away from descriptions towards explanations. Clarke *et al* [1990] has characterized this as a movement from "communication about mathematics", through "communicating mathematics", to "using mathematics to communicate".

Is there a connection between how tasks are interpreted and the classification of journal writing?

The various interpretations that students have of the assigned task is mirrored in the form of the writing they produce. Following is a list of the writing processes engendered by the tasks. The sub-categories under each heading give sub-processes arranged in order of increasing difficulty

i) SUMMARIZING
   a) Writing précis:
   b) Writing down ideas to help formulate them;
   c) Reviewing learning so that new and old knowledge is connected;
   d) Overviewing what's been learnt so that new questions are suggested, or holes recognised.

ii) COLLECTING EXAMPLES:
   a) Using an example as a means for recording how an algorithm is done or applied;
   b) Collecting re-worked mistakes;
   c) Using examples to model structure of argument;
   d) Building a set of paradigms.

iii) QUESTIONING:
   a) Used for finding out how to do;
   b) Used for looking for reasons that connect;
   c) Used for discovering the logic of;
   d) Used to initiate a line of inquiry.

iv) DISCUSSING:
   a) Mulling an idea over or wondering;
   b) Formulating an approach to a problem;

c) Monitoring the development of a logical argument;
d) Generating arguments.

From teachers' observations of journals it appears that there is a strong connection between the writing processes a student brings to bear in accomplishing a task, and whether their journal writing is categorized as Recount, Summary, or Dialogue.

RECOUNT
When students are writing in this mode they interpret the tasks in terms of concrete things to be done: to write a summary means *record*; examples show how to *get* answers; questions relate to how to *do*; and discussion means talking about what *happened*. The text shows an emphasis on objective description, first of external facts; and when more advanced, of internal facets of learning. The writing processes seem to fixate on: Summarizing a,b; Collecting examples a,b; Questioning a; and Discussing a. While students' writing remains in a RECOUNT mode the journal seems to be of little use in advancing their learning. It does serve the purpose of reinforcing facts.

SUMMARY
When students are writing in this mode they interpret the tasks as requiring involvement. The involvement is utilitarian. Describing gives way to *stating* and *organizing*. The main focus of a student's work is on integrating external facts into an internal system of knowledge. Journals show students trying to form an overview. There is a movement from describing what happened to explaining the personal sense of the content. While writing in this mode the writing processes seem to stop at: Collecting examples, c; Questioning, c; and Discussing, c. When in this mode students learn the technicalities of mathematics well. They begin to appreciate the explanatory power of mathematics and to want to understand the underlying ideas.

This category seems to apply to the majority of the texts produced by students. The basic feature of a summary is the recognizing and ordering of important ideas. As with RECOUNT, there are levels of sophistication in SUMMARY. The elements to look for in a "summary" are the use of appropriate examples, the ability to précis, and thoughtful annotation or commentary.

DIALOGUE
When writing in this mode, students see the tasks as requiring them to generate mathematics. Learning moves from being externally directed to being internally directed. Summaries are about *integrating*; questions are about *analyzing* and *directing*; examples are *paradigms*; and discussing is about *formulating* arguments. The emphasis is on explaining external phenomena or internal contradictions. The major limitation that students meet when writing in this mode is an inability to express ideas and relations. The developments of and practice with symbolic language remains paramount. Students writing in this mode are creating. Their learning is being shaped by their inquiries. Without necessarily wanting to be mathematicians they can appreciate what it is that a mathematician does, and even why he does it.

Dialogue is the most technical category used in looking at the text of the journals. It refers to those sections of text where what is expressed is what is happening as it is being expressed. This expressing is different to the expressing of a stream of consciousness because it arises from consciousness structured by questions and, for our purposes, by the discipline which is mathematics. Very few students seem to be capable of sustained dialogue. What is often seen in journals is the start of a dialogue which is cut off in mid-flight because the students haven't the control over language, or their thinking, or the material, to carry the dialogue through.

At each level of the classification students are able to undertake the various tasks but their focus seems to be different.

Assessing students' journal writing

It is in the notion of assessment that much of what has been said so far is practically embodied. For this reason the process of developing assessment procedures for journals at Vaucluse College will be outlined. Staff at Vaucluse College have approached the question of assessing journals by developing a set of progress descriptors for journal writing. These descriptors arose from reflection on the tasks involved in keeping a journal — to summarize, to collect examples, to question, and to discuss; how students at different levels seem to interpret these tasks; and how different writing processes dominate in different modes. The progress descriptors are set out in Table 1. It is intended that each descriptive comment in the table flows from an understanding of the set tasks; an appreciation of how students tend to develop in their interpretations of the tasks; and, their ability to translate their understanding of the tasks into writing. These descriptors are used by teachers to communicate to students how their journal writing is progressing. Each time the journals are collected, at most one of the elements in each of the columns is circled. This serves the purpose of telling the student what they are doing at the moment, while at the same time giving them a description of what would be counted as an improvement.

It should be noted that "DISCUSSION" does not label a set of descriptors even though it is nominated as a task. It was argued that "discussing" is a sort of "meta" category. Producing a discursive summary is both summarizing and discussing. As teachers moved through progressive drafts for the descriptors, the column of descriptors that had been headed "DISCUSSION" slowly dispersed and items moved across into the other columns; for example: "a sequence of connected ideas", "explore consequences or extend ideas", "illustrate points in the discussion of an idea". It was felt that the resulting descriptors would be easier to apply because SUMMARY, EXEMPLIFICATION, QUESTIONING, and APPLICATION have a far more concrete embodiment in journal texts.

Once such a bank of descriptors has been developed, it is a relatively easy matter to produce a set of grade descriptors. Journals are marked on a five point scale, A to E, where each of the grades is meant to be a reflection of a level of achievement. In addition to the A to E symbols two other symbols are used in reporting to parents. These are UG and N. UG stands for ungraded and reports the fact that the student isn't working at a gradeable level. N

SUMMARY.	EXEMPLIFICATION	QUESTIONING	APPLICATION
Able to regularly copy part or most of the board notes into the journal.	Includes examples copied from board or from exercises, but unable to connect them with the journal entry.	Asking questions that are unfocussed. e.g. How do you do Algebra ?, How do you do these things?	Writes short entries about occasional lessons.
As above but also able to describe important aspects of what was done in class.	Able to choose appropriate practice exercises as examples to illustrate the content of the lesson.	Able to ask focussed questions to get help with particular difficulties.	Writes brief entries regularly.
Able to record some of the main ideas of a lesson, and able to write some thoughts about them	Able to use examples to show how a mathematical procedure is applied.	Able to ask questions about mistakes or misunderstandings that lead to a discussion of the underlying idea.	Maintains regular entries that give an adequate coverage of the day's lesson.
Able to isolate and record in own words a sequence of connected ideas from a lesson, with an emphasis on expressing mathematical learning.	Able to choose important examples and show clearly how the example illustrates the mathematics being used and how it works	Able to ask questions that explore consequences or extend ideas - e g. What if ?	Maintains regular entries that often explore or extend the material covered in class.
Able to formulate and state an overview of the material covered in a lesson, text, or topic with appropriate use of formal language and vocabulary.	Able to choose relevant mathematical examples to illustrate points in the discussion of an idea.	Able to ask questions that are aimed at linking one part of mathematical learning with another.	
Able to extrapolate from material presented in class, or in a text, and reshape it in terms of own learning needs.	Able to choose examples that summarize important aspects of a topic, idea, or application. These examples are fully annotated to show their relevance.	Able to pose clearly mathematical questions, i.e. questions that are appropriate to the discipline of mathematics, in a mathematical way.	

Table 1
Progress descriptors for assessing journals

reports the fact that the journal hasn't been done. At Vaucluse, grades are only used from Year 10 on. Following are the grade descriptors used for reporting at Year 10.

GRADE DESCRIPTORS FOR JOURNALS

A makes excellent use of her journal to explore and review the mathematics she is learning. She use mathematical language appropriately and asks questions that focus and extend her learning. She can think through the difficulties she encounters.

B maintains regular entries and is able to record a sequence of ideas. She uses examples to illustrate and further her understanding, and is able to use formal language to express ideas, but is yet to develop mathematical explorations.

C maintains regular entries that adequately summarize a day's lesson. She records important examples and explains their importance. She asks questions that focus her difficulties. She still has trouble forming an overview of the work.

D writes brief entries regularly. She can record the main point from a lesson and copy appropriate examples. She is starting to ask questions that focus her difficulties but is unable to form an overview of the material.

E writes brief entries regularly. She is able to copy relevant sections of the board notes and includes worked exercises. She occasionally uses examples to show how a procedure is applied. She doesn't ask question or form an overview of the lesson.

These grade descriptors appear on reports as one of four grade descriptors. The other categories of assessment reported on are: project work; problem solving; tests and set exercises. The descriptors for journal work contribute valuable information to parents on problems that often underly a student's achievement in mathematics.

Supporting journal development in the classroom

Once the tasks involved in writing journals were understood and there was a sense of the levels at which these tasks can be undertaken, then elements of classroom practice tended to change. Teachers become far more aware of themselves as communicators, and they paid more attention to how learning is organized; that is, they became aware of, and began to address, elements of learning that haven't traditionally been part of mathematics instruction. For example, how students take notes, how an overview of a topic is developed, how an important example is recognised. The journal, as a teaching and learning tool, brings these elements of instruction into focus for the teacher, as well as revealing them as elements of learning to the students. Following are some suggestions that seem to help most students in making the transition from one mode of writing to another.

Helping students make progress in writing summaries: At the junior level the emphasis is learning to take notes and to identify key ideas. To support this, use point form summaries as a lesson progresses, and/or give five-minute summary overviews at the end of the lesson. At the middle level emphasise the logic of the development of a topic. An essential feature of teaching in this phase is the use of semantic maps. These can be used to both preview and review a topic with a class. It is also important to emphasize the independent use of a text book. At the senior level, emphasize formulating connections so that a mathematical intuition can begin to work. When presenting a topic, try to move from the global to the local concepts; cross topic boundaries; and set wide reading (there are many excellent popular expositions of modern mathematics).

Helping students make progress in collecting examples: At the junior level treat examples (textbook exercises) as instances of argument, and emphasize clear setting out. This leads on at the middle level to treating examples as models for doing mathematics. The emphasis is on the content of the example. It is often useful to give students a scheme for analyzing examples: e.g. What's given, what's to be found, and how it is found. Such an approach reinforces a schema of thinking mathematically. For seniors, identify examples that are paradigmatic of the topic. The emphasis is on the pattern portrayed by an example. (Paradigms have always served this purpose in mathematics.)

Helping students make progress in asking questions: At the junior and middle levels insist that questions of understanding are specific. Don't accept a blanket, "I don't know how to do this". Value good questions as much as right answers. Help students distinguish between questions that initiate a topic — being able to point to the historical question behind a topic is an important aspect of this — and questions that reveal the logical development of the topic. For the seniors, focus their wide reading on how questions are formulated and addressed by mathematicians.

Helping students make progress in discussing: At all levels oral work can be used as a precursor of written work. Use plenty of group work; get students to speak to, and annotate, examples; require a spoken or written analysis of mistakes. Particularly at the middle level encourage students to *play* with ideas. In particular encourage play with formal language: stress the use of implication and set tasks that require definitions. Ask for clear written statements of key ideas, e.g. what is a gradient and what is its relevance in interpreting graphs? Make sure they write formal reports on investigations. For both middle and senior level students develop topics from various standpoints, and encourage students to live with confusion. Model thinking by taking personal investigations to class, or work at a topic with a class that is new to both you and the students.

Conclusions

Our experience has led us to the following general and pragmatic conclusions about implementing journals:

(1) A clear understanding of what is intended by "keeping a journal" must be communicated to the students.
(2) Class and homework time has to be given over to supporting the use of journals.
(3) The journal work must be seen to be valued as highly as more traditional aspects of mathematics learning: journals need to be assessed and reported on.
(4) Ideally journals need to be introduced at the level of department policy.

The experience at Vaucluse has been that journal writing enhances students' mathematics learning — but the processes engendered by journal use happen over years, and only happen with a conscious teaching effort.

The experiment reported here is important because it can serve as a precursor to further research. In terms of ongoing research, what has been gained so far is: *firstly*, an extensive corpus of students' journal writing — nearly a thousand journals each representing a year's work in mathematics by a student; and, *secondly*, a well-defined prac-

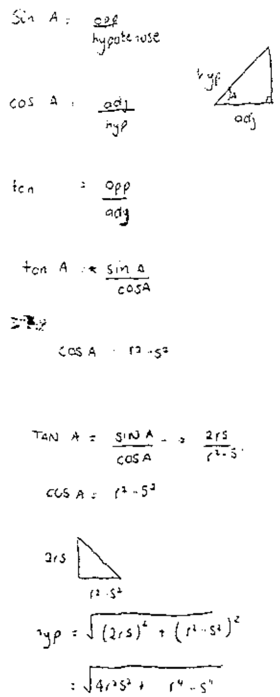

Figure 1

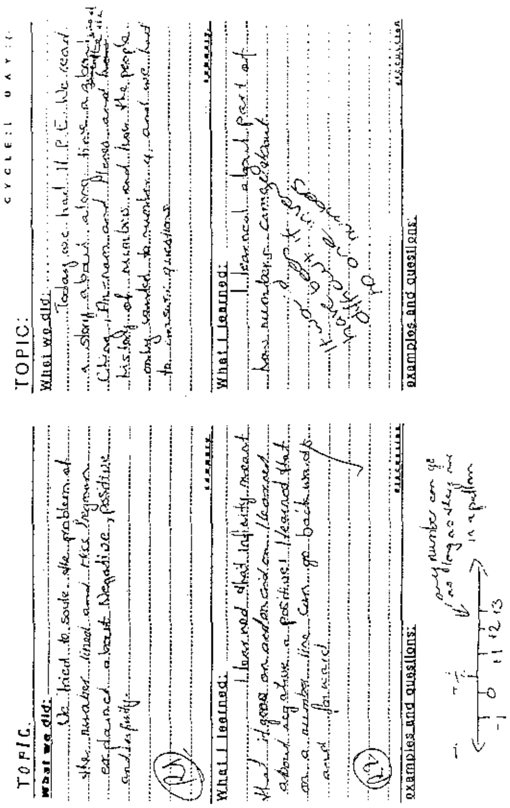

Figure 3

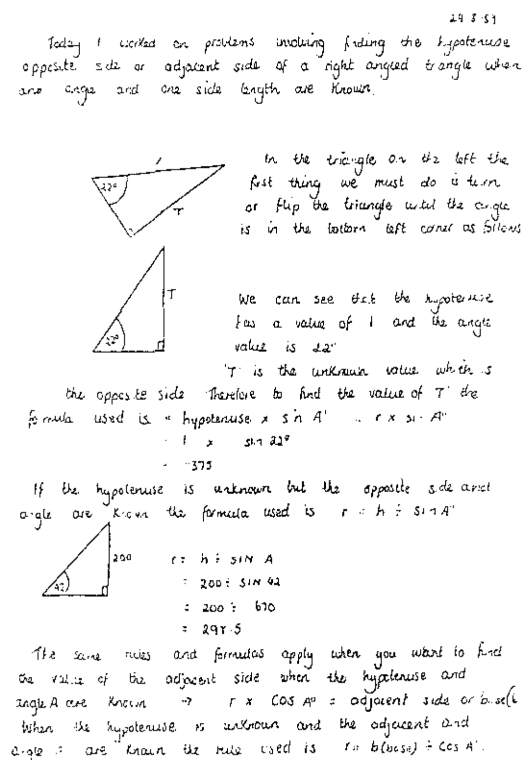

Figure 2

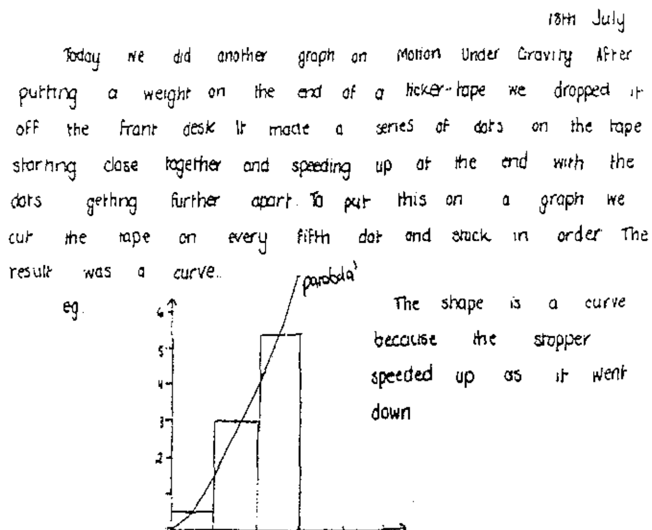

Figure 4

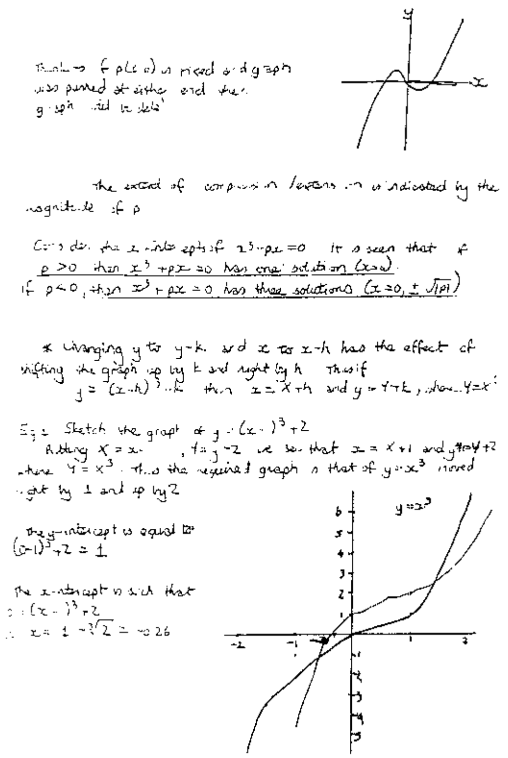

Figure 5

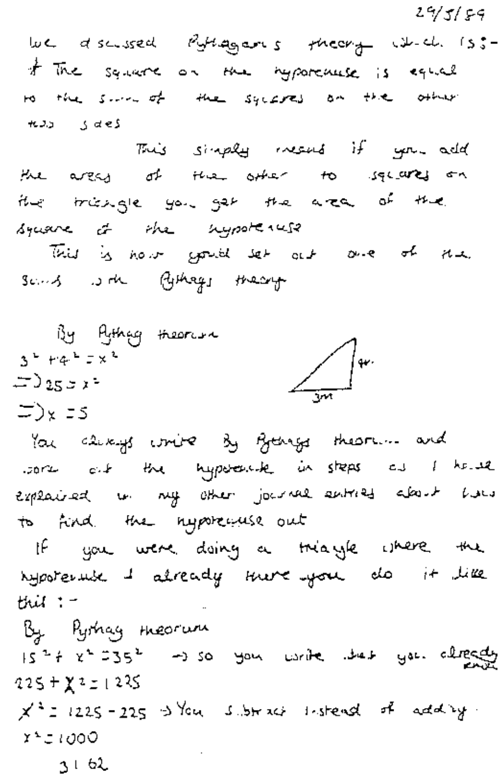

Figure 6

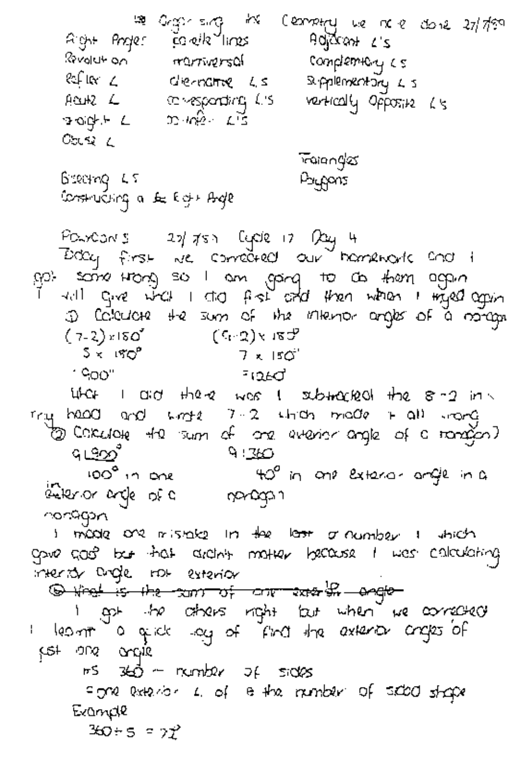

Figure 7

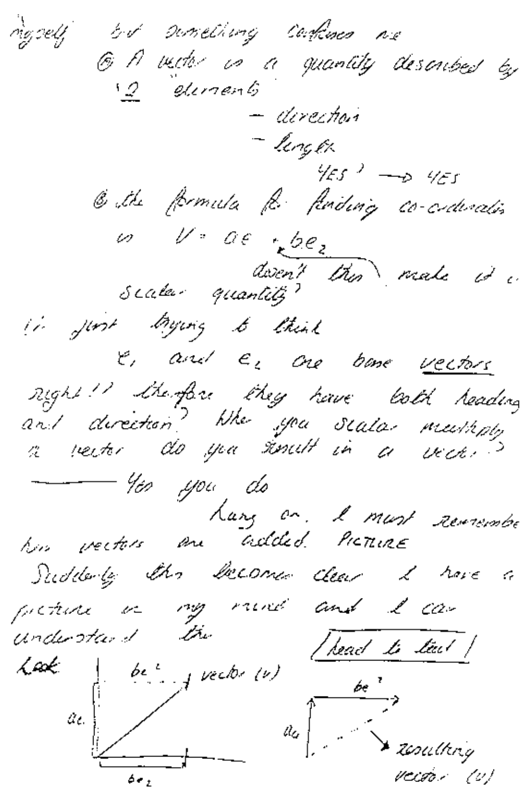

Figure 8

tice that allows individuals' journals to be compared across the same year level as well as between different year levels. The second of these gains is most important because it gives a basis for controlling unwanted variables like writing proficiency and language background — students who can't produce a syntactically correct sentence can still be seen to be performing the macro-language acts that we have emphasized as reflecting how students orchestrate their learning. From such a large data set it would be possible to answer many of the questions posed by Pimm [1987]:

> In conclusion, considerable attention needs to be paid to questions of how children record mathematics spontaneously, and what they find worthwhile to record in a particular context where both the purpose and the need to record are clearly imposed by the constraints of the situation. In particular, what are the purposes to which disembodied language in the form of written records is put, and how might these purposes be conveyed to pupils? What do pupils find useful to record? Is the audience clear and known? Is the purpose known? What conventions are operating which govern the form in which the records should be written? These and many other questions seem to me to be central to an understanding of the place of writing in mathematics. [p 137]

The present work, as well as that of researchers like Borasi and Rose [1989], is making progress towards answering the questions posed by Pimm. However, answers to these questions don't give enough of a foundation to answer the questions, of even greatest interest, about the relation of writing to learning, and in particular to learning mathematics. It must be the aim of any further research project to define the connection between writing and learning, and then to extend it to writing to learn mathematics. Such a research project would address the two underlying claims for journal writing: *firstly*, journal writing enhances mathematical learning. Each of these claims require justifying, though the first is often taken for granted.

To justify the strong claim for journal writing, that writing is instrumental in learning mathematics, a particular connection needs to be established. Are there features of the learning of mathematics that require learning to be orchestrated differently, or supplemented by other strategies, to other disciplines? Common sense dictates that something like this must be true because it is just these sorts of features that demarcate different disciplines. If these features exist then they should inform the tasks set for journal writing in mathematics and, if they can be recognised in the written record, they would mark learning-to-do-mathematics — as distinct from learning-about-mathematics. Such specific mathematical activity can be recognised in journals. To give an example: if students are using their journals successfully they will be writing in what has been called "dialogue" mode: that is, they will be actively involved in constructing knowledge for themselves. In so far as they are actively constructing mathematical knowledge one of the key activities they will be engaged in is spontaneously calculating. Calculating is a key marker of mathematical activity, just as playing music is a key marker of musical activity. The categorisation that has been outlined allows the analysis of writing to learn mathematics to proceed taking cognisance of instances of mathematical activity within the textual record

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