Didactic Engineering, Research and Development Tool: some Theoretical Problems linked to this Duality

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I Introduction

The notion of "didactic engineering" emerged in what can be called the French school of didactics of mathematics in the early 1980's; see for example: [Chevallard, 1982] and [Brousseau, 1982]. The term was introduced to label a form of didactic work: work comparable to that of an engineer who, in order to carry out a particular project, uses scientific knowledge, agrees to submit to a scientific type of control, but who at the same time finds himself obliged to work on objects which are much more complex than the refined objects of science and which he therefore must attack in a practical way and with all the means at his disposal, problems which science does not want to or cannot take care of

This labelling was seen as the way to approach two crucial questions related to the state of development of the didactics of mathematics at the time:

— the organization of a rational relationship between research and action on the teaching system,

— the role which should be given to "didactic productions" in classrooms within the methodology of didactic research

The stakes were therefore two-fold and this brought Y. Chevallard to make a distinction in the text quoted above between "engineering, action for research" — whereby a question arising from theory is put to the test, didactic engineering being the instrument for the experimental staging of this test — and "engineering, action for action" — whereby a request stemming from the educational system has to be replied to, and the work of engineering aims directly at action.

The current polysemy of the expression "didactic engineering," used both to refer to a research methodology and to productions developed for teaching, was directly inherited from this initial duality. It is a convenient polysemy as it implies that the border between research and action, often difficult to locate, does not have to be marked out clearly. However it is also a dangerous polysemy as it hides the diversity of the economies administering to research and to action.

II Didactic engineering, research methodology

Didactic engineering has developed over the years and, through research, into a real research methodology. We shall not describe the characteristics of this methodology in detail here; the reader interested in this point could refer to [Douady, 1987] or [Artigue, 1989a], for instance. We shall limit ourselves to pointing out how it is differentiated from other classic methodologies dealing with classroom observations or experimentations such as comparative methodology or ethnographic methodology. Didactic engineering contrasts with the former in its register: case study, and by its method of validation. This is not *external*, based on the statistical comparison of the achievements of experimental groups and contrrol groups, but is fundamentally *internal*. Didactic engineering differs from the second through the *a priori* control of the experimental situations it requires. This control is essential, as validation in didactic engineering is based on the confrontation between *a priori* analysis and *a posteriori* analysis of the corresponding didactic situations.

Let us clarify this point by referring to the distribution in time of engineering work. It consists of several phases: preliminary studies, conception and *a priori* analysis, experimentation, *a posteriori* analysis and validation. The second of these phases is of particular interest to us here. In the conception stage the researcher decides to act on a certain number of command variables of the system, variables which determine the global and local organization of the engineering

The validation process starts right from this phase via the *a priori* analysis of the didactic situations which result from these decisions. It is an analysis which is inseparable from the conception and, like this conception, based on the theory of didactic situations initially developed by G. Brousseau [Brousseau, 1986]. It is difficult to explain in a few words what the theory of didactic situations is. However for people who are not acquainted with this theory, we stress the following point: the constructivist theory sets down the principle of the pupil's investment in the elaboration of his personal knowledge through interactions (generally social interactions) with a certain environment. The theory of didactic situations relies on this and aims to form a theory for the control of relations between the meaning of such constructed knowledge and the situations in which it occurs

In this theory, the organization of interaction between the pupils and the environment is expressed in terms of the devolution of an a-didactic situation to the pupils, that is to say a situation whose didactic intentions have been momentarily erased: the behaviour of the pupils is dictated by the mathematical problem to be solved and the weight of the didactic contract softens momentarily, even though it is clear to everyone that this is a learning situation.

The aim of the *a priori* analysis of a didactic situation in these conditions is to determine, a priori, how the global and local choices made can allow an *internal* control of the pupil to be carried out, that is to say a control of the significance of his behaviour. In research published up to now, this analysis is usually focused on the a-didactic component of the didactic situation. It includes a descriptive part which presents the aims of the situation, the didactic choices made on the local level (generally linked with global choices), the resulting characteristics of the situation, and a predictive part in which an attempt is made to clarify, according to these characteristics, the real stakes of the situation for the pupil, the types of behaviour which could appear and the meaning which could be given to these. It tries in particular to show that the expected behaviours, if they do appear, really result from the knowledge the situation aims to develop.

III Didactic engineering and development of teaching products

At the present time, most didactic development products stem directly from engineering products constructed by researchers for research needs. This kind of work, although not initially designed for action, irresistibly incites the transition to action: it is based on teaching sequences and can therefore easily be seen, provided the corresponding sequences are not too exotic, as "ready to use" products for teaching. On the one hand, teaching is kind to such products, and on the other hand, the researchers themselves, impatient to have some effect on the system, often encourage the premature exit of teaching sequences constructed within the experimental framework of research. Such an illusion of closeness between research and action is certainly more difficulty to achieve with research which calls on methodologies more external to the classroom.

However, it is well known that the effectiveness of the transmission of the products of didactic engineering is not self-evident. Several researchers have encountered this problem within the research process itself despite the privileged conditions in which such a process takes place: they do not usually do the teaching themselves, so in some way their engineering partly escapes them at the time of the experimentation. Even when the teachers carrying out the experimentation have closely participated in its development, when experimenting they frequently take unforeseen initiatives which disturb the functioning of the research process.

These distortions increase strongly when one leaves the controlled framework of the research. This must raise questions for the researchers. Are the difficulties encountered linked to the fact that the adaptation necessary to transform a research product into a development product was not taken into account? Do they have more profound causes? It is not, of course, our aim to answer such questions in a few lines here. We will limit ourselves to indicating a certain number of facts which, in our opinion, it seems necessary to take into account when studying such questions.

1 The external conception of reproducibility expressed in didactic texts

The designer of a situation aims at its *internal reproducibility*, that is to say a reproducibility which is situated at the level of meaning. Despite this, the descriptions generally given of engineering products emphasize above all the external characteristics of reproducibility: individual or collective behaviours, their evolution in time, what is observed on the surface, as if this *external reproducibility* alone guaranteed the internal one. Such an implication is not self-evident. Various recent researches, for instance [Arsac, 1989], have highlighted the *macroscopic* effect of decisions which can be qualified as *microscopic* if one refers to the level of observation, and the bifurcation in the dynamics of a classroom which can be caused by an apparently innocent remark, or even a simple movement or expression by the teacher. They clearly prove that the teacher can exert a close control over the dynamics of situations, at this microscopic, nearly invisible level, in order to reproduce what he perceives as necessary, or at least important, to reproduce through the description given to him.

Moreover it appears that this external reproduction often occurs to the detriment of the internal one, as if the two levels of reproducibility, instead of going hand in hand, tend to exclude each other in most situations. This can be easily understood if we abandon the vision of the classroom as a classic stable dynamical system, and see it more as a chaotic system. But it shows clearly that work on transmission has to reject an over-simple assimilation between internal and external reproducibility.

In our opinion, the predominance of descriptions of an external type is not only linked to naïve didactic ideas about reproducibility, but equally reflects the difficulty we encounter in creating description methods adapted to our theoretical knowledge and compatible with the requirements of publication and communication. For instance, it must be stressed that the slide towards external description, and the reduction of the place given to the *a priori* analysis that allows internal control, increases when a publication is intended for a public of non-didacticians. Moreover it is then, for obvious reasons of communicability, accompanied by a flattening-out of scientific didactic language into the common language of teaching. It is not at all certain that, by doing this, we really reduce the problem of transmission. We give an illusion of communicability — but only an illusion. In fact we encourage naïve interpretations and therefore possibly make internal reproducibility more difficult to obtain.

2 The closure of research engineering products

Research engineering products often appear as products with strong constraints, designed according to precise experimental conditions and taking into account the specific constraints of this experimentation. This is not due to chance: in engineering research, the confrontation between the *a priori* analysis and the *a posteriori* analysis being the basis for the validation, it is desirable that the developed system should possess only a few degrees of freedom which escape the *a priori* analysis.

On the contrary, a development product is a product which will have to be adapted by the user to relatively varied situations. If this adaptability is not planned for in its conception, it will take place unofficially, in general through real-time decisions taken by the teacher trying to get around unforeseen difficulties It is therefore necessary to envisage, in passing from research to action, an opening up of the engineering, an opening up which allows adaptation while still guaranteeing maximum internal control This is not at all easy

M. Artigue was confronted with this type of problem within the framework of a research while working in higher education on the teaching of differential equations [Artigue, 1989b]. Indeed, teachers at this level often refuse to enter into an experimental setting if the proposed objectives are too constrained. Therefore already at this experimental level, a space allowing freedom compatible with internal control has to be organized

This led her to design and then progressively adjust description schemas of the corresponding didactic engineering:

— at the global level: analysis of the epistemological, cognitive and didactic constraints which contribute to explaining current teaching in the area and its effects; then a well-argued description of the global choices proposed in the engineering in order to substantially modify this set of constraints and to allow for a more suitable equilibrium in the teaching,

— at the local level of didactic situations: first, the identification of a set of conditions to be fulfilled by each of these situations with their corresponding justifications — a situation satisfying these conditions being given in each case as an example; secondly, the identification of the main cruxes of the situation with, for each crux, a range of *a priori* "reasonable" decisions to be taken by the teacher and an analysis of the possible consequences at both internal and external levels of each of them

3 The relative weakness of the theory

The theory of didactic situations emphasizes the importance of pupils functioning in near-isolated situations, that is to say in situations in which the conditions for the organization of interaction with the environment are enough to produce the adaptation and regulation processes aimed at In this theory, the role of the teacher is mainly:

— to organize the pupils' interactions with the environment so that this adaptation brings into play the acquisitions aimed at in the learning process;

— to then allow the pupils to accept responsibility for solving the set problem, in an a-didactic functioning mode, and to maintain it through a process of devolution,

— finally, to link certain acquisitions brought into play during the solution process to institutional knowledge, through the process of institutionalization, the opposite of the devolution process

Didactic research has shown that it is possible to construct and bring into existence such learning processes, for various mathematical fields, in particular in the elementary school It has also shown their effectiveness However, such situations, fundamental in learning, can only rarely correspond to the whole of the teaching in a given field under standard conditions, even when such functioning is theoretically possible, due simply to time constraints. The conception of development engineering (or even research engineering, in certain conditions) will often pass therefore through situations in which it is impossible to guarantee pure a-didactic functioning, but in which one tries nonetheless to guarantee the pupils maximum responsibility for finding the solution. This is the case, for example, in the research on differential equations quoted above.

We have to confess that the study of the regulation phenomena at work in such situations, and their control according to the respective roles of the pupils and the teacher, is still at an embryonic stage: we have at our disposal a lot of work on the interaction between pupils and environment in a-didactic situations, but much less work on the role of the teacher in its non-trivial interweaving with the role of the pupils.

IV Incompatibility between the theoretical principles of engineering products and teachers' conceptions

The engineerings produced by research are based, at their conception, as we have already pointed out, on a theory of didactic situations. Research currently carried out on the *metacognitive representations* of teachers [Robert, Robinet, 1989], for example — that is to say, the conceptions teachers have about mathematics, teaching, and learning in this area, conceptions considered in relationship to the professional practices of teachers — leads us to set up a hypothesis that at least parts of these representations are incompatible with the foundations of the theory of didactic situations. However, these representations play an essential role as they condition the interpretation which teachers are going to make of the texts given to them, the choices they will make within their space of freedom, the decisions they will take in real time in order to face up to unforeseen phenomena. So incompatibility, if it does exist, is a real obstacle to the transmission of engineering products, undoubtedly the ultimate obstacle.

Moreover we cannot hope to reduce it by providing completely determined products, for at least two reasons:

— it is impossible to determine a didactic situation right down to the microscopic level, and the knowledge which we currently have of the didactic system shows that decisions taken at this level regularly give rise to macroscopic effects,

— the dominant conception of teachers about their role is itself incompatible with the acceptance of strongly-determined products.

We would like to develop more particularly here the analysis of these incompatibilities by using to this end research carried out by M J Perrin [Perrin, 1990] and [Perrin, forthcoming] on pupils in difficulty. The fact that we place ourselves on the edge of the usual functioning will be used to reveal phenomena which, in standard conditions, would more easily pass unnoticed.

This research began in 1984, at a level involving the end of elementary school and the beginning of middle school (age group: 10 to 12 years), in classes mainly made up of pupils from low social classes who had repeated at least one course (60% to 70%). At first, the research was undertaken with the intention of using engineering products on decimals, ratios, and areas developed in previous researches [Douady, 1984], [Douady, Perrin, 1989] that had already proved their efficiency in standard conditions. With classes in difficulty it turned out to be very difficult at the elementary level, and practically impossible at the middle school level, to put the didactic engineering to work — not only for reasons which at first sight seemed to be cognitive This was the starting point for a new research which attempted to understand the real nature of the difficulties encountered, and the way in which the different actors in the system (pupils and teachers) contributed to these difficulties

Concerning the teachers (and here we shall limit ourselves to this aspect) the data gathered (observations of classroom sequences, interviews) show up the primordial concern of the teacher to see his class succeed vis-à-vis the outside world as well as relative to the internal life of the classroom. They also show the conceptions that teachers have of this success and of the role they have to play in order to obtain it. We shall try to present here some characteristics of these conceptions which are widespread in the population we studied

Success vis-à-vis the outside world is measured by the success of the pupils in exams, or in tests taken in common with several classes at the same level, and through the opinion which the teacher who has them the following year forms of the pupils and the teaching they have received. Success within the classroom refers to a classroom which is harmonious, in which the pupils are attentive, motivated, active, interested during sessions and do their homework

Teachers think that they have an active role to play in this success in the following ways:

1 They must guarantee external success by providing solid bases, by preparing the pupils for the types of exercises they will encounter in external assessment, and by favouring their adaptation to further teaching,

2 They must guarantee success within the classroom:

— by managing to motivate the pupils, by capturing and keeping their attention, by making them participate,

— by making the learning easier through complete and clear explanations, through a well-organized grading of difficulties, through systematic correction, and if possible prevention, of errors,

— by keeping control over what happens in their classroom and by managing to direct these classrooms, in real time and through the decisions they take, towards the aims they themselves fix

Working with pupils in difficulty at school particularly highlights the incompatibilities which can exist between the metacognitive representations of teaching and learning which underlie these convictions and the principles which underlie the theory of didactic situations, specially for the following reasons:

— the teachers' lack of confidence in their pupils' abilities,

— the constraints linked to the management of this type of class

We want to point out here some of these potential incompatibilities and try to explain why working with classes in difficulty contributes to throwing light on them.

A The status of errors

Errors do not have any function in learning They are always dangerous The appearance of errors must be prevented by warning actions and, failing this, errors must at least be corrected systematically Pupils must get rid of them as soon as possible. The risk of allowing errors to thrive in pupils' minds is greater with pupils in difficulty: they are more inclined to make errors, and they have less chance of correcting them spontaneously. Therefore everything must be checked, even more than with standard pupils. This is not easily compatible with the management of open situations which leave a lot to the initiative of the pupils

B Continuity of learning

The ideal teacher is one who is able to handle learning in such a way as to allow a continuous progression without any breaches to take place, who only adds a little difficulty at each step, which if necessary he will help the pupils to overcome This is particularly true with pupils in difficulty, who need to be made to feel secure and successful and who, in the opinion of the teacher, can only reproduce what has been taught with few variations.

C Pupil's activity

Success within the classroom is currently strongly associated with the ideologically dominant image of an active classroom in which the pupils "participate" But what is the mathematical level of this activity? Does it imply that the pupils have to take real responsibility or initiative for their own conceptualizations? Working with pupils in difficulty, it becomes evident that this level may remain very low Thinking that conceptual tasks are too difficult for them, teachers tend more than in standard conditions to take charge of everything of a conceptual nature, leaving only execution tasks to the active register.

D Research activities

The status of pupils' activity is linked to the status of the research activities Interviews show that the latter do not have a sound function in the learning process for teachers and therefore appear to be secondary when compared with the acquisition of basic skills and the adaptations required for external success. With classes in difficulty, this is particularly visible because:

— The problems encountered with the memorization and consolidation of knowledge increase the amount of time spent on practicing basic skills,

— Most often, teachers cannot count on homework providing this practice and therefore try to organize the whole learning within school time, which brings them up against stronger time constraints,

— Even if the devolution of such activities is successful, the pupils work more slowly and the impression of didactic time consumed is therefore increased,

— Moreover, these are clearly activities which cannot be brought to life if the teacher wants to control what is going on completely.

In fact, research activities are most often considered as a means of motivating the pupils at the start of learning a notion and, more generally, as a means of cultivating an interest in mathematics Strictly from the point of view of mathematics learning, they appear as luxury activities which the teacher can only use efficiently with high level pupils.

It is obvious that major incompatibilities exist between what we have just described and the didactic theory In the theory, learning necessarily includes breaches, and the reorganizations which mark these breaches are essential cruxes of learning. According to the theory, teaching should not try to avoid the conflicts and errors associated with these breaches but, on the contrary, through a careful choice of situations and manipulation of didactic variables, should create the conditions for their appearance and allow them to be managed efficiently.

It is clear, moreover, that in engineering products the researchers invest most of their energy in designing these key situations. Contrary to what happens in standard teaching, they supplant all the familiarization, maturing, and practice work which then, partly outside the classroom and in a more continuous way, allows for the consolidation of knowledge and makes it really operational And often the organization of this part of the teaching is more or less left to the initiative of the user.

Finally, there is also a distortion between the theory and the characteristics described above for the piloting of didactic situations Theoretical piloting is anticipated piloting, as much as if not more than real-time piloting It is not at all sure that, even if this anticipated piloting works effectively, a teacher who is used to permanent real-time action will easily adapt.

It can therefore be expected that teachers, faced with the products of didactic engineering, will attempt, consciously or not, to reduce the distance between what is proposed and their usual way of functioning, this reduction being easier if they teach high level pupils. Their conception of the pupils' learning modes as well as their role as teacher is indeed, in this case, more likely to adapt to didactic theory The following extracts from interviews tend to prove this:

Extract 1.
"I consider this to be important (research activity) but there are technical competences in basic calculus which take priority over this type of teaching There are the constraints of the curriculum, the constraints of requirements, the pupils who at the end of middle school will start work, who will do a CAP (short technical training), there is a minimum of techniques to be learnt, and I give priority to them I do use research exercises, but only when the class has made good progress at the technical level"

Extract 2.
"When you have to explain proportionality in the third class (14-15 years), or even the application of percentages, even the good pupils are completely lost. However, they know how to factorize a polynomial perfectly well, or how to develop an expression, because *that* is part of the collection of things which they will be asked about on the day of the "brevet" (exam taken at the end of the junior cycle); and that is part of our constraints, and also of our wish to see them pass an exam which may be the only one they will take in their life."

Extract 3:
"The thing that worries me is the number of pupils: I have 32 pupils this year in the first year of junior cycle, the ideal would be between 20-25. Otherwise I always regret that, at the end of the session, about 10 pupils are not under my control"

Extract 4:
"Good mathematics teaching for me is demonstrated by what the pupils have assimilated of the curriculum. The ideal is to go a bit beyond that and to help them like maths — that would be better — but if they have acquired a solid basis, it's not too bad."

Extract 5:
"With a pupil who keeps up well one tends to ask him things which are more difficult, to give him an extra difficulty On the other hand, with a pupil who is in difficulty, although perhaps it is wrong, I would tend to stick more closely to the curriculum For example, we were using a cut-out which transformed a rectangle into a square and we discovered that they didn't have the same area

In a good class, I think I would use it, because it is a good exercise, whereas in a weak class, they might like it but I am afraid of everything that disturbs what little I could have obtained through something which is a bit repetitive."

Extract 6.
"For it to be worthwhile (group work), I could envisage giving different problems in the class, but afterwards, to bring everything together, it would be difficult as some are obliged to listen to the others, or else the same type of difficulty in several problems has to be found, because otherwise they remain passive afterwards, there is always a moment when the children are passive and that's what I don t like."

Extract 7.
"I generally give tests every two weeks, so I try to assess what they have understood over two weeks. At the end of the term I like to give them a longer test in which I take an inventory of what I did over the term and tests just on the lesson, which, for weak pupils, works well. just on 5 points very fast, they give me the definition — this is "by heart" learning. this could be once a week, it depends on the lessons, and I ask again and again until they get the right answer and, with weak pupils, that's where they pick up points fastest.. I manage to save a few that way only for all that, they don't know how to use it any better It s my first step [ ] I always fix things so that a pupil who has listened but who isn't really good gets the average [. . .] I also like to have 4 or 5 points which are quite a bit more difficult Between 16 and 20, I need more difficult exercises to stimulate them The good ones have to come up against some difficulties"

V Conclusion

This analysis tends to confirm that there is a gap between research and action. So an effective methodology for research, as didactic engineering has proved to be, cannot be directly transposed into a means of action on the educational system, even when it deals with teaching in classroom situations. There is no doubt that relations between research and action give rise to difficult problems that the current state of the theory does not allow to be solved in a satisfactory way. But at a time when there exists a large and consistent body of didactic knowledge, research cannot avoid the theoretical challenge these problems pose

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My body (I cannot help it) is not plunged into a single specified space It works in Euclidean space but it only works there It sees in a projective space; it touches caresses and feels in a topological space; it suffers in another; hears and communicates in a third; and so forth as far as one wishes to go Euclidean space was chosen in our work-oriented cultures because it is the space of work — of the mason the surveyor or the architect Hence the cultural idea of the practical origins of geometry that is a tautology since the only recognized space is precisely that of work of transport My body therefore is not plunged into a single space but into the difficult intersection of this numerous family into the set of connections and junctions to be established between these varieties This is not simply given or is not *always already* there as the saying goes This intersection these junctions always need to be constructed And in general whoever is unsuccessful in this undertaking is considered sick His body explodes from the disconnection of spaces My body lives in as many spaces as the society the group or the collectivity have formed: the Euclidean house the street and its network the open and closed garden the church or the enclosed spaces of the sacred the school and its spatial varieties containing fixed points and the complex ensemble of flowcharts, those of language of the factory of the family of the political party and so forth Consequently my body is not plunged into one space but into the intersection or the junctions of this multiplicity [ ] The identity of a culture is to be read on a map its identification card: this is the map of its homeomorphisms

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