Greek Mathematical Diagrams: Their Use and Their Meaning

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I begin with a word of clarification. This article introduces a wider project, that of Netz (1999). There, I develop a more detailed description of Greek mathematical practice. But, my intention here is different, namely to look at the role of diagrams in mathematical signification.

There is a growing interest in the role of the visual in the creation and the teaching of mathematics, partly as a backlash against the more verbally-oriented mathematical ideals of the 20th century and partly as a reflection of the new potentialities opened up by computers. It is in this very general context that I wish to point out the central role of diagrams to deductive mathematics in its very inception, in Greek mathematical writings. I shall return later to this large-scale historical perspective, but concentrate mainly in this article on the Greek data.

Before going on to discuss these Greek data, something must be said about the evidence. The problem of the evidence is always difficult for ancient mathematics (see Fowler, 1998, Chapter 7, for an excellent introduction to this type of difficulty), but this problem is especially difficult for the subject of this article, Greek diagrams. There are all the usual difficulties about the transmission of Greek texts, but for the text itself (as opposed to the diagrams) there is a very good tradition of modern palaeographic research. There was hardly any research into the palaeography of diagrams. The diagrams in the editions have in general nothing to do with the diagrams in the manuscripts.

There are exceptions: for instance, Mogenet's (1950) edition of the work of the very early astronomer Autolycus is very useful. Another highly scholarly edition (but one based on a single manuscript only) is Jones (1986). This provides an edition of a book by Pappus, the 4th century AD - i.e. relatively late - mathematician from Alexandria. But these are isolated exceptions, and in general the palaeography of diagrams has not really begun.

I therefore must stress immediately that what I offer now assumes nothing about how diagrams *look* in the manuscripts. I work throughout on the basis of the texts and the texts alone. One therefore has to devise ways to make the texts speak on behalf of the diagrams. One must devise new methods of interrogating the texts. In the following, part of the interest lies in what I have learned from Greek mathematicians, while another part, I think, is in the methods of interrogation I have devised. I shall therefore start from a certain method of interrogation.

1. Determination of objects is done through the diagram

By 'determination', I mean the determination of objects by letters: that is, a proof may start with something like 'Let there be a line *AB*'. Then the idea is that *A* and *B* are two points, each on one of the ends of the line *AB*. As a rule, letters represent points, and objects other than points are represented by a combination of the points on them. Now, points have to be determined somehow. To explain this, I shall give a case where there is a perfect determination. Suppose you say (Fig. 1):

'Let there be drawn a circle, whose centre is *A*.'

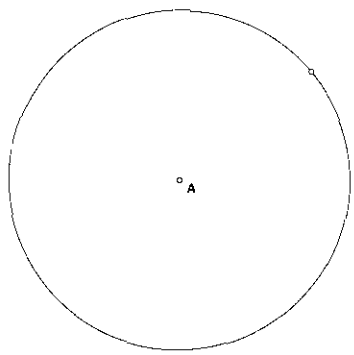

Figure 1

A is perfectly determined, since a circle can have only one centre. I need a term with which to call perfectly determined letters, so I will call them *white* letters.

Another possible case is something like the following (Fig. 2):

'Let there be drawn a circle, whose radius is *BC*.'

This is a more complicated case. To begin with, I'll make clear that I need not make any fuss about the fact that a circle may have many radii. It may well be that for the purposes of the proof it is immaterial which radius you take, so from this point of view saying 'a radius' may offer all the determination you need. Notice how liberal I am concerning determination: I allow indeterminacy in so far as it is mathematically immaterial. But even granted all of this, a real indeterminacy remains here. For we cannot tell here which of *B* and *C* is which: which is the centre and which touches the circumference. So *B* and *C* are under-determined by the text.

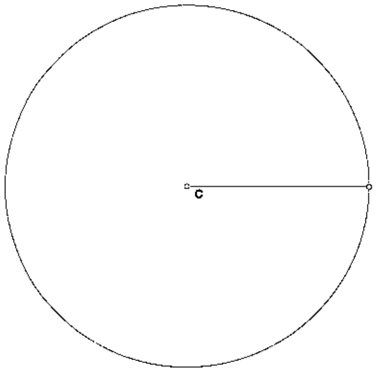

Figure 2

Needless to say, as we may see for ourselves, once a diagram is present, no problem of determination remains and, in fact, it is difficult to realise that something is missing in the text, when the diagram is present as well I have repeatedly found it very difficult to *un*see the diagram, to teach myself to disregard it and to imagine that the only information there is that supplied by the text. Visual information compels itself in an unobtrusive way. But the main thing is that there is a component of visual information in the determination of such letters, so these are under-determined letters and I call them *grey*.

Finally, imagine that the above example continues in the following way (Fig. 3):

> 'Let there be drawn a circle, whose radius is *BC* I say that *BD* is twice *BC*.'

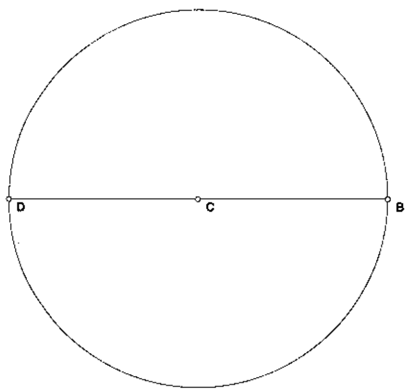

Figure 3

D in this example is neither determined nor under-determined Here is a letter which gets no determination at all in the text, which simply appears out of the blue. I therefore call it a *blue* letter, one that is completely undetermined

Yet another possible class of letters, for which I shall not give an example now, is what I call *pre-determined* letters: they are first introduced and only later do they get a determination. They start as blue or grey, and then they become grey or white.

So we saw four possible classes – determined, under-determined, undetermined, and pre-determined – and we can approach them in a slightly more formal way. The text defines a certain locus, and the mathematical situation demands another locus These two loci may vary in their relationship:

- *determined* (locus defined by text = locus demanded mathematically) – 'white';
- *under-determined* (locus defined by text > locus demanded mathematically) – 'grey';
- *undetermined* (no locus defined by text) – 'blue';
- *pre-determined* (locus gradually defined by text).

In all cases except the case of white letters, the determination is done through the diagram, and thus it is important to know how many letters belong to each class. I have checked this with two texts, Apollonius' *Conics* Book I and Euclid's *Elements* Book XIII. In all, there are 838 letters introduced in these two works, 370 of which are determined, 247 are under-determined, 66 are completely undetermined, and 155 are pre-determined. So completely undetermined letters are a minority, but they are far from being a negligible minority. In fact, most propositions have one or more completely undetermined letters And, more generally, we see that most letters are not completely determined: that is, most often the reader must supply the determination from the diagram

This shows that Greek mathematics relies upon diagrams in an essential, logical way Without diagrams, objects lose their reference; so, obviously, assertions lose their truth-value. *Ergo*, part of the content is supplied by the diagram, and not solely by the text. The diagram is not just a pedagogic aid, it is a necessary, logical component. So assertion 1 is significant

Assertion 1: Determination of objects is done through the diagram.

2. The writing down is preceded by an oral dress rehearsal

So far, I have outlined one method of interrogation, the one in which the determination of objects by letters is checked. The next method to be presented here uses the sequence of baptisms in a proposition. What is meant by 'baptism' has already been seen. Points are assigned letters – they are baptised. I propose we look more closely at the way in which they are baptised, that we look at the names – the Greek letters – which are assigned to them.

First, there is an overwhelming tendency to baptise according to an alphabetical sequence: that is, the first point to be baptised would be called *alpha*, the second one would be called *beta*, etc. We must remember that this is not the only possible way to baptise points. When I draw a diagram, I usually use a *spatial* baptism principle: I give the letter A to the top-most right point, and proceed leftwards and downwards, following the general convention of writing in European languages. And, in general, the number of possible sequences of baptisms with *n* letters is *n*! of course. But this large range of possibilities is hardly touched upon in Greek texts.

While the alphabetical sequence is an overwhelming tendency, it may often be broken by more or less minor deviations from the alphabetical sequence Such deviations serve various purposes: sometimes one wishes to stress the importance of a certain point in the space of the diagram, and therefore give it a non-alphabetical name. Or one might wish to emphasize the interrelationship between various letters in different propositions, and therefore give this group of points related letters. Such deviations from the alphabetical sequence happen and are, in fact, quite frequent.

However, one principle is maintained very often, though not strictly always: this is the compactness principle, which says that, if a proposition contains, say, eight acts of baptism, these would involve the first eight letters of the Greek alphabet. The order may not be strictly alphabetical, but once the baptism business is over, there are no gaps left, the alphabetical sequence is compact.

A final principle involves the determination of objects mentioned above. The principle is the following: completely undetermined letters are almost always baptised as if they were defined alphabetically at exactly the moment where the text should have referred to them. To take two examples (Figs 4a, 4b):

'Draw the line *AB*. Then *AC* equals *CB*.'

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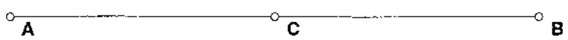

Figure 4a

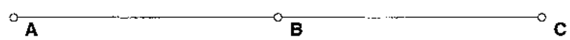

Figure 4b

In both cases, there is a completely undetermined letter. In Fig. 4(a), this is *C*; in Fig. 4(b), it is *B*. In Fig. 4(a), the completely undetermined letter is alphabetical. In Fig. 4(b), the completely undetermined letter is not alphabetical, but would have become alphabetical had the text been complete at exactly the moment it was produced. The second case is far more common in Greek mathematics. So a baptism occurred there; the author did write down the relevant letter, he only omitted this from the text.

The omission may in this or that place have been due to a textual corruption, but on the whole I think it is rash to think all blue letters are due to such textual corruptions. But something general needs to be said about the textual position here.

Readers may be sceptical about my use of baptism. This scepticism is, of course, well-founded, because we can not tell how many stages a diagram went through before we got the present one. It may be that the author drew many different diagrams, each with different sorts of baptisms, with different attachments of letters to elements in the diagram. Alpha may have been one thing, and then another: so the final diagram is accidental, as it were.

This is a legitimate criticism, but I think it is wrong. I actually think it was very rare that anyone changed the lettering in a diagram after it was first drawn, and my argument for this is simple. If you change a diagram, you must change your text accordingly. You must go through some proof-reading. And as many readers will know from experience, this is a very, very difficult sort of proof-reading to do. It is very difficult not to leave behind mistakes, it is very difficult to correct all the letters from the first draft. Here and there, mistakes always occur, with letters referring to the wrong objects, to what they did in the first draft. This is inevitable to some extent and it will take very, very careful proof-reading to avoid this altogether.

My impression is that such discrepancies *never* occur in Greek mathematics. There are many mistakes in the manuscripts for letters, but they are all obvious scribal mistakes. The truth is that proof-reading a mathematical proposition with Greek technology of writing would have been hell, and just to avoid it, there must have been a very strong tendency to be as conservative as possible when doing any revisions. So I think that in the clear majority of cases, the diagrams we possess correspond to the original diagrams.

To resume, there are three baptismal principles:

(i) baptism is alphabetical;

(ii) even when not alphabetical, baptism is compact;

(iii) completely undetermined letters would have been alphabetical had they been determined at the appropriate moment.

The first, the alphabetical nature of baptism, proves that lettering did not come before the formulation of the argument. It is impossible that the author first lettered the diagram and then proceeded to think how to argue about it. The author must either have inserted letters into the diagrams while he was formulating the argument, or he may have first formulated the argument and then inserted letters into the diagram. (The term 'formulation of the argument' is intentionally ambiguous between an oral and a written articulation of a line of reasoning.) So, my first point: formulation of the argument cannot come after the insertion of letters.

The second principle, the compactness, proves an almost contrary result. The writing down of the text must have come after the formulation of the argument. The author knew exactly how many letters he would require. If the first letter used was the seventh in the alphabetical sequence, he already had to know that he would fill in at least six more letters. Therefore, he must already have known, when writing down the proof, how many letters he would require.

The alphabetical principle is the thesis: it shows that the insertion of letters into diagrams could not have come prior to the formulation of the argument. The compactness principle is the antithesis: it shows that the writing down of the text must have come after the formulation of the argument. Now we move on to the synthesis, and this is furnished by the way in which undetermined letters behave.

This behaviour shows that, at some moments, there were letters which were inserted into the diagram *without* this being recorded in the written text. This means that those letters were inserted into the diagram as part of the formulation of the argument - but not into the written text. This leads to the following hypothesis: first, the author drew a diagram; next, the author formulated the main outline of the argument and simultaneously inserted letters into the diagram; and finally the author wrote down the proof.

These three stages are:

(i) drawing a diagram;

(ii) a dress rehearsal in front of the diagram, in which the diagram is dressed, i.e. letters are inserted;

(iii) a full production, writing down the proof.

I will just present the following controversial claim without arguing for it in detail. Greek mathematicians came from a culture which was more oral than ours, and specifically they had means for expressing themselves precisely and briefly through oral formulae. What they did not have was anything like the modern mathematical systems of visual notations. Therefore, when a Greek mathematician wished to express himself briefly and precisely, he would use the oral medium. If a dress rehearsal were not the full production, it could not have been written, because *there was nothing between the oral and the written*, there were no short-cuts such as the modern visual symbols. I therefore suggest that the dress rehearsal was oral. So my hypothesis is that before proceeding to write down a proof, the outline of the proof would be formulated in an oral dress rehearsal in front of the diagram.

In fact, this argument can be even simpler. Since proofreading was such a trouble, it would have been very natural to try to have a very clear notion of what you wanted, before sitting down to write: my argument for the oral dress rehearsal can be as simple as that.

Assertion 2: The writing down is preceded by an oral dress rehearsal.

3. Letters are indices, not symbols

I now move on to yet another method of interrogation, once again using the letters of the text. But this time I look at them not in isolation, but in their combinations. A Greek diagram is most often like a graph; it is a set of partly-connected vertices. Letters, as a rule, are attached to points, and other objects, which are essentially a connected set of points, get their names through these points. So a line may be called *AB*, a rectangle may be called *ABCD*, etc. This is the most typical principle of how names are assigned to objects.

What is clear, whichever precise principle is adopted, is that the same object may in principle get more than one name. The line *AB* may equally be called *BA*. The triangle *ABC* may equally be called *CBA*. And, in fact, we see that Greeks quite freely change names in this way. About 15% of the time, the name given to an object is not identical with that which was last given. If at some point in the text the 'line *AB*' is mentioned, it is about 85% probable that the next time this line gets mentioned, it will be called '*AB*', and 15% probable that it will be called '*BA*'. 15% is a considerable percentage, although of course it is significantly below chance level. 15% means that almost every sentence contains an occurrence of a change of a name. There is some preference not to change names - which is natural, after all repetition comes very naturally when writing down a text - but this is no more than a preference.

There are some interesting constraints on such changes. For instance, it is very rare to get a non-linear name. What I mean can be shown like this: a square such as that in Figure 5 has four points and therefore can in principle be named in 24 different ways. However, something like '*ACBD*' is forbidden. The sequence of letters in a name must follow a linear survey of the object. So actually only 8 out of the theoretical 24 names are allowed (four allowed starting-points multiplied by the two possible directions).

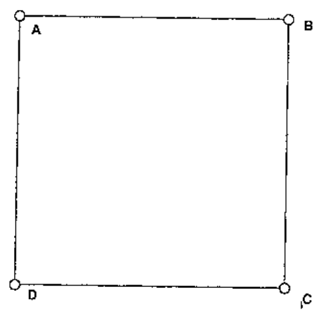

Figure 5

Another interesting constraint is the isodirectionality of parallels. Very often, we can say for certain, through our understanding of the mathematical situation, what the relative direction of lines in the ancient diagram was. So we can say that the relative direction of letters in the diagram was as in Figure 6 (I insist on 'relative direction': it is possible, of course, that the entire diagram has a different orientation in the original - indeed this is common in Greek/Arabic transformations, for instance; but the internal relative orientations are often constrained by the mathematical situation).

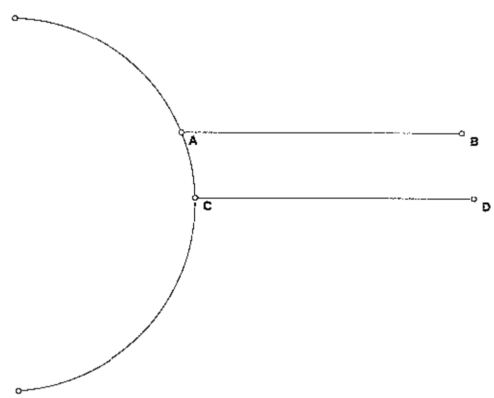

Figure 6

So the principle is the following: when such parallels as those of Figure 6 are mentioned *together* in the text, they will most often be mentioned in such a way that makes the 'direction' of both parallels the same. The text would say '*AB* is parallel to *CD*', not '*AB* is parallel to *DC*'.

So what we see is that visual considerations act as constraints on the naming of objects This, in fact, is the main outcome of the basic fact that names may change. If names change, if *AB* is taken automatically to be equivalent to *BA*, this means that the basic principle of naming is a visual, and not a symbolic one. The identity of the object is not the identity of symbols, but the identity of the visual line in the diagram.

A dramatic example for this is the following. Sometimes, quadrilaterals get a name with two letters only. Thus, the square of Figure 5 could in principle be called *AC*. This is not yet the dramatic fact What I am so excited about is the following: in the very same proposition, such a quadrilateral may be called, at one place, *AC* and at another place, *BD*. So it is taken for granted that *AC* and *BD* are equivalent - and this is possible only through the visual identity. This, in a sense, is a continuation of Assertion 1 What we come to see is that the identity of objects is visual and not symbolic. Two completely different symbols are taken to mean precisely the same thing and this is done in a completely matter-of-fact way, as if the choice of symbols were immaterial. Of course, this is because the object is given not by the combination of symbols, but by the object itself We refer to it, the quadrilateral, directly - its presence guarantees the possibility of the communication. Geometrical names, as it were, turn out to be rigid designators (Kripke, 1980)

But what I would like to claim is something more specific, going to an earlier philosopher of language. Peirce, the American semiotician and philosopher, offered a famous distinction among three kinds of signs. The most obvious and natural sign is probably an *icon*, something which designates an object by virtue of being similar to it. A portrait is the prime example of an icon Another sort of sign is the *index*. An index designates an object by pointing to it in some way, by some sort of proximity. A sign showing the way to this room would be an index of this room. When we attract attention to an object with an arrow, we use that arrow as an index:

(This is something to which I wish to point your attention.) → (This is something to which attention should be drawn.)

Finally, the last sort of sign is the *symbol*, which designates an object by virtue of some convention. Words are, in general, symbols.

I return now to Greek mathematics. There has been much speculation about the remarkable fact that in some early Greek mathematical texts, the expression used was not 'the point alpha' but rather 'the point at which alpha' (see e.g. the excellent article by Federspiel, 1992). My hypothesis, based on Peirce's analysis, starts from such texts I can not analyse them in this article, but I argue that in general, in Greek mathematics, *letters are indices, not symbols* The letter alpha is not like a variable in modern mathematics. It is simply an index signifying a point, and the signification is based simply on the fact that the letter is written next to that point. So it is obvious why the identity of the object should be visual and not symbolic. The signs used are no more than visual aids: in the technical sense, there are no *symbols* involved at all in the process of the signification

Assertion 3: Letters are indices, not symbols.

4. The diagram is the metonym of the proposition

I have so far made a few claims based on my textual methods of interrogation. I now move on to what is more in the nature of cultural history

What are the symbols of modern science? This is a strange question, perhaps, but it has answers Einstein is a symbol of modern science, he is a metonym of science. Einstein is probably the single person who represents science more than anyone else, in the popular imagination. And what is the symbol of Einstein? This is almost certainly $E=mc^2$ Everyone knows this Most will not know what E, m and c stand for, but everyone knows that this is Einstein, that this is science. This formula is a metonym of modern science

The one person who symbolised ancient science was Archimedes Very interestingly, Euclid took this position in the Middle Ages and has kept it since. We now think of Greek mathematics as Euclid - but Euclid was completely faceless for the ancients, whereas Archimedes was a household name, the sort of name which may crop up in Cicero, say.

The external histories may be relevant: the siege of Syracuse dramatised Archimedes for the ancients, just as the atom bomb dramatised Einstein in the twentieth century. But this is a digression. The person coming closest to being a metonym of science in antiquity was Archimedes, and we know also what was the metonym of Archimedes. It was a diagram showing a sphere inscribed within a cylinder This is what Archimedes chose for his tomb, what Cicero could rely on his readers to recall in connection with Archimedes (as is told by Cicero in a work by him: he, Cicero, as a Roman official in Sicily, found and restored Archimedes' tomb with its diagram - very much like a modern government taking pride in its preservation of the original manuscripts of a famous physicist)

So this is a graphic, striking comparison The metonym of modern science is a formula, a symbolic text The metonym of ancient science is a diagram, a visual representation. You may say this is no more than popular imagination; but I think that the popular imagination captures here the essential truth. There is much more detail which can be used to argue for this, for the role of the diagram in antiquity as a metonym

Pappus, when commenting on earlier mathematical treatises, estimates the size of a treatise by the number of diagrams. He does not say only 'this work has 73 propositions': rather, he often says 'that work has 73 diagrams'. Of course, this shows first of all something about the Greek word *diagramma* which Pappus uses. It means much more than just "a diagram" It means something much closer to "a proposition" or "a proof" (see Knorr, 1975, pp. 69-75).

This is a notorious fact about Greek practice: it is generally difficult to tell whether the authors speak about drawing a figure or about proving an assertion, and this is because the same words are used for both. And this again is because the diagram is the proof, it is the essence of the proof for the Greek, the metonym of the proof.

Xenophon, for instance, tells us that Socrates advised his pupils to do mathematics, but only as far as the simple diagrams, and it is clear that what he meant is simple mathematics. And Aristotle, most interestingly, when he wants to say that something would be best understood mathematically, says literally that the thing is best understood through the diagrams. The plural 'diagrams' (*diagrammata*) simply means "mathematics" in that instance.

So this is all about cultural history and the use of language, having nothing to do with textual methods of interrogation. So, to conclude this point, I wish to present briefly a question I have devised concerning this issue. If the diagram is the metonym of the mathematical proposition, we should expect it to be idiosyncratic to the mathematical proposition. Otherwise, if the same diagram is used for many different proofs, the diagram can hardly be a metonym.

In modern mathematics (as well as in Chinese mathematics), it is customary to have a number of proofs about a single diagram, and this is (in the modern case) because letters are assigned to objects in a stereotypical manner. *R* is always the radius, *O* is always the centre, etc. Such conventions mean that many diagrams are identical. But we have seen that baptism in Greek mathematics works differently. Baptism works alphabetically, so letters are always scrambled when there is a new proposition, the cards are newly distributed every time there is a new proposition. And this means that, in fact, every proposition in Greek mathematics has an idiosyncratic diagram. There are a few exceptions, especially in Ptolemy, but this is the clear rule. Every proposition has its own diagram and the diagram is like a fingerprint of the particular proposition. In this way, diagrams are not only possible metonyms for mathematical propositions, they are also natural metonyms.

Assertion 4: The diagram is the metonym of the proposition.

5. The inter-subjective object of mathematics

I will go quickly through the assertions I have offered so far. Just above, I have argued that the Greek perception is that mathematics is mainly about diagrams: that is, the Greek perception is that the *object* of mathematics is the diagram. In Assertion 3, I argued that the 'significatory apparatus', to use a cumbersome but useful expression, employed in the text of the proposition – namely the letters – refers to the diagram as an *index*, not as a symbol. In other words, the text does not take upon itself any significatory value. All the signification is contained in the diagram and the text is parasitic on this signification; it uses this signification via the use of letters as indices. In Assertion 2, I suggested that the formulation of the mathematical argument was done in front of the diagram alone and that written aids other than the diagram were of less importance for this formulation of the argument. And, in Assertion 1, I made the fundamental claim, the starting-point for all the other claims, that the object of the proof was logically determined by the diagram and not by the text alone.

So far I have been a historian and I do not claim to be more than a historian. But it will be more interesting to put this discussion in a more philosophical framework. I believe we approach here a fundamental issue concerning mathematics in general and this should be spelled out. Of course, I do not aim to define mathematics here, but I think it is correct to describe mathematics as the field in which we apply reason to inter-subjectively definitely given objects. (Notice that I insist upon the word 'reason' – I do not think we necessarily apply *logic* in mathematics.) The most important thing about mathematics is that we have a definitely given object. It is not just the vague, familiar, empirical world, something about which our knowledge must be fuzzy, but it is some perfectly determined object, an object whose properties are definitely given, and these properties are definitely given in an inter-subjective way: it is not just my own intuition, it is something I can share, I can communicate. So the presence of a definitely given, inter-subjective object is crucial for the development of mathematics.

In modern mathematics, the object is made definite and inter-subjective by the use of axiomatic systems which capture once and for all the properties of the object under discussion: we capture the beast with the nest of verbal formulation. This is the modern approach and it is distinctively modern. Axiomatics are far less important in pre-modern mathematics. The Greeks may on occasion play the axiomatic game, but it does not have for them the same central logical function. And this is because the definite, inter-subjective object of Greek mathematics is the diagram.

I have already said that the study of Greek diagrams as they actually are in the manuscripts is still in its infancy. I will now give a case where it is clear that the modern editions are misleading and this case throws a dramatic light upon the Greek construction of the mathematical object. In many of the manuscript traditions of Euclid, the definitions at the start of the book are accompanied by diagrams. The text is 'a point is that which has no part', and next to this Greek sentence there is a point drawn in the manuscript, and so on with all the rest of the definitions. You will never even get a hint of this in modern editions. This is the axiomatic shrine, the holiest of the holiest. Here, we the moderns expect the Word to be enshrined alone; no other presence should defile the purity of the abstract verbal formulation. But when we enter the temple we see there enshrined the picture, the diagram. This, and not the Word, is the central object of Greek mathematics. So I now make the following, tentative assertion. I do not claim of course to have *shown* it; but I might as well make it, as a starting point for further discussion.

Assertion 5: Mathematics requires an inter-subjectively given object. This is supplied in modern mathematics by conceptual systems and in Greek mathematics by the diagram.

6. The role of writing in Greek mathematics

At this stage, readers may feel that I have already gone too far, and that I should by no means add a sixth assertion, one even more general and speculative. And indeed I conclude with what is less philosophical, but is still quite conjectural. I wish to offer, very briefly, a guess concerning the origins of the very central place of the diagram I have described so far This is not a complete explanation, but I think it identifies an important relevant factor.

I have mentioned orality in Assertion 2 I described what I called there an oral process, that of thinking without written aids, in front of a diagram. It should be understood that I use orality in a very limited sense: Greek mathematicians were highly literate persons, and they relied on writing to communicate their results However, this does not mean that whatever they did was based on writing. This article is based upon a lecture (as I suppose has been obvious already) [1]: when giving the lecture, I relied on a written text - the basis of this text - which I performed to the best of my ability. So it is interesting to see that even when speaking, I used a written aid, the prepared text of the lecture. This is an example of the way in which our culture relies heavily upon writing. It is not clear that the Greeks were like this.

It was once thought that the Greeks in general relied little on writing, that they never read texts silently, for instance: this is now known to have been an exaggeration. The Greeks read silently, and they read a lot; the Greek élite was a literate élite. However, the Greeks did not develop many systems of signification based specifically on the advantages of written symbols Most importantly, for our purposes, they did not develop mathematical notation. Because of this, they relied much more heavily upon oral aids, such as oral formulae. The verbal was typically used in the oral mode. In mathematics, then, on the one hand, the Greeks did not develop anything like our modern written symbols; on the other, they did develop a system of formulaic expressions, one which is essentially oral They were therefore in a sense oral both in what they did not do and in what they did do. I am convinced that much of their mathematical reasoning took the shape of a silent soliloquy, unaided by writing, in front of a diagram. We all do this: I suspect the Greeks did this much more often. This is all I mean when I claim that there was a strong oral component in Greek mathematics

The meaning of this for the construction of the fixed object is clear. If the word is typically used as the spoken word, not as the written word, then it is natural that the word will not provide the fixed object for the argument The spoken word is a quicksand, it appears and disappears. It cannot serve for foundations: only the written word can serve in this way. It is therefore natural that modern mathematics, which relies so much on printed, visual symbols, will also fix its objects through the written text - but the one fixed, solid object in Greek mathematics was not the word, but the picture. This was actually drawn, not just spoken, and therefore it was actually out there, it was the physical reality, the inter-subjective definite object This then is my guess, which I write as Assertion 6.

Assertion 6: The verbal could not be the fixed object in Greek mathematics because of the more oral approach taken in Greek mathematics.

Mathematics is essentially a diverse field, its diversity determined by its cultural setting. I write this article in the hope of making a contribution to this general point of view.

Note

[1] This article is based on a talk given at the Third International Conference for Greek Mathematics, Delphi, Greece, July - August 1996

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