

FROM LOGICAL CAUSATIONS TO LIKELY CONNECTIONS

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When a Year 7 student physically reacted to a prompt of another student by anxiously drumming the desk with his ruler, exclaiming “*uuuuhh*”, the initial thought of the observing researcher, Laura, was: “this is an interesting account”. This started a reflective journey of first applying robust research methodologies to the episode we initially saw as a linear, logical system, then slowly developing understanding of the complexity of the interaction, as well as our position as complex observers of a complex entity. Thus, the journey evolved into a second-order complexity narrative (the complex entity being the first-order, the complex observer being the second order; Tsoukas & Hatch, 2001) that we tell here. In this essay, we explain how exploring a complexity perspective helped us to make sense of a collaborative problem solving (CPS) situation that creates so much non-deterministic variation that it becomes irreducible to linear reasoning only.

According to Bruner (1986), there are two modes of thinking: one that operates on logic to justify arguments and seek truths, and another that builds up a narrative and can inform about likely related accounts. We used both modes to analyse data that included the Year 7 student’s reaction. We found out that seeing educational theories, as well as all the human and non-human agents involved as adding to the complexity of CPS research (Fenwick & Edwards, 2010) can give us tools to discuss intuitive interpretations and likely associations of what is present in the data.

In such a view, ideas such as ‘video data’, ‘decision making’, ‘theory’, ‘methods’, ‘digital technology’ and ‘time spent on research’ each have their unique role in the process of CPS research. To tell a good story (in Bruner’s terms), we will introduce the role of these ideas in our journey. We will start with introducing the video data, describe how several decisions were made in terms of theories, methods and technologies that led to sound research findings; we will emphasise the length of dwelling on this research, and finally, we will move on to complexity and discuss what we were able to accept when looking at the *journey*—not just the data—as second-order complex observers.

To position ourselves, we believe that mathematics education research should have an impact on teaching practices. This is what motivated us to examine the episode with Bruner’s two modes of thinking; we wanted to leverage our journey to benefit a variety of educational contexts. We reflect upon our journey reducing aspects of reality into data and finding it problematic since, as Messick (1995) put it, the data are not just the student’s reaction to the stimulus provided, *but also to the entire context*. We stress that the experiences individuals bring to any activity, the overall

dynamics of the group, the context, the short term and longer-term goals of students and the teacher are intertwined, and all contribute to the ways in which students can learn from CPS. We acknowledge the impact of ourselves as researchers when attempting to determine what is meaningful in CPS. We begin by introducing the video excerpt that motivated us.

The video excerpt

Laura faced the challenges of understanding the nature of CPS while participating in the Social Unit of Learning project that focused on ‘the essentials of social learning’. She was invited to work on the rich video data collected as part of the project, and which was described in detail by Chan, Clarke and Cao (2018).

She was given access to the video material and became intrigued by an excerpt showing four Year 7 students (two girls, Anna and Pandit, and two boys, John and Arman (pseudonyms), aged 12–13 years). The students were working together on an open-ended 20-minute task called ‘Fred’s apartment’, the prompt being: *Fred’s apartment has five rooms. The total area is 60 square metres. 1) Draw a plan of Fred’s apartment. 2) Label each room and show the dimensions (length and width) of all rooms*. The video excerpt allowed an outsider to observe how the participants engaged in mathematical CPS but did not reveal whether the students did or did not know each other well, or anything else that they brought to the situation. Undoubtedly, they had varying prior experiences, personalities, expectations, wishes and needs.

Laura started to analyse the excerpt independently. A brief moment between John and Pandit caught her attention, as described in her logbook:

Pandit gestures and calls for the guys (“Guys guys what are you doing?”), gets an unclear utterance (audible, but vague) from John and looks a bit uncomfortable, then she gives a mathematical answer to John (“It’s not the same area”). She gets an unclear answer from John, gets confused, laughs and bends down, pressing her face between her arms, towards the table. She straightens herself, seems emotional (smiling and confused), turns back to the task and Anna. John says something unclearly to Pandit, then again smiling and trying to look confident; he nods his head repeatedly. John has problems with his language. It looks like he notices that his communication with Pandit has failed.

It seemed that John showed willingness to engage in the CPS, but he struggled with the language as a second language English speaker and failed to get his ideas across.

Pandit listened to John, but it was as if there was too much socio-emotional load for both students to bear. They had different cultural backgrounds, different communication skills, different genders and no doubt many other differences. Right after the episode something noticeable happened. In the discussion below notice that not much is identifiable from what the students say:

- John* Just to make sure it's a [unclear] same area or [unclear]
- Pandit* It's not the same area.
- John* You sure?
- Pandit* Yeah.
- John* What's [unclear]

A more holistic picture can be constructed from Laura's additional notes about the interaction shown in the video (we regret the video excerpt cannot be made public because of the ethical commitment made in the project), revealing the socio-emotional tensions arising after the first disruptive episode between John and Pandit:

In this episode, Pandit asks what the guys are doing, John gives an appropriate answer, Pandit reacts to that in a normal way (they have a tiny mathematical discussion), then John seems to react normally, but it is almost like this was too much for them both, because then Pandit, without much reason says "Oh my God", and bends her head inside her arms on the table, pulls back from the communication; John surprisingly starts to talk nonsense (weow, weow, weow, weow) and laughs and he also pulls back from the communication seemingly excited and confused and smiling. He says: "uuuuhh" and keeps on hitting the table with the ruler (rapidly). Then he ruler fights with Arman again and calms down. He continues with the task. (Pandit looks less excited, but she also gets a bit confused, starts to bend the rulers and says: "we got some of the bendy rulers". Anna acknowledges the episode, and the girls discuss their relationship with the boys very quickly.)

The episode peaks in John's exclamation "uuuuhh" at which point he is wildly drumming the table with his ruler. Pandit turns away from the situation. How these tensions impact the rest of the interaction was not easily identified in the transcript but evident in the non-verbal interaction: John's efforts are no longer taken seriously. John continues to propose mathematical ideas, but others only reluctantly listen to them. Phenomena like this are rarely 'mathematical enough' to be acknowledged in rigid analyses. For example, Nieminen, Chan and Clarke (2021) removed all such excerpts from their analysis of mathematical CPS, deeming them insignificant. However, in these 'non-mathematical' moments lies much of importance for students and we wondered about how to make appropriate interpretations of such occurrences.

Rational mode: logical causations

Laura applied Goldin's (2017) motivating desires as a framework to understand the episode. The analysis uncovered

many motivating desires, such as willingness to show off, commitment and need for attention, but it was also found that just a slight modification in the analytical unit changed the overall interpretation of the situation (Tuohilampi, 2018). Framing the data with a certain analytical unit resulted in interpretations of participation looking favourable, whereas choosing another framing of the analytical unit revealed the participation was perhaps less optimal. Another approach was to discuss the concepts of participation and identity, which according to Lawson and Masyn (2015) can develop independently. Laura was able to identify indicators of participation from John, but it seemed clear that to determine his identification, that being a more emotional aspect, a longer term follow up would have been needed.

These analyses left Laura feeling that the students' interaction was yet to be fully explained. Had she been observing CPS that was beneficial, neutral or counterproductive? At this stage Juuso jumped in and we started contrasting our interpretations with the overall progress that was ongoing within the Social Unit of Learning project. Our attempts, both formal and informal, were as follows:

Add theoretical perspectives. After the first analysis in 2015, more approaches, tools, perspectives, concepts and analytical units were applied to the 20-minute problem-solving session. We were able to establish the quality of CPS: the students in the excerpt negotiated how to approach the problem which made them progressively more efficient at making decisions about the task. They arrived at a single solution. According to Yackel, Cobb and Wood (1991), we would have concluded that the CPS had been completed successfully.

Discuss the data. In addition to typical ways to discuss the process, such as conference presentations and articles, the Social Unit team created many opportunities to discuss the data that included seminars and meetings, as well as dinners, coffees, walks and celebrations. We even discussed the data at a football match.

Standardise the unit of analysis. The team made an effort to conduct different analyses using the same analytical unit but it proved impossible. Whereas a unit, such as a negotiative event (Chan & Clarke, 2019), served well for one approach, another approach identified more than one essential chunk of units within the chosen negotiative event.

Multimodality. Even though we were not able to address single units through several lenses, we managed to weave together different approaches by having team members focusing on certain pieces of data, such as making discursive analyses of the transcript, or considering gestures evident in the video. Díez-Palomar, Chan, Clarke and Padrós (2021) noted, from the same data, that the four students had applied 5 of 7 different types of talk, and that because of power distribution across the group members, some mathematical elaborations might have become narrow.

Theorise. For Chan and Sfard (2020), theorising is a way to generalise knowledge from small-scale case data, as stated in their study on mathematical CPS: "The presence of theory made the claim on the generalizability of results quite compelling in spite of its being grounded in just two empirical instances" (p. 15). This suggests that theory lies at the heart of producing knowledge about mathematics education; the stronger the theorisation, the more robust the knowledge.

Meaning-centred mode: likely connections

Over the years, the Social Unit team expanded into a global network of researchers. More data from similar settings were gathered, adding different perspectives and findings allowing us to discuss the data more thoroughly and deeply. There were theory-based indicators to look for in the data: smiling, agreeing, arguing, negotiating, sharing, explaining. But there was something that had triggered Laura so strongly that was yet to be uncovered. It seemed that John was left as an outsider, emotionally challenged and alone. There were feelings of empathy, a wish to rectify something, helplessness at realising that sometimes ‘everything is done right’ but it still does not look like enough. At some point we realised the Social Unit team had developed a shared speculative understanding of the many meanings the student interaction might have for the students short and long term that wasn’t based on rigorous analysis-based findings. This intuitive understanding was based on the verbalisations and actions that students had made available in the video data, but eluded logical interpretations and well-defined methodologies.

On sharing our thoughts with Kim, we entered an iterative process of discussions that helped us gradually understand more and finally led us toward complexity as a means of making sense of the episode.

Complex systems (*e.g.*, cells, organs, organisms) are not simply more complicated than other systems but are qualitatively different, demanding different analytical tools (Davis & Simmt, 2003). A complexity approach sees CPS as a complex system comprising interacting complex systems—the students themselves. This constitutes the conditions where collective characteristics, that are not attributable to particular actions, are likely to emerge (Davis & Simmt, 2003).

The idea of a specific actor not being directly responsible for the collective characteristics of a group in which they are situated ontologically challenges the traditional mechanistic approach where causal actions can be identified and supported or discouraged. This suggests a narrative mode instead of the rational one. We can see a complex system as an incubator of actions with particular likelihoods of intertwining in a way that might be interpreted as progress. What is more, in a complex system a minor occurrence might make a sudden transformation in the system without a clear (or none whatsoever) cause and effect relationship. As described by Tsoukas and Hatch (2001): “Small causes may give rise to large effects. Non-linearity is the rule, linearity is the exception” (p. 988).

In human contexts, such as CPS, the leader (*e.g.*, a teacher) can organise the environment to support patterns of interaction more likely to give rise to desired outcomes (Beswick, Watson & DeGeest, 2010). However, emergent phenomena cannot be completely controlled or predicted.

Some optimal conditions for complex emergence were proposed by Davis and Sumara (2005):

- 1) sufficient *diversity* among the agents of the system (*i.e.*, the ways of thinking of the collaborating students);
- 2) adequate *redundancy* among the agents for effective communication to occur among them (*e.g.*, students are similar enough in their mathematical backgrounds to understand one another);

- 3) that agents’ *control is decentralised* (*i.e.*, students are able to think independently and choose the nature and extent of their participation);
- 4) that *neighbour interactions* occur (*i.e.*, students’ ideas are brought together and reinforced or challenged in the process); and
- 5) *enabling constraints* that provide a focus for the system’s activity (*i.e.*, there is a problem to be solved within a time frame).

The first, fourth and the fifth condition seemed largely met in the students’ interaction, unlike the second and third condition. Investigating the conditions in the group could have opened us another door to explore, however the CPS situation of the students *not having met* all conditions probably was the very reason for the tension, triggering us to start our meaning-making journey.

Looking back through the complexity lens at our previous attempts to understand the episode we were able to explain some inconsistencies. For example, being challenged by how much the choice of the analytical unit impacted the interpretations (Tuohilampi, 2018) became less critical when we learned that a complex system can be considered a fractal, whereby the true answer depends on the measurement scale (Tsoukas & Hatch, 2001).

To Laura, it looked like the rational mode needed to be traded for the narrative mode to be able to discuss the likely connections of a complex event. The narrative itself would serve as a second-order complex story, causing reactions in its readers that are subject to the readers’ interpretations of the narrative. Writing this essay is our attempt to represent the narrative mode. However, we had to question whether the complexity approach was interchangeable with the narrative mode, suggesting complex analyses not be rational—*i.e.*, to be irrational. We concluded that there can be an overlap. Within the narrative mode, the rational mode can be used to choose approaches, to make interpretations, to tell the story. Yet, the goal of the narrative mode is different from the solely rational mode. As described by Bruner (1986) “One leads to a search for universal truth conditions, the other for likely particular connections between two events—mortal grief, suicide, foul play” (pp. 11–12), suggesting that instead of investigating what might *cause* John’s exclamation (or, as in Bruner’s example, grief being caused by the death of the Queen), the narrative mode aims to seize the event and open a door for multiple complex interpreters to zoom in.

To further analyse the indeterminacy of CPS, we reviewed our experiences in terms of Messick’s (1995) classic and widely used framework for establishing the “adequacy and appropriateness of interpretations” (p. 741). In Table 1 we introduce and reformulate Messick’s notions to recommend how researchers could understand data concerning mathematical problem-solving as a complex endeavor.

On a quest to understand the nature of the four students’ interaction in CPS, we first started by seeking ‘the optimal’. We used rational mode to create a criterion for what could be optimal short or long term for students’ growth as mathematics learners and human beings. As researchers, we can afford

Table 1. *Messick's aspects of validity and conditions for complex emergence, reframed for the purposes of complex mathematical problem-solving research.*

Recommendation for research that acknowledges the complexity of CPS based on Messick's aspects of validity
<i>Content aspect:</i> content relevance, representativeness and technical quality. Complexity extends our view beyond the episode. We can define the system as the worlds in which CPS participants live; it is all relevant.
<i>Substantive aspect.</i> Looking for patterns that appear to increase the likelihood of desired outcomes, as well as observed consistencies in data along with empirical evidence that the theoretical processes are actually engaged by respondents.
<i>Structure and fidelity.</i> Even if there is no clear coding structure, one can interpret observations as emerging from a complex system and avoid making narrow linear conclusions.
<i>Generalizability aspect.</i> To determine the extent to which interpretations generalize to and across population groups, settings and tasks, treating outcomes as more likely to happen again if the conditions are as close to the same as possible <i>but</i> recognising that the system is inherently unpredictable so repeatability is not guaranteed.
<i>External aspect.</i> Includes convergent and discriminant evidence from multimethod comparisons, as well as evidence of criterion relevance and applied utility as part of the complex interpretation of a complex system.
<i>Consequential aspect.</i> Appraises the value implications of interpretations as a basis for action especially in regard to sources of invalidity bias, fairness and distributive justice. Recommended actions are framed as influencing the probabilities of desired outcomes rather than linearly causal.

to entertain the idea of cause and effect, of linearity, the rational mode. We have the privilege of reducing reality to well behaving pieces, where the existence of a criteria, when fulfilled, lands in logical findings like a train that does not get derailed by small cracks on the track.

For a student participating in CPS the inherent complexity is unavoidable. The complexity approach sees all tiny cracks as possibly holding an enormous meaning, having a power to stop the train, change its course or transform it into something different. This is why we needed to tell a story of what we saw as likely connections. We felt that in the well completed CPS of these four students, where not all the conditions suggested by Davis and Sumara (2005) were fulfilled, there were tensions that had meaning. On one hand, the tensions might not have arisen if only something small would have occurred, such as the teacher intervening at a particular moment with a lighthearted joke. On the other hand, we speculated that the students probably forgot about the episode soon afterwards. Yet, the moment of awkwardness experienced by Pandit and John likely stayed and created its own meaning for each individual.

Conclusions

Holistic approaches, such as arts-based approaches or thick descriptions of CPS have the potential to highlight relationships among various aspects of phenomena. For example, Tossavainen and Dubois (2021) discussed how the prior experiences of two 5-year-old girls impacted their interpretations of the task at hand, and how their observations connected with mathematical concepts: when one of the girls was prompted to figure out whether a painting depicted a day or night time activity, she was able to infer correctly

that going to the cinema as shown in the painting was an evening event because her parents had gone to the cinema in the evening. In this essay, we have attempted to take a holistic view by combining the two modes of thinking suggested by Bruner.

We have elaborated on our engagement in data, what we saw and what we experienced. When we started, we were interested in determining how to judge which of the students' actions are productive or counterproductive in terms of solving the problem—or further mathematics-related development. We learned all the things we have communicated here and want to summarise three provocations for future studies to acknowledge.

Provocation #1. Single moments of interaction can be significant to the overall interaction—and vice versa. Although John's struggle initially drew our attention, the roots of that moment could likely be tracked to the beginning of the sessions—or even earlier. The state of affairs in a particular moment cannot be separated from a larger picture, nor can the big picture be elaborated without describing certain moments in detail.

Provocation #2. It is difficult, maybe impossible, to determine the nature or the significance of singular moments without reducing reality. Even while John was participating and resilient, he did not seem to be progressing mathematically or socially. However, perhaps he was on a path to become a member of this group—or a more fitting group—in the longer run.

Provocation #3. The one-dimensional trade-off approach, wherein investing in one activity takes time from something else (as exemplified by Yackel, Cobb and Wood, 1991, in their discussion of CPS), *might not be true.* Yackel, Cobb

and Wood based their suggestions for productive CPS on the assumption that previous negotiations can be applied to subsequent problems, making the CPS more streamlined iteration by iteration. In that sense, we would assume that *the more* we can identify mathematically sophisticated expressions, *the better* the mathematical quality of future conversations. Having seen the episode between John and Pandit we speculated that different areas of student development or learning are not necessarily in competition for a finite resource (time) but can occur simultaneously. We propose that in a complex system a particular type of development *may not take space from another*: students might be able to develop their identities through non-effective looking chatter while learning about others' mathematical reasoning, holding an incorrect idea and contrasting it with one's previous knowledge, all simultaneously.

Viewing CPS as a complex system acknowledges that deterministic classroom research outcomes are challenging to translate. No matter how well the teacher has prepared the activity; no matter how knowledgeable or well-intentioned they are, and even if an observer could not fault the lesson there is no guarantee that the desired learning will happen. The students in our study seemed willing to participate and persist and seemed genuine in their efforts to collaborate. CPS entails uncertainty of outcomes as well as a variety of outcomes for different agents. We suggest more discussion of the *likelihood* of certain interactions.

In our excerpt, the students worked to find a solution that everyone could agree upon. This is in accordance with Yackel, Cobb and Wood's recommendation to have everyone responsible for the solution and being able, if asked, to explain the process. A teacher might wish to see such a neat process in their classroom and aim to establish an environment in which students could have mathematical negotiations resulting in an expected product. Our journey suggests, however, that seeing CPS as a zero-sum game in which certain actions are seen as less favourable, taking space from others, might result in the promotion of practices that suppress features of the episode that on the surface seem contradictory, but which could be enormously enriching. As problem-solving scenarios are understood as complex systems, 'off task' actions are part of a broader system and, as such, can become important to the ultimate mathematical goal. This idea challenges rational research that artificially focuses on 'the mathematical' (e.g., Nieminen, Chan & Clarke, 2021).

For teachers our story holds some practical implications. We encourage teachers to accept the uncomfortable variability within a particular timeframe that is inherent in the indeterminant phenomena of CPS. This is not to advocate a laissez-faire approach but rather to admit that human interaction, and learning are wonderfully complex. Teachers can stack the odds in favour of learning so that it is very likely to develop in the expected directions, particularly over the longer term. Attention needs to be paid to a wider progress: what has been *ignited*, not just what has been *completed*. Teachers need to be permitted to acknowledge without fear of judgment that not every activity is immediately productive in terms of the goals they are responsible to attain.

As educational researchers, we should not be satisfied by

the rational mode only. Complexity offers a way for mathematics education researchers to rethink how to provide likelihood-based recommendations for CPS practice. We challenge researchers to consider whether there is a risk of seeing a disciplined use of theories and methodologies as a panacea preventing invalid interpretations. For example, we could have analysed the video through networked theories (Bikner-Ahsbahr & Prediger, 2014): perhaps by drawing on multiple theoretical lenses we might not have felt the need for the complexity we now call for. As second-order complex narrators, we propose the selection process of theory, the used theory, as well as CPS research itself as a complex system with complex occurrences. While our journey has been human-centred, future analyses might benefit from post-humanist approaches that further unpack the role of non-human agents involved in CPS and CPS research (Fenwick & Edwards, 2010).

We concur with Messick (1995) that validity is an evolving property of research. Validity is not simply about measurement principles but social values (the consequential aspect). What we see as valid influences our evaluative judgements and the decisions we make (Zumbo & Hubley, 2016). As researchers, we intervene in participants' lives, make hypotheses about their experiences, collect data, observe and interpret, and then make evaluations, speculations and recommendations.

Here we have attempted to honestly reveal the convoluted journey that we went on having been captured by John's "uuuhhh". Maybe we knew that it was an important moment for John, in that specific situation, among those participants, in that time and space. Maybe it was something he would forget about soon after, just a snap moment to live through. Or maybe it became an important memory. A mundane educational setting where you interact daily, an interaction where you reflect upon your status and construct your identity, could be a socio-emotional moment that scars one of the participants more deeply than others. At least we know that the interaction of these four students was witnessed by us, it moved us, it did happen. It was all true for a blink of an eye.

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An alabaster sculpture that combines geometry and architecture. Originally given to the Bodleian Library, University of Oxford by Sir Clement Edmondes in 1620. History of Science Museum, Oxford.