

ETHNOMATHEMATICS AND EDUCATION: SOME THOUGHTS

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Two and two make four/ They never make five
And as long as we know it/ We can all survive
(Graham Nash, *Songs for Beginners*)

In 1996, Barton writes, “Very little of the ethnomathematical literature is explicit about its philosophical stance”, which he considers to be “one of the areas” that “must be addressed if the subject is to gain wider legitimacy in mathematical circles” (p. 201). Seven years later, Adam, Alangui and Barton (2003) take issue with a recent, critical investigation of ethnomathematics, especially of its relationship with academic mathematics regarding teaching and learning. They dismiss the largely philosophical questions raised by Rowlands and Carson (2002) by suggesting that “the role of ethnomathematics in mathematics education is now predominantly an empirical matter” (p. 327).

What has happened in the interim that has contributed to this seeming shift? Is it true, now that the political and philosophical/conceptual questions have been taken care of, that ethnomathematics is used successfully “as a framework in the teaching of mathematics” (Adam *et al.*, 2003, p. 334)? This view appears to rely for its plausibility partly on the work discussed in the first two International Congresses on Ethnomathematics (Contreras, Morales and Ramírez (eds), 1999; de Monteiro (ed.), 2002) and by Barton himself (1999b) and partly on studies in so-called indigenous knowledge. The underlying idea is that a philosophical, conceptual inquiry regarding ethnomathematics is now dated – and that any critique must address the empirical issues around curricular reliance on ethnomathematics and evaluate the results.

My aim in this article is twofold. I argue that any dismissal of philosophical engagement with the plausibility and viability of ethnomathematics on the grounds alluded to above smacks of mere verbal legislation. Furthermore, I will illustrate, through an analysis and critique of recent perspectives, that the philosophical debate around *ethnomathematics* is both alive and warranted – indeed, crucial. I argue that ‘ethnomathematics’ makes, at best, limited sense, namely insofar as it is understood as describing indigenous mathematical *practices*.

Seeking refuge in research on indigenous knowledge is misguided, in that the idea of ‘indigenous knowledge’ faces serious and potentially fatal objections (see Horsthemke, 2004a; Horsthemke 2004b).

Moreover, very few philosophical debates have dated. Very few philosophical puzzles and problems have been resolved. (The freedom/ determinism debate and the mind-body problem may be among these few.) So, to suggest that ethnomathematics has shifted towards predominantly

empirical matters is to proceed in terms of unwarranted verbal arbitration. Insofar as ethnomathematics continues to involve a questionable understanding of knowledge and truth, any such ruling is likely to beg the question of the validity and conceptual soundness of the ethnomathematical enterprise.

Shifts in definition and direction, and current developments

In his 1996 article, Barton traces definitional shifts within ethnomathematics by examining chronologically the work of D’Ambrosio, Gerdes and Ascher, respectively. He explores the development of ethnomathematics ‘into a research programme, with a broader referent’, that

now includes: a) the formation of all knowledge (D’Ambrosio), b) mathematics in relation to society (Gerdes); and c) mathematical ideas wherever they occur (Ascher). (Barton, 1996, p. 210)

Building on these ideas, Barton presents the following definition:

Ethnomathematics is a research programme of the way in which cultural groups understand, articulate and use the concepts and practices which we describe as mathematical, whether or not the cultural group has a concept of mathematics. (2004, p. 214)

D’Ambrosio’s understanding is strikingly similar but also contains an emphasis on both philosophy and pedagogy:

Ethnomathematics is a research programme in the history and philosophy of mathematics, with pedagogical implications, focusing the arts and techniques (*tics* [from *technē*]) of explaining, understanding and coping with (*mathema*) different socio-cultural environments (*ethno*). (D’Ambrosio [2], 1985)

Barton’s article constitutes an “attempt to note [...] the current directions in ethnomathematics” and “other well-established developments”: the “use of resources derived from other cultures in mathematics education” (1999a, p. 32) or what might be called the cultural resource direction, one that appears to have a predominantly practical or empirical focus; D’Ambrosio, Gerdes and the “humanistic mathematics” direction; and “the academic debate concerning the philosophy, legitimacy and relationships with other disciplines and theories”, that is, “the critical mathematics direction” (*ibid*, p. 32). These different “directions” are clearly intimately linked and the boundaries between the cultural resource direction, the humanistic mathematics

direction and the critical mathematics direction overlap, as Barton acknowledges later (2004, p. 22).

Three years later, Rowlands and Carson publish a critical review of some of the ethnomathematics literature. In response to the question, “What would an ethnomathematics curriculum look like and where would formal, academic mathematics fit in such a curriculum?”, they argue that

it is only through the lens of formal, academic mathematics sensitive to cultural differences that the real value of the mathematics inherent in certain cultures and societies be understood and appreciated. (Rowlands and Carson, 2002, pp. 80, 79)

In their response to Rowlands and Carson, Adam *et al.* contend that

debate on cultural issues in mathematics [...] must be based on contemporary writing in the field, and should not focus on extreme views within the political justification for ethnomathematics [and assert that] the role of ethnomathematics in mathematics education is now predominantly an empirical matter. (Adam *et al.*, 2003, p. 327)

This contention evidently begs the question of the validity and soundness of the concept of ethnomathematics. It mistakenly assumes general agreement on what ethnomathematics is and on its “legitimacy in mathematical circles” (1996, p. 201), to use Barton’s earlier phrase.

After noting two perspectives on ethnomathematics, the “political perspective”, “now more often related to writing on indigenous knowledge” (Adam *et al.*, 2003, p. 328), and the philosophical perspective (including the question of “mathematical relativity”), the authors assert,

Whether [...] an ethnomathematical] perspective helps fulfil the aims of conventional mathematics is no longer a question that is debated on ideological lines. For some time it has been regarded as an open empirical question by most of those working in ethnomathematics. (Adam *et al.*, 2003, p. 330)

Does this not amount to verbal legislation? The claim that this is the view of “most of those working in ethnomathematics” fails to establish anything.

The authors state, further, that

[e]ducational research is so far demonstrating the success in conventional terms of at least one ethnomathematical approach to the curriculum. Any critique of this field must address such results and evaluate them as research. It must enter current debates. It cannot be ideologically directed, nor is it helpful to address antiquated or extreme positions. (Adam *et al.*, 2003, p. 333)

Adam *et al.* cite the work done in Alaska (described in Lipka, 2002) as an example of “students who have been taught using such an ethnomathematical curriculum [and who] perform better on conventional mathematics tests” (2003, p. 333). Yet, they miss the essential points of the critique of ethnomathematics: which is to interrogate what is “culturally specific” or “unique” about this approach and – if indeed it is so specific or unique – whether it is “mathematics”. This

critique has little, if anything, to do with “ideology”. On the contrary, it is informed by a plea for conceptual clarity and argumentational rigour.

Finally, in their endorsement of

an integration of the mathematical concepts and practices originating in the learners’ culture with those of conventional, formal academic mathematics’, (p. 332)

the authors do not address Rowlands and Carson’s concerns. The latter do not “claim that mathematics should be taught in an artificial setting without relating it to the real-world whatsoever” (Adam *et al.*, 2003, p. 333; Rowlands and Carson, 2004, p. 336). Instead, they argue that

a teacher ought to be sensitive to cultural experience [...] but] to confuse the boundaries between cultural experience and high-order abstract concepts in mathematics is to confuse different cultural systems [...] All good teachers should be aware of the cognitive state of their students, but that awareness can be achieved by how the student responds to the mediation of high-order concepts. This does not mean connecting high-order concepts with cultural experience. (Rowlands and Carson, 2002, pp. 96, 97, 98)

It might be argued (as Alangui did, in conversation with me at the recent Auckland ICEm-3) that sensitivity to cultural experience and awareness of students’ cognitive state are insubstantial and woefully inadequate for addressing the deeper concerns, like the effects of physical and mental colonization. My question is, however, what work does a focus on ‘ethnomathematics’ do that a rights-based approach does not or cannot do? ‘Rights’ may be an occidental idea (and certainly not a failsafe one at that), but it is arguably a useful tool for addressing issues of social justice and redress.

In a recent article, Barton usefully distinguishes between “mathematical” knowledge and “practical” knowledge (for example, the “mathematical” practices of artisans):

There have been many studies done on these issues: studies in ethnomathematics, studies in mathematics education, studies in situated cognition, studies in anthropology, studies in the history of mathematics and studies in indigenous knowledge. [...] Writing in one area has been criticised as if it was from another. Rowlands and Carson’s (2002) critique of ethnomathematics as if it is an educational movement is a case in point, the rebuttal (Adam, Alangui and Barton, 2003) differentiating between the open educational questions, and the ethnomathematical issue of relativity in mathematical thought. (2004, p. 22)

Again, Barton misses the (meta-)issue here. The critique concerns the very plausibility of ethnomathematics and indigenous knowledge. In order to make sense of ideas like ‘relativity’ and ‘cultural specificity’ in mathematics (education), reference to the distinction between ‘mathematical’ and ‘practical’ knowledge is crucial. Yet, such reference is usually unavailable within ‘studies in ethnomathematics’. Tellingly, Barton notes that

the boundaries between these areas of study overlap – the differences are often ones of emphasis and focus

rather than distinct features. Furthermore, many writers deliberately address more than one of these areas in the same article. (2004, p. 23)

The confusion between theoretical and practical knowledge seems to lie at the heart of the defence of ethnomathematics and of indigenous knowledge in general. I attempt to address this confusion, with particular reference to the ideas of relativity and/or cultural specificity of mathematics.

Knowledge and truth: the problems of relativity and cultural specificity

After presenting his definition of ethnomathematics, Barton announces,

Both “mathematics” and “mathematical” are culturally specific because their referents depend upon who is using the terms. (1996, p. 214)

He claims that there are four implications of his definition:

(a) ethnomathematics is not a mathematical study, it is more like anthropology or history; (b) the definition itself depends on who is stating it, and it is culturally specific; (c) the practice which it describes is also culturally specific; and (d) ethnomathematics implies some form of relativism for mathematics. (1996, p. 215)

Before I examine the idea of mathematical relativism in greater detail, I want to comment briefly on Barton’s elaborations of points (b) and (c). He writes,

The definition of ethnomathematics is culturally specific: it is written from the point of view of one culture or social grouping, namely a culture or social grouping which has a conceptual category named “mathematics”. [...] Part of the purpose of ethnomathematics is to *challenge the universal nature of mathematics*, and to expose different mathematical conceptions. If this is successful, then ethnomathematics is also specific to one particular concept of mathematics. Thus a universal definition is not possible. (1996, p. 216; emphasis added)

Barton accepts that his preferred definition of ethnomathematics, too, is culturally specific, so he cannot be accused of inconsistency in this regard. However, this move renders ‘other’ mathematics opaque or unintelligible, and perhaps useless, to anyone outside the specific culture. Even more seriously, ethnomathematics is rendered immune to interrogation from without, a sleight of hand I find deeply disturbing intellectually. Yet, Barton is inconsistent in another regard. At bottom, he appears to be unaware of the tension between the claim that the definition of ethnomathematics is culturally specific and the claim that part of its purpose is to challenge the universal (or transcultural) nature of mathematics. Is this purpose also culturally specific? Moreover, the verdict that a universal definition is not possible presumably has universal purchase. Or is it culturally specific, too? Barton is silent on these issues. Finally, if it is correct that ethnomathematics is “more like anthropology or history”, it is unclear how its definition can be culturally specific. Barton’s is a theoretical definition and, as such, is subject to transcultural evaluation. A theoretical definition can be seen to be more or less useful – and can also be out-

right incorrect, unlike a stipulative definition. The assertion that the practice described by ethnomathematics is culturally specific makes good sense. However, Barton adds,

If the practice of ethnomathematics is carried out with integrity, there will be cognisance of those aspects of the practices and concepts which are other-culture based and which may not, initially, be considered mathematics. (p. 217)

Yet, who judges whether “the practice of ethnomathematics” is “carried out with integrity”, and on what grounds? Will this not also involve a culturally specific judgement and/or set of criteria? It would appear that those who are universalists (or transculturalists) about definition and judgement find themselves on logically more compelling grounds.

His definition, Barton says, ‘implies two senses in which mathematics is universal, and two senses in which it is relative’ (p. 218). I take the former two to be uncontroversial, indeed commonsensical, and will not discuss them here. My focus, rather, will be on the alleged senses in which mathematics is relative.

Regarding the first of these, Barton says that

mathematics must be changing. This change needs to be more than just an evolutionary building on what has gone before, it must be revolutionary. (p. 218)

There are well-documented problems with this kind of relativism regarding (revolutionary) change, in mathematics as elsewhere (see Horsthemke, 2004b, pp. 575, 576). To provide just one example, if successive or ‘alternative’ paradigms are incommensurable, then a new or ‘other’ paradigm cannot be established to be superior. Barton writes,

Ethnomathematics must admit the possibility of other mathematical concepts which are not subsumable by existing ones, or by some new, overarching generalization. This is not to say that all ethnomathematical study will generate alternative mathematics. What is necessary is the idea that it could happen: that new ideas could transform the way mathematics is conceived. (p. 218)

And presumably still be able to call it mathematics (see Barton’s point that mathematics “exists as a knowledge category” (p. 218)). It follows from this that there is no such thing as ‘alternative mathematics’: it either is or is not ‘mathematics’ proper. (This will become clearer in what follows.)

Regarding the second sense in which mathematics is relative,

there must be a recognition that mathematics is not the only way to see the world, nor is it the only way to see those aspects of the world commonly referred to as mathematical, i.e. having to do with number, shape and relationships. What is more, there needs to be a recognition that alternative ways of seeing these phenomena are legitimate and valid. For if they are not legitimate, then there will be no point in trying to study them, there would only be point in trying to “educate” those who do not see it in the “correct” way. (p. 219)

First, this is no argument for seeing alternative ways (of seeing aspects of the world commonly referred to as mathe-

mathematical) as legitimate or valid. Indeed, one might simply acknowledge that, on epistemological and truth-functional grounds, they are not legitimate and that there is no point in studying them, other than as anthropological curiosities (like numerology, and the like). Second, reference to “alternative ways of seeing” seems to be misconceived. If a particular alternative procedure can be shown before “the community of mathematicians” (Barton, 2004, p. 23) to work, then it is not called alternative anymore. It is just mathematics.

Barton claims that the

use of ethnomathematics as a theoretical tool can be seen as a practical way of acknowledging the reflexivity of [the] relativistic viewpoint: it is the differing conceptions of the field which make it a valuable tool in on-going political and educational debates. We acknowledge that our own conceptions are context-derived, but use that knowledge to continue our work. (1999a, p. 34)

Here, as elsewhere, he appears to be unaware of the tension between the ‘relativistic viewpoint’ and universal knowledge claims. He writes, “It is an assumption of ethnomathematics that thinking about quantity, relationships and space may vary between cultural groups” (1999a, p. 34). Okay – but this does not, indeed cannot, mean that the various views are all equally valid, or that they indicate fundamental differences in mathematical orientation. Barton’s final point may indicate a somewhat conciliatory approach, but it contains the mistaken assumption that ethnomathematics is a unique and distinct “field of knowledge”.

Adam *et al.* write that the “political perspective on ethnomathematics [...] is now more often related to writing on indigenous knowledge”, before claiming that “[p]rivileging some peoples’ ideas in the discourse of mathematics while denying others’ is colonialism” (2003, p. 328). “Such views”, according to the authors,

justify the need for indigenous mathematicians to engage in ethnomathematics because indigenous knowledge and value systems are under attack. Decolonisation involves reclaiming, protecting and valuing the unique ways of indigenous knowing and doing [...] Indigenous mathematicians engage in ethnomathematics because we know that our peoples have complex knowledge systems that are valuable and could teach the outside world *alternative ways of knowing*. (2003, pp. 328, 329; emphasis mine)

The idea of indigenous ways of doing being unique is certainly plausible. However, apart from relying on the unwarranted assumption that ‘indigenous knowledge’ is an unproblematic notion, this perception is mistaken in a further respect. If anything, ‘privileging’ or ‘denying’ views happens on the basis of the respective knowledge or truth content, not on the basis of who holds them. Moreover, if these are genuine “ways of knowing”, then they would no longer be alternative. They would be part and parcel of ‘knowing’ as such. If Adam *et al.* wish to preserve the qualification ‘alternative’, then what they are referring to, presumably, is not ‘knowing’ but ‘believing’. When the

authors assert that ‘alternative systems of relationships and their meanings [...] are important to the growth of mathematical knowledge’, they take this to indicate not that ‘another world is still possible for indigenous people’ but that such a world ‘already exists’ (2003, p. 329). Does this mean that all indigenous worldviews, however ill-founded, are equally valid?

According to Adam *et al.*, the

political perspective is just one of many in the ethnomathematical field. The philosophical issues are also far-reaching and widely debated. To name just one example, the question of mathematical relativity is implied by ethnomathematics and needs justification. (2003, p. 329)

The authors bring this perspective to bear also on their discussion of ‘rationality’:

Greek rationality is only one form of rationality, and [...] the particular form of mathematics that traces its trajectory through a Greek tradition (and a few others) serves particular functions and has particular consequences. [...] However, to use this particular form of mathematics as the standard by which mathematics is to be judged misses the point. (2003, p. 330)

What point?, one might ask. Clearly, in the absence of any kind of argument in favour of (for example) culture-specific rationality, the authors’ claim here amounts to little more than bald assertion. Moreover, the claim that there are several (equally valid) forms of rationality renders it impossible to evaluate competing knowledge claims. Even more seriously, any kind of behaviour or worldview could be accounted for and rendered immune to condemnation, in terms of employing or engaging a “different form of rationality”.

Barton explains that

[e]thnomathematics has its focus *firmly fixed* on mathematical knowledge – its aim is the illumination of this knowledge, its methods are to expand the ambit of what can be legitimately regarded as mathematics, *by including* mathematical practices and systems wherever they occur, and, in particular, where they occur in specific contexts. (2004, p. 22; emphasis mine)

Barton’s initial emphasis of the distinction between mathematical knowledge and practical knowledge notwithstanding, this statement exemplifies the basic conceptual confusion underlying the defence of both ethnomathematics and indigenous knowledge, namely what is in the final analysis a conflation of theoretical and practical knowledge.

Regarding the question, “How does ethnomathematics extend mathematical knowledge?”, Barton points to “some examples of direct contributions from culturally specific knowledge to the general body of conventional mathematics” (2004, p. 23; as an example, he cites Ascher, 2002). I suggest that the idea of “culturally specific knowledge” makes sense only with regard to practical knowledge or “mathematical practices” – but not when it is taken to refer to theoretical (mathematical) knowledge. Theoretical, factual or propositional knowledge cannot be culturally specific or relative. Neither can truth. Mathematical truths hold tran-

sculturally. My hunch is that when ethnomathematicians and indigenous knowledge apologists speak of culturally specific knowledge or of truth being relative, they are actually referring either to practices or to beliefs.

An evaluation of the research around and application of so-called 'alternative' mathematics is necessarily and correctly conducted against the background of 'formal, academic' mathematics. Having said this, I do not share Rowlands and Carson's view that

[the] conversation [between critics and defenders of ethnomathematics, [...] in addition to purely mathematical issues, [...] involves questions of historical injury and contemporary relationships between *cultural groups whose values are incommensurable*. (2004, p. 329; emphasis added)

In fact, I would suggest that it is precisely the pernicious cultural and ethical relativism invoked here that would make 'conversation' impossible. On the contrary, I wish to argue – and my recent ICEm-3 experience strongly bears this out – that the degree of convergence between values and priorities is striking and that, despite some historical and cultural divergence in approaches, there is a common commitment to discussion and argument, as well as to standards of reasoning about matters that concern us most. [2]

Notes

[1] Taken from Ubiratan D'Ambrosio's (2006) powerpoint presentation 'Ethnomathematics: the scenario 30 years after', plenary presentation, *Third International Conference on Ethnomathematics (ICEm-3): Cultural Connections and Mathematical Manipulations*, Auckland, New Zealand.

[2] On this note, I wish to thank Bill Barton and his team for making ICEm-3 the intellectual and human success it was. My heartfelt thanks go to all the delegates with whom I had personal conversations, especially Gelsa Knjnik, Ubiratan d'Ambrosio, Willy Alanguí, Bill Barton, Ivan Reilly and to my dear friend and colleague Marc Schäfer.

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[These references follow from page 45 of the article "Leveraging epistemological diversity through computer-based argumentation in the domain of probability" that starts on page 39 (ed.)]

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