WHAT SORT OF SCIENCE IS DIDACTICS?

BRENT DAVIS

> Can didactics say how to teach? Should didactics be a normative science?

Gascón and Nicolás (2017) present these queries as provocative, shorthand versions of a question that they posed to a small group of prominent mathematics education researchers:

> To what extent, how, under which conditions, can (or must) didactics set value judgments and normative prescriptions in order to provide criteria about how to organize and manage study processes?

In this commentary, I discuss my efforts to make sense of both these questions and their analyses of the responses that they received. I qualify my remarks by confessing that my first and lingering response to their article was that either such questions have simple and obvious responses or they are patently unanswerable. I lean in the direction of the latter.

Let me start by offering some definitions, a step made necessary because neither 'didactics' nor 'normative science' are common terms in my immediate discourse communities (comprising mainly English-speaking North Americans). Regarding *didactics*, based on what I have read and heard, I understand it as a domain that encompasses those insights and inquiries intended to inform the broad spectrum of a mathematics teacher's responsibilities, including both practical knowledge (*e.g.,* regarding lesson design, teaching approaches, and assessment practices) and conceptual knowledge (*e.g.,* of mathematics, human development, and cognition).

As for *normative science,* this phrase was originally employed as a contrast to *descriptive science.* It has been around since at least the mid-1800s, and the underlying distinction is much older, as signaled by Kant in the late 1700s:

> Two things fill the mind with ever new and increasing admiration and awe, the more often and steadily we reflect upon them: *the starry heavens* above me and the *moral law* within me. (5:161.33, from Guyer, 1992; my emphasis)

Kant's choices of topics here were not random. In the centuries before he composed this sentence, the starry heavens shifted in status from a main source of mystic enlightenment to an exemplar of the descriptive sciences in their mechanical predictability. In contrast, moral law had become an exemplar of normative science as it shifted from a set of rules assumed to be inscribed in the universe to a subject confounded by personal, social, and cultural interests.

Those reversals were complete a century ago, evidenced in the way Sabine (1912) characterized the distinction between normative sciences and descriptive sciences:

> It is quite natural […] that ethics and other sciences that imply valuation should have been called normative, and that the sciences which approach their subject matter with a more disinterested attitude should have been called descriptive. (p. 433)

In brief, then, whereas a descriptive science was originally seen to be focused on *the way things are*, the normative sciences were characterized as concerned with *the way things should be*—and the latter were assumed to rely on systems of beliefs and values to orient their conclusions. Such qualities are obvious aspects of didactics and so, if appropriately construed as a science at all, didactics *must be* a normative one. More than a half century of critical scholarship in education has driven home that point. But I believe matters to be more subtle—given, for example, that scholars including Kuhn (1962) and Lakatos (1976) have demonstrated that normative forces are at work in the descriptive sciences. As such, in order to engage with the questions posed by Gascón and Nicolás, it would seem useful to look first at more recent and nuanced distinctions among types of science. I will do so rather expeditiously, as my purpose is to utilize rather than explore these distinctions.

Types of descriptive science

The phrase 'Scientific Revolution' is commonly used to refer to the rise of descriptive science in the modern era. It is often thought to refer to a singular historical turn in sensibilities. However, as was noted long ago (*e.g.,* Dewey, 1910), the assumption of a lone tectonic shift is incorrect. There has been more than one scientific revolution over recent centuries.

To appreciate this point, it is useful to consider the original meaning of the word *science*, which is derived from the same Indo-European root as *scissors, incisor, shed,* and *schism*. *Science* originally had to do with separating, splitting, and categorizing—that is, with parsing up, classifying, and labeling all forms and phenomena. Initially an entirely descriptive enterprise, science was focused on discerning the universe's natural order. Consistent with beliefs carried forward from various mystical, religious, and philosophical systems, this natural order was assumed to be fixed. It is not surprising, then, that the paradigms of early science featured lines and rectangles—including, for example, arrows to represent cause and effect relationships and grids to lay out typologies and taxonomies. The Linnaean taxonomy, developed in the 1730s to classify animals, plants, and minerals is an exemplar of this paradigm. Directional lines and rectangular grids were so well fitted to the scientific project of the times that they still serve as paradigms for the physical sciences.

Although rectangulated charts were originally developed to foreground distinctions, as observations became more subtle, attentions shifted from the delineation of differences to the mechanics of differentiation. This shift in emphasis began to unfold in the 1700s and by the mid-1800s emerged as a full-blown science of evolution. As Dewey (1910) recounted, by the end of the 1800s, evolutions of forms (versus distinctions among forms) became the dominant foci among almost all the descriptive sciences, as the paradigm of the distinction-highlighting rectangular grid gave way to a completely different scientific sensibility, illustrated in the ascendant paradigm of the relationship-tracing tree (Lima, 2014). This device continues to be a prominent paradigm in biology and its subfields.

Another shift of equal magnitude unfolded through the latter half of the 1900s with the rise of complexity science. As yet there is no unified definition of 'complexity', mainly because formulations tend to be anchored to studies of specific phenomena. One thus finds quite focused definitions in such fields as mathematics and software engineering, more indistinct meanings in chemistry and biology, and quite flexible interpretations in the social sciences (*cf*. Mitchell, 2009). Efforts toward a coherent, unified description of complexity revolve around common qualities across diverse research foci, and such terms as *emergent* and *self-organizing* are prominent. As for images, the decentralized network is a paradigmatic characterization of complex systems (Watts, 2004; Lima, 2011).

Other emergences in the descriptive sciences could be cited, but these three (and their paradigms)—that is, *physical sciences* (directional lines and rectangular grids), *life sciences* (bifurcating trees), and *complexity sciences* (decentralized networks)—suffice for my present purpose of situating Gascón and Nicolás's commentary within the current landscape of the sciences.

Types of normative science

Early characterizations of normative science tended to be denigrating, at least in relation to descriptive sciences. The normative sciences were cast as values based, policy driven, and context specific, leading to *created* laws that were very different from the *discovered* ones drawn from assumed-to-be unbiased, fully confirmable and universal facts of the descriptive sciences.

The disparagement of normative sciences has diminished greatly over the past century, in large part because of pragmatic insights generated by those sciences. In particular, the *human sciences* and the *social sciences*, by far the most prominent branches of the normative sciences, have contributed greatly to scientific knowledge at the same time as they have helped undermine confidence in the descriptive sciences. Since their emergence in the early-20th century, the human sciences have helped to unsettle confidence in the descriptive sciences by shifting the focus from the *described* to the *describer*. That is, by providing insight into human perception and consciousness, the human sciences have demonstrated that human knowledge is not only riddled with bias, it actually relies on bias. In a parallel manner, with the emergence of the social sciences in the mid-20th century, the discursive and cultural dimensions of descriptive science have been drawn into clearer focus.

The line between the human sciences and the social sciences is not a sharp one. Psychology, sociology, and anthropology tend to be included on both of their lists of disciplines encompassed. But they do have different inclinations. The human sciences tip strongly toward studies of *persons* (*e.g.,* perception, consciousness, and identity), and the root metaphors and prevailing paradigms of the human sciences tend toward the organic, origin-conscious, and possibility-seeking tropes that are common in the life sciences. In contrast, the social sciences lean in the direction of studying *peoples* (*e.g.,* collective dynamics, ethics, culture), and so the prevailing metaphors and paradigms tend to revolve around such notions as contracts and participation. Consequently, extended lists of the human sciences typically include philosophy, neuroscience, and phenomenology (among others), and extended lists of the social sciences typically comprise history, economics, law, linguistics, and political science (among others). Paradigmatically, this difference commonly appears in icons developed around stylized unities (*e.g.,* trees and individuals, tipping toward the human sciences) and icons developed around groups of people clustered around a shared interest (*i.e.,* flagging the more collective foci of the social sciences).

This distinction is critical to the current discussion. I read in Gascón and Nicolás's questions a strong alignment with the foci and interpretive strategies of the social sciences specifically, not the normative sciences generally. As I develop below, that alignment has limiting implications.

A different question

I return to Gascón and Nicolás's orienting question,

> To what extent, how, under which conditions, can (or must) didactics set value judgments and normative prescriptions in order to provide criteria about how to organize and manage study processes?

—or, as they abridged it, "Should didactics be a normative science?" My reasons for the above review of one way to subdivide science—and, in particular, for pointing to the more prominent paradigms associated with the resulting categorizations—are to develop means to surface and interrogate some of the assumptions about science that might be at work in Gascón and Nicolás's questions.

For me, everything about what was asked and how it was asked pulls from and pushes to the space of *peoples*. The interests and images of the social sciences are made the figure. The human sciences are pushed to the background, and the physical sciences, life sciences, and complexity sciences are rendered an un-interrogated ground. It is that move that, I think, makes the question seem like a simple one. The foci, subject matters, methods, assumptions, and intentions of didactics make it a normative science. It cannot help but be.

But the answer must change when the distinction between descriptive and normative sciences is recognized to be simplistic and outdated. Clearly, the human sciences and the social sciences are core to this realm of inquiry. But, just as clearly, elements of the physical sciences, the life sciences, and the complexity sciences are relevant to and invoked within didactics. So the more useful question is perhaps how didactics might be located in the increasingly nuanced

landscape of contemporary sciences. That is, perhaps the important question might be an expansive, outward-looking "What sort of science might didactics be?", rather than the constricting, inward-gazing "Should didactics be a normative science?". So let me try a different tack, informed by the categories of science mentioned above.

Perhaps didactics is better included among a growing set of hybrid domains that sit across the no-longer-tenable line once drawn between descriptive sciences and normative sciences. Didactics would seem, for example, to be in the company of the *cognitive sciences* and the *learning sciences* as it collects insights from psychology, philosophy, linguistics, anthropology, neuroscience, computer science, and so on—and as further evidenced by the prominence of mathematics education research in texts primarily situated in the cognitive sciences (*e.g.,* Lakoff & Núñez, 2000) and the learning sciences (*e.g.,* Sawyer, 2014).

Across these hybrid domains, one of the major bridges spanning the descriptive sciences and normative sciences is the realization that humans are not principally logical creatures, as had long been assumed by scientists and educationists alike. Rather, most learning and reasoning happens by association—in time, in space, in conceptual form, and such. In particular, with regard to the learning and teaching of mathematics, likenesses in conceptual form—that is, reasoning by analogy—is of especial importance (see Lakoff & Núñez, 2000; Davis & Renert, 2013). Oriented by this realization, the balance of this discussion is explicitly informed by two key conclusions of the cognitive sciences: firstly, as just mentioned, that analogy is a principal mode of reasoning among humans; secondly, that associative thinking proceeds not as collections of links but as ecosystems of connectivity. Kelly (2010) made the point more poetically:

> Ideas never stand alone. They come woven in a web of auxiliary ideas, consequential notions, supporting concepts, foundational assumptions, side effects, and logical consequences and a cascade of subsequent possibilities. Ideas fly in flocks. To hold one idea in mind means to hold a cloud of them. (pp. 44–45)

Such thinking has been central to my research for the past few decades, through which I have worked to decipher some of the diverse flocks of association at work in formal education (see Davis 2004; 2017; Davis, Sumara, & Luce-Kapler, 2015). For the purposes of this writing I have extended the analysis to different sorts of science, as summarized in Table 1 [1]. Each column of the table is meant to present a distinct flock of associations, affording form and coherence across prevailing cultural sensibilities and emergent types of inquiry/science.

With reference to Table 1, I take the normatively phrased question, "Should didactics be a normative science?" as roughly equivalent to "Does didactics reside in the human and social sciences?"—and this is how and where I come to a different answer to the seemingly obvious "Yes." As I look across the foci of inquiry, the research lenses, and the varied criteria for claims to truth (*i.e.,* the bottom rows of Table 1), it seems to me that almost every journal article I have read and every conference session I have attended has drawn on and advanced the sensibilities represented across all four columns of Table 1. Further, acknowledging that what I am about to suggest could well be skewed by my own narrow interests, the late-20th-century dominance of the human and social sciences in mathematics education research is eroding ever faster. As tools and techniques are made available to us, many emphases and insights of the empirical sciences seem to be more and more relevant and useful.

On the topic of this growing embrace of the empirical sciences, and as already intimated, as I examine Gascón and Nicolás's article for explicit emphases and strategies of interpretation alongside its implicit metaphors and implicit structures of inference, I interpret that their thinking most strongly aligns with the cluster of associations in the Social Sciences column, which in turn preconditions their phrasings of questions and their readings of others' work to be similarly located. (I favor Human Sciences and Complexity Sciences columns, and have made a career of deliberately invoking the latter to interpret educational phenomena.) Stated more bluntly, to my reading, it feels that Gascón and Nicolás are almost engaging with a tautology as they pose questions, make interpretations, and mount arguments in terms drawn exclusively from the social sciences, and then use those to answer a question about value judgments and normative prescriptions.

That said, they do present at least one escape route from this circular thinking—namely turning to the question of pragmatics. That takes me to another of Gascón and Nicolás's questions, mentioned first in their title.

Can didactics say how to teach?

I take this query as a paraphrase of Gascón and Nicolás's reference to "criteria about how to organize and manage the study process" in the longer version of their orienting question. The rephrased question involves an interesting rhetorical move, as it positions *didactics* rather than *teaching* as the site of contestation.

This sleight of hand matters because 'teaching' is an entirely problematic and unsettled notion, as evidenced in the fact that it has literally hundreds of synonyms in English. Based on much more extensive traces of the mystical, religious, rationalist, empiricist, structuralist, post-structuralist, and other roots of many of these synonyms (Davis, 2004) along with their respective flocks of association that lend coherence to notions of knowledge, learning, and such (Davis *et al.,* 2015), I have contrasted some of the major modern (and some pre-modern) discourses on education in Table 2. As might be inferred from the Emergent Metaphors of Teaching row of that table, the practical and conceptual entailments of various synonyms diverge dramatically. By implication, we might expect that the question "Can didactics say how to teach?" will evoke quite different responses when posed to adherents of different sensibilities.

That is precisely what happens, revealing that the real question here is not whether didactics can or should say anything about teaching, but what the interlocutors might assume about the natures of knowledge, learning, and teaching. It is around these issues that I get myself into some frustrating tangles with Gascón and Nicolás's article: they

Table 1: An overview of four major paradigms in modern science. (In this table, I follow a convention in the cognitive science literature in the use of small caps to signify metaphors.)

Frame		Physical Sciences	Life & Human Sciences	Social Sciences	Complexity Sciences
Cultural Context	Rise to Prominence	1700s	late-1800s	mid-1900s	late-1900s
	Prevailing Philosophy [2]	Humanism	Liberal Humanism	Socialist Humanism	Transhumanism
	Influential Domains	Physics & Industry	Biology & Structuralism	Sociology & Economics	Ecology & Systems Theory
	Prominent Notions and Paradigms	MECHANICAL; DIRECTIONAL	ORGANIC; BRANCHING	CONTRACTUAL; COLLABORATIVE	ECOSYSTEMIC; NETWORKED
Prominent Attitudes in Educational Research	Main Foci of Inquiries	THINGS (*e.g.,* brain function, visible and measurable behavior, competencies/skills)	PERSONS (*e.g.,* attitudes, self-identifications, perceptions, understandings)	PEOPLES (*e.g.,* shared values, meanings, ethics, collective processes; traditions)	SYSTEMS (*e.g.,*natural, economic, cultural, technological communicational, epistemic ecosystems)
	Research Lens	Empiricist—FINDING through, *e.g.,* randomized controlled experiment	Interpretive—INFERRING through, *e.g.,* teaching experiment	Participatory—CO-CREATING through, *e.g.,* critical action research	Emergentist—CO-EMERGING through, *e.g.,* design-based research
	Criteria for Truth Claims	valid, reliable, rigorous, replicable, generalizable	resonant, enabling, reasonable, useful, relevant, viable	corroboratable, just, shared, ethical, awareness-raising	modellable, mimicable, robust/healthy, fitting, situationally coupled

Table 2: A selection of contemporary metaphors of teaching, organized by their historical-conceptual roots (based on Davis, 2004 and Davis, Sumara, & Luce-Kapler, 2015).

Educational Sensibility	Scholasticism	Standardized ('Traditional')	Authentic ('Reform')	Democratic Citizenship	Systemic Sustainability
Rise to Prominence	1000 CE	1600	1900	mid-1900s	late-1900s
Influential Discourses	Platonism & Christianity	Physics & Industry	Biology & Structuralism	Sociology & Economics	Ecology & Systems Theory
Emergent Metaphors of Teaching	NURTURING FOSTERING DISCIPLINING INDOCTRINATING	INSTRUCTING INFORMING DELIVERING	FACILITATING GUIDING	LIBERATING EMPOWERING CONSCIENTIZING	DESIGNING ENGAGING CHALLENGING

do not say what they mean by 'teach'. In consequence, I can render their analyses as wrong, nonsensical, or correct, depending on what I imagine teaching to be.

In North America, we witness this point regularly through persistent invocations of the 'Math Wars', an ongoing public dispute between those who wish to preserve more traditional/standardized sensibilities and practices in school mathematics and those who advocate for more reform/authentic approaches. As might be expected, traditionalists press for curricula, teaching methods, and teacher education models that focus on well-delineated topics, clear explanations, and objective measures. That is, they would likely imagine 'didactics' to be a descriptive science, one that is informed by sound observations, comparisons, and measurements. Thus, to the question, the traditionalist/standardized crowd would answer the question, "Can didactics say how to teach?", with something along the lines of, "What's the point of it if it can't?".

In contrast, reformists tend to be more modest in their answers, attentive to the conviction that mathematics learning is dependent on, but not determined by mathematics teaching. As this discourse strips causality from the teacher/learner dynamic, it leans toward the conclusion that didactics cannot actually say how to teach in any prescriptive sense. But it does offer some proscriptive advice, cautioning teachers from actions that might be anchored in assumptions of causality. That is, its answer to the question, "Can didactics say how to teach?", would be something like, "No, it can't tell the teacher what to do, but it can inform the teacher of what can't be done.".

The preceding two paragraphs cover responses that are likely to be the most coherent within the sensibilities signaled in the Standardized Education (*i.e.,* "Yes.") and Authentic Education (*i.e.,* "Not in any causal sense.") columns of Table 2, and I have already mentioned one common and fitting response for the Democratic Citizenship Education column. Within that sensibility, there is an ethical obligation to be open about advice, explicit on intentions, critical of entrenched assumptions, and cautious of possible effects. I might rephrase that answer as, "Didactics *must* attempt to offer pragmatic suggestions on how to teach while attending to cultural realities and social consequences of those suggestions." (I believe this response would also come close to fitting a Systemic Sustainability Education sensibility, with a minor edit of attending to ecological and other ecosystemic concerns.)

Where do Gascón and Nicolás land? Using this rubric, my strong sense is that they straddle the tension that appears between an Authentic Education sensibility and a Democratic Citizenship Education sensibility. The first pulls them in the direction of a qualified, descriptive "No, it can't," and the second pulls toward an ethical, normative "Yes, it must." One need not dig deeply into their data and their interpretations to encounter this tension. In fact, the conflict is already evident in their orienting questions.

To recap then, making any sort of headway on questions such as "Can didactics say how to teach?" and "Should didactics be a normative science?" is only going to be possible by rendering opaque what has slipped into transparency. That is, the critical issues are not the definitions of *didactics* and *normative*, as I first assumed when I read these questions, but the meanings of *teach* and *science*. What is it that we imagine we're doing when we are teaching mathematics and when we are studying mathematics learning? Until we are explicit in those answers—that is, until we have settled for ourselves whether teachers are nurturing, instructing, delivering, guiding, facilitating, enculturating, and/or whatever, and whether researchers are finding, inferring, co-creating, co-emerging, and/or whatever—we will get tautological responses to whatever questions we ask of didactics. The answers would not be answers at all, but echoes of unstated assumptions.

And so?

For the past seven years, I have been leading a partnership of researchers, school districts, a not-for-profit publisher, and business-based funders in a longitudinal study of mathematics teaching. Because our financial supporters afforded us the luxury of time, we have deliberately been slow to publish our findings and interpretations. This is not the place to go into detail, but the team's foci are informed by theoretical and research literatures of not just education, but of neuroscience, psychology, sociology, mathematics, and other domains. We have striven to be as comprehensive as possible in excavating assumptions, challenging our orthodoxies, and accumulating evidence for our suspicions.

While we are assured by our results and confident of our message, we are holding back on 'going public' until completing the current phase of the project, focused on developing and testing strategies to scale up. Seeking additional partners in this stage, some months ago we invited decision makers from all nearby school districts to review our results and examine our models. Wanting neither to misrepresent the data nor to understate the project's underpinnings, we devoted a full day to the session.

The invitation was met with enthusiasm, albeit qualified and cautious. We have since heard from multiple districts indicating varied levels of interest. Recently, one of these districts sent an information-gathering contingent to one of the schools currently involved in the project. These curriculum leaders sat in on a few lessons and, based on their very limited observations, decided their district would not join us. Why? They saw no small group work, and some conceptual elements of the lessons seemed 'reductive' and 'procedural'. Moreover, not all the young children could 'explain their thinking' when asked, cold, by a stranger to do so.

It was a frustrating moment. Years of data that demonstrates robust and sustained impacts and that is supported by comprehensive reviews of substantial literatures were allowed to be eclipsed by minutes of superficial observation that were framed by expectations of group process and rich inquiry.

To be clear, my frustration in this circumstance was not with the observers, but with a professional literature that is riddled with near-useless and disconnected soundbites about mindset, cognitive load, learning styles, collectivity, teacher knowledge, guided discovery, problem solving, and whatever other obsessions are current—notwithstanding that the evidence bases of many of these emergent orthodoxies are often somewhat dubious. And so my point is not that these

curriculum leaders should have just trusted what we had to say, but that the profession (at least where I live) does not seem inclined—much less obliged—to distinguish between popular rhetoric and robust science.

In other words, it seems that the only results of educational research that reach the audience they are intended to reach are those fragments that resonate with the sensibilities of the moment. Where I live, for example, if a research claim does not incorporate personal strategies or collective process, or if it dares to recommend a routinized procedure or teacher-centered practice, it is dismissed as regressive and uninformed. I am troubled by this realization, and I worry the problem is not that educational research (*i.e.,* didactics) is poorly understood, but because educational research is, in the main, not a science. This point is revealed in the mere fact that the field can be caught up in wondering whether didactics should be a normative science or whether didactics can say how to teach. Such questions are hardly reflective of a domain devoted to scientific knowledge.

To re-emphasize, as I would hope is clear from my opening discussion of various sorts of science, I am not using 'scientific knowledge' to refer to the 16th-century notion of inert, knower-independent truths that are inscribed in the universe. Rather, scientific knowledge is a moving form that evolves by re(de)fining its terms as precisely as it can and by rendering its assumptions as explicit as possible—fully aware that neither task can ever be finished. Definitions will continue to develop; assumptions will continue to be detected. However, when these requirements are not met—when, for example, someone engages in debate on the scientific character of didactics without defining constructs as fundamental as *teach* and without interrogating assumptions as fundamental as interlocutors' conceptual frames—the discussion seems less than scientific.

What, then, is the field's obligation to be scientific? What sort of science might didactics be? These are vital questions, and my sense is that the field's relevance in upcoming years will be hinged to its ability to engage them seriously. Such engagement will involve reviews of the many phenomena and issues that mathematics education researchers must study, alongside analyses of the sorts of science their study entails. Even a hasty scan will reveal that these issues and phenomena span research in the physical sciences, the life sciences, the human sciences, the social sciences, and the complexity sciences.

The insights of this hybrid science are both descriptive and normative, as they must be. After all, scientific research is a not a contest between ideas and ideologies, but a conversation among them. As flagged in Table 1, each coherent realm of science organizes itself around its own construct of truth. In consequence, each has a particular relationship to the pragmatic obligations of didactics. I cannot help but think that this added layer—that is, of the pragmatic obligations of didactics, specifically, and formal education, generally—renders educational research more complex than many other hybrid sciences. And this may be why I back off from questions that, to my ear, seek to reduce the complexity of didactics. I would argue that the domain must consciously and continuously situate itself among domains of inquiry, and this responsibility can only be accomplished by attending to the distinct characters of different scientific domains. To me, to wonder about didactics' relationship to a singular domain—and, more troublesome, to frame that wondering in terms of the foci and anxieties of just that domain—is antithetical to contemporary trends in research.

Notes

[1] I recognize the irony in presenting this information in a distinction-amplifying table against early remarks of newer paradigms for scientific inquiry. To that end, in Davis (2004) I present a bifurcation-based tracing of similar discussions.

[2] See Harari (2014) for concise descriptions of and cultural-historical details surrounding these types of humanism.

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