The Social Ideologies of School Mathematics Applications: a Case Study of Elementary School Textbooks

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1. The mapping of mathematical constructs to real-world situations

It can be maintained that any mathematical construct is defined in the context of a mathematical theory - a set of axioms and a number of deduced theorems - in which it is embedded. And as a consequence, the propositions of a mathematical theory function as meaning postulates for any construct embedded in that theory. Thus, any mathematical construct acquires its specific mathematical meaning from a particular mathematical theory.

The construct of addition, for example, is defined within a theory of numbers founded on Peano's axioms by the following two propositions: $a + 0 = a$ and $a + f(x) = f(a + x)$, where $f(x)$ is the successor of the number x. These two propositions on the one hand define generally the sum of any two numbers, and on the other assert certain propositions - usually called 'properties' or 'laws' - about the addition of numbers. That is, they define what is meant by the arithmetical operation of addition according to that theory: addition is an operation on numbers that satisfies these two propositions. This is the specific mathematical meaning of addition within that mathematical theory.

Mathematical constructs, however, just like many other mental constructs, are used to describe, or are 'applied to' as it has come to be called, real-world situations. Any such mathematical description of, or any such mathematical application to, any real-world situation may be considered to be a micro-theory of that particular aspect of the real world, in which case any implicated mathematical statement may be interpreted as a statement about the aspect of the real world concerned.

The mathematical construct of addition on integers, for instance, may be applied to and considered as a reasonable micro-theory of simple financial transactions, describing income or credit by positive integers and expense or debt by negative integers. In such a case, any mathematical statement implied by that particular application of addition to a particular situation of financial transactions may be interpreted as a statement about that particular aspect of the social reality.

A description of a real-world situation in terms of a mathematical construct or an application of a mathematical construct to a real-world situation is set up on the basis of a mapping between that construct and the real-world situation. Every such mapping, though, is indispensably mediated both by a class of non-mathematical concepts that circumscribes the class of real-world situations to which the mathematical construct is applied and by a set of associated linguistic and more generally symbolic expressions that signify these concepts.

The non-mathematical concepts associated with their symbolic expressions assign another non-mathematical meaning and simultaneously specify a referent for the mathematical construct within the particular description or application. This meaning may be considered as constituting the applicational or the referential meaning of the mathematical construct in the real world. The mathematical construct of addition, as an example, may be used to describe or may be applied to a class of real-world situations circumscribed by concepts of change, combination or comparison.

Particular instances of these concepts (e.g., the growth of a particular quantity as an instance of change, the union of two particular quantities as an instance of combination, or the difference of two particular quantities as an instance of comparison), in conjunction with their signifying symbolic-linguistic expressions, attribute to the applied mathematical construct of addition another meaning differentiated from its mathematical one, specifying at the same time particular referents of addition within its particular applications to particular real-world situations. Any one such meaning constitutes an applicational or referential meaning of the mathematical construct of addition on numbers.

The referential meanings assigned to the mathematical construct of addition by different instances of the concepts of change, combination, or comparison are not, however, identical in every aspect. When addition, for example, is applied to and interpreted as describing the growth process of a population group over a given time interval, the first addend describes and refers to the population size and the second addend to its increment during the given time interval. As a consequence, the addends in this case are not actually interchangeable, since the growth of a population size X by an increment Y is not the same process as the growth of a population size Y by an increment X, the rates of growth, for instance, not being identical. This means that in such a case addition cannot be commutative, although the mathematical construct of addition, by definition, is.

In contrast, when addition is applied to and interpreted

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as describing the process of uniting two population groups, the first addend describes and refers to the size of either one and the second addend to the size of the other population group. In such a case, the process of uniting a population size X with a population size Y is identical to uniting Y with X: the addends may thus be interchanged and the addition can be commutative, compatible with its mathematical definition. Therefore, a particular meaning of the growth or of the union concept specifies how the applied mathematical construct of addition refers to and acquires meaning by a particular mapping to a particular real-world situation.

In summary, it may be held that any mathematical construct can acquire multiple referential meanings beyond its specific mathematical meaning, as it is assigned several different mappings in several different applications to several different real-world situations.

2. Real-world situations utilised in school mathematics applications

Adopting the view that real-world situations acquire their meanings through the implicated human activities which are always meaningful, since intentional, the position may be defended that any real-world situation and its representation bear meanings that are never value-free and so are not ideologically neutral. Going one step further, it may be claimed that the selected real-world situations, and consequently the associated referents of the mathematical constructs to those specific aspects of the real world which are used as examples, applications, questions, or problems-to-be-solved in the teaching of school mathematics, are never value-free and bear - in any feasible case - a more or less definite, even though not always clear, ideological orientation. They thus assign to the mathematical constructs corresponding ideologically-orientated referential meanings.

The ideological orientations of the referential meanings assigned to the mathematical construct of addition, for instance, when applied to and interpreted as describing a growth process of profit in a situation of commercial dealings, or a growth process of nuclear waste in a situation of environmental pollution, are not the same. The two situations highlight different aspects of human activity, and implicitly emphasise different attitudes and patterns of thinking towards human activities, support different life values and ultimately transmit different social ideologies.

From this point of view, school mathematics, just as is the case with many other school subjects, may not be considered as an ideologically and hence a socially neutral subject of knowledge which is derived from a conformably neutral scientific mathematical activity. It has to be conceived as a school subject composed of selected mathematical topics bearing ideologically-orientated referential meanings assigned to mathematical constructs by their selected mappings in selected applications to selected real-world situations.

In this sense, school mathematics, along with other school subjects, advocates the dominant system of values and patterns of thought in a particular society: it is used to a considerable extent as a vehicle for the indoctrination of children in a specific societal ideology.

3. The case of Greek elementary school mathematics

The mathematics curriculum for the Greek elementary school is centrally prescribed in extensive detail by the Ministry of Education. The present curriculum includes the study of the natural numbers and fractions with their four fundamental operations and the associated algorithms plus the elements of geometry and the basics of measurement. Applications in both mathematical and everyday life activities are emphasised for any topic. A unique mathematics textbook playing essentially the role of a workbook is used for each of the six elementary school grades, accompanied by a compatible teacher's book dictating in detail every teaching unit, its content, teaching method, and learning tools. Both sets of books are produced and established at a national level by a State institution. Since the contents along with the teaching methods and materials are centrally prescribed and controlled, the teaching of mathematics in the Greek elementary schools may be considered uniform in all of its main aspects.

The related question encountered in this paper is in two parts. First, what are the prominent characteristics of the real-world situations prevailing in the teaching of Greek elementary school mathematics as referents for the application of mathematical constructs to the real world? And second, what is the social ideology, if any, advocated by the meanings these referents assign to the mathematical constructs? In other words, which aspects of the real world are selected and in which patterns are they structured and nominated by the Greek elementary school as the prominent real-world objects for mathematical activity?

Passing over the presentation of detailed quantitative data, a content analysis of Greek elementary school mathematics textbooks yields the conclusion that two classes of situations are prevalent in the examples, applications, and problems-to-be-solved as referents for the mathematical constructs to the real world:

(a) *Artificial and socially indefinite situations coming out of a supposedly child-like and ostensibly abstract, natural or social world*

This world is made up of material objects (mostly playthings and pieces of furniture), plants (mostly flowers and vegetables), fruits (mostly apples and oranges) and animals (mostly birds and cats) that, situated side-by-side in collections without any meaningful real-world context, are at the immediate disposal of any mathematical activity, usually incited for its own sake by the questions "How many?" or "How much?"

As well, coins are treated as an indispensable element of this world, being indirectly attributed an almost natural existence, and their manipulation is introduced as an outstanding referent of mathematical constructs as early as in the first teaching units of the second elementary school grade.

The human beings involved in this world are vaguely indicated by ordinary personal names (e.g., George, Helen), family relatives (e.g., mother, brother), or sex identifications (e.g., a girl, a boy) who, detached from any social setting, are engaged in counting, comparing, measuring, or calculating activities which, as a rule, have no clear reason beyond mathematics itself.

This class of situations is the prevailing referent for mathematical constructs in the lower grades of the Greek elementary school, comprising the context for more than 85% of the examples, applications, and problems included in the mathematics textbooks for the first and second elementary school grades.

The number of these situations included in the elementary school mathematics textbooks gradually decreases as the school grade ascends. They are substituted for by the following type.

(b) *Financial, and especially commercial, situations devoid of any pertinent social relationships*

Buying and selling of commodities, money returns and payments, business profit and loss accounts, individual incomes and expenses, household expenditures, consumption bills, debit and credit accounts, and any other kind of relevant financial transactions, comprise the prevailing referents for the mathematical constructs in the upper grades of the elementary school. Even calculations of interest on capital are included as a distinct teaching unit in the mathematics textbooks for the sixth elementary school grade.

These transactions derive as a rule from socially abstract forms of material production and distribution or the rendering of professional services introduced by statements of the type: "A factory manufactures ...", "A farm produces ...", "A store sells ...", or "Bottles are packed ...", "Apples are sold ...", "Drinks are canned ...", to all appearances existing on their own beyond any human agency and away from any space, time, or social structure.

On the other hand, whenever persons are implicated in such situations they are either presented as socially indefinable agents (e.g. a *producer* retails ..., a *worker* earns ..., a *consumer* spends ...) or indicated by an occupational identity (e.g. a *bookseller*, a *grocer*, a *confectioner*, a *craftsman*). In this second case, the induced image of the economically-active Greek population is entirely distorted in as much as retail trading is implied to be its dominant occupational activity.

Likewise, money manipulations as referents for the mathematical constructs belong to the standard repertoire of examples, applications, and problems included in the mathematics textbooks for the upper grades of the Greek elementary school, even in quite unrealistic cases which could not be justified by any reason.

> You had 8/10 of a ten-drachma coin and spent 5/10 of it. How many tenths of the ten-drachma coin have you got left?

This problem, for example, is included in the mathematics textbook for the 4th grade as an application of decimal fractions, and such problems are not rare in the textbooks for other school grades.

The class of financial and commercial situations outlined constitutes the context of more than 70% of the examples, applications, and problems included in the mathematics textbooks for the fifth and sixth Greek elementary school grades.

In conclusion, it may be argued that the commercial market is covertly endorsed by Greek elementary school mathematics textbooks as the prominent field of mathematical activity, and, moreover, as the dominant aspect of the real social world.

4. The social ideology of real-world referents for mathematical constructs

The foregoing referents for mathematical constructs in real-world situations that are included in elementary school mathematics textbooks being apparently not exclusively dictated by the mathematical activity *per se* may not be considered as ideologically-neutral choices. The choices covertly reinforce and promote fundamental aspects of a specific ideological conception of mathematical knowledge and its real-world applications, along with a specific image of worthy modes of human activity.

The real-world situations included as referents for mathematical constructs in the Greek elementary school mathematical textbooks, and in consequence in the teaching of mathematics, are embedded either in a socially indefinite context or in a context mainly concerned with commercial transactions. These two classes of real-world situations are in their turn implicitly presented as the principal objects of mathematical activity. In addition, commercial transactions are covertly nominated as the dominant mode of human activity while commodity production and distribution are abstractly presented as impersonal and socially neutral, therefore as essentially technical activities. The background to such choices may be ascribed to the predominant ideological orientation in Greek society that, according to the relevant literature, may be presumed to be a version of middle-class mercantile ideology. These options in their turn assign ideologically conforming referential meanings to the applied mathematical constructs.

5. School mathematics and its relation to mathematical knowledge

On reflection, it may be claimed that one traditional relation between school mathematics and mathematical knowledge that formally constitutes its subject matter is the practical relation of usage. This tells us that the relation between school mathematics and its subject matter has as its prime function not the learning of the mathematical subject matter itself but rather the assimilation of rules, procedures, and approaches aimed at establishing a specific relationship between children and mathematical subject matter.

Anyway, the declared objectives of elementary mathematics education in Greece and in many other countries point to this intention in various wordings. In the endorsed-by-law Greek elementary school curriculum, for example, it is stated that:

> the broad aims of teaching mathematics are the systematic training of pupils in rational thinking and logical reasoning as well as their initiation into the deductive processes of mathematics, the advancement of their overall intellectual growth, the development of their abilities in conceiving quantities, properties, and relations, especially those that are necessary for

the comprehension and solution of real-world problems and the awareness of contemporary technological, economic, and social reality, their familiarisation in putting their thoughts into words clearly, precisely and neatly, and finally the development of their appreciation of the role that mathematics plays in different sciences.

In my words, this says that the primary purpose of mathematics teaching in Greek elementary school is the learning of rules and procedures appropriate for the use of the mathematical subject matter. The headings of the teaching units included in the mathematics textbooks for the fifth and sixth elementary school grade emphasise this fact by their phrasing, which is invariably "How to ..." or "How we ..." plus some verb implying usage, as for instance, 'write', 'compare', 'form', 'find', 'measure', 'count', 'do', 'calculate', 'check', 'distinguish', 'solve', etc

This relation of usage that the teaching of mathematics establishes with its subject matter of course contributes to the construction of mathematical knowledge. This knowledge, however, is primarily a practical knowledge concerning mathematics usage, while alongside it is the academic knowledge of the mathematical subject matter

In my view, this may be considered the essential meaning of the conceptually vague Greek term 'mathematical *paideia*', roughly translated as "mathematical enculturation"; mathematical knowledge is wholly invested in the practical knowledge of its usage. For this among other reasons, mathematics traditionally constitutes a fundamental component of any pedagogy that aims at the socio-cultural indoctrination of children Mathematics trains children towards 'the correct' modes of thinking, 'the correct' modes of deduction, 'the correct' modes of decision-making. A relation of this type between school mathematics and its subject matter may not in any case be considered as essentially a learning relation

It is above all a relation of ideological indoctrination of children, which on the back of a commonly esteemed subject matter habituates them to particular standpoints and impresses specific patterns of behaviour towards mathematics on them, aiming through the mathematics at the associated prevailing social values

To sum up, the teaching of mathematics (as with the teaching of many school subjects) incorporates a double relation with its subject matter: a *scientific relation* in as much as it is a vehicle for the theoretical knowledge of its subject matter, conjoined to an *ideological relation* in so far as it is a vehicle for practical knowledge about this subject matter – a practical knowledge that essentially refers to patterns of behaviour towards the theoretical and the social function of the subject matter being taught. In this sense, the teaching of mathematics in schools is a vehicle for mathematical knowledge itself, but is also a vehicle for an ideology concerning mathematical activity and its outcomes; that is, for an ideology dealing with mathematical knowledge based on a specific conception of the place and function of mathematical activity and its outcomes in present-day social reality.

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