

WORD PROBLEMS AS SIMULATIONS OF REAL-WORLD SITUATIONS: A PROPOSED FRAMEWORK

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The idea of including the out-of-school world in mathematics education is emphasised in policy documents in many countries (Palm, 2005; NCTM, 2000; Utbildningsstyrelsen, 1994), implying that some focus be put on real-life applications. Consequently, many teachers, textbook writers and assessment developers are struggling with the development of mathematical school tasks that resemble out-of-school situations. In many of the tasks encountered by students in school mathematics the situation described in the task, the *figurative context* (Clarke and Helme, 1998), is a situation from real life beyond school (Palm and Burman, 2004). These contextualised tasks, word problems, include both short 'dressed up' tasks and larger projects. The purpose of this article is to propose a framework for the concordance between word problems and task situations in the real world beyond the mathematics classroom.

Why do we need such a framework?

An issue raised by many researchers (and students) is the lack of realism experienced in these contextualised tasks (e.g., Boaler, 1993, 1994; Cooper and Dunne, 2000; Greer, 1993; Niss, 1992; Reusser, 1988; Verschaffel, De Corte and Lasure, 1994). The concern is that many of them are not 'real' simulations of out-of-school situations but merely ordinary school mathematics tasks 'dressed up' with an out-of-school figurative context. Due to their lack of realism they might have a negative impact on the construct validity of assessments that include such tasks as well as on students' learning, attitudes and beliefs. Support for such concerns can be found in studies by, for example, Cooper and Dunne (2000) and Boaler (1994).

There may be several reasons for this situation. Some of the word problems are, of course, not intended to emulate real-life situations but include real life objects only as support for students' thinking about concepts and models. When developing tasks intended more closely to resemble out-of-school situations, a number of other problems may occur, e.g., finding such suitable out-of-school situations may take a lot of time.

Many task developers are not certain what they would consider the constituents of a realistic task to be. Word problems can be seen as a genre "written in imitation not of life but of other word problems" (Gerofsky, 1999, p. 37). This tradition of what a word problem is makes it difficult to develop applications that carry a different set of properties. An awareness of the influence from tradition is an important insight when developing more realistic word problems.

A detailed framework describing aspects to consider in the development of tasks emulating real-life situations would be helpful.

Are there existing frameworks?

Several studies deal with the concordance between mathematical school tasks and the corresponding out-of-school situations:

Archbald and Newmann (1988): introduced the term *authenticity* into learning and assessment; described the intellectual qualities considered necessary for many significant human accomplishments beyond success in school; seven standards for tasks (including mathematical tasks) that promote "authentic achievement" such as *Problem connected to the world*,

The task asks students to address a concept, problem, or issue that is similar to one they have encountered or are likely to encounter in life beyond the classroom. (Newmann, Secada and Wehlage, 1995)

and *Audience beyond school*.

Wiggins (1993): uses the term *authenticity*; concerned with the particular mastery of "the various 'roles' and situations that competent professionals encounter in their work" (1993, p. 202); factors involved in authentic assessment (independent of school subject) include tasks being either replicas of or analogous to the kinds of problems faced by adult citizens and consumers or professionals in the field and "faithful representations of the contexts encountered in a field of study or in the real-life 'tests' of adult life" (1993, p. 206). In such representations,

formal options, constraints, and access to resources are apt rather than arbitrary. In particular, the use of excessive secrecy, limits on methods, the imposition of arbitrary deadlines or restraints on the use of resources to rethink, consult, revise, and so on – all with the aim of making testing more efficient – should be evaluated and minimized. (p. 206)

Niss (1992) and the mathematical literacy framework of the OECD's Programme for Student Assessment, PISA (Organisation for Economic Cooperation and Development, 1999): use the term *authenticity* for a link between the two worlds; focus on the figurative context, that it should truthfully describe a situation from real life that has occurred or might very well happen,

We define an authentic extra-mathematical situation as one which is embedded in a true existing practice or subject area outside mathematics, and which deals with objects, phenomena, issues, or problems that are genuine to that area and recognized as such by people working in it. (Niss, 1992, p. 353)

Alrø and Skovsmose (2002): different milieus of learning differ from one another partly due to the kind of ‘world’ the meanings of the activities carried out in the milieus may be related to; tasks may have references to pure mathematics, a semi-reality or to real life, where semi-reality is described as a world that is fully described by the text of the task and in which all measurements are exact.

Realistic Mathematics Education (RME): the concordance between tasks with an out-of-school figurative context and the corresponding real-life situations does not seem to be an important issue, rather,

the task context is suitable for mathematization – the students are able to imagine the situation or event so that they can make use of their own experiences and knowledge. (Van den Heuvel-Panhuizen, 2005, p. 3)

and for this purpose,

the fantasy world of fairy tales and even the formal world of mathematics can provide suitable contexts for a problem, as long as they are real in the students’ minds and they can experience them as real for themselves. (Van den Heuvel-Panhuizen, 2005, p. 2)

Towards a framework

When the aim is to facilitate an experience of mathematics as useful in real life beyond school or when the aim is to practise solving problems that require respecting circumstances that need to be considered in real life, fairy tales and the formal world of mathematics are not sufficient as contexts.

I am interested in developing a framework for the concordance between mathematical school tasks and situations in the real world beyond the mathematics classroom, extending the factors suggested by Wiggins. I want a detailed, fine-grained description of what constitutes *Problems connected to the world, authentic tasks* or *tasks in a milieu with real-life references*.

Before I describe the framework itself, some technical terms and ideas underpinning the framework will be described. Three short examples of word problems will be used to illustrate aspects and help exemplify how the framework can be used to analyse tasks. The use of the framework for research purposes will be exemplified at the end of the article.

Example 1 (developed for this paper): In a bakery you see a 20cm long cylinder-shaped Swiss roll. A dissection straight through the cake produces a circular shape with a diameter of 7cm. The points of time in a day when the Swiss rolls are all sold are normally distributed with mean 5.30p.m. and standard deviation 15 minutes.

a. What is the volume of the Swiss roll?

b. What is the probability that the Swiss rolls are all sold before 6.00p.m., when the bakery closes?

Example 2 (from National Pilot Mathematics Test Summer 1992, Band 1-4, paper 1, see Cooper, 1992): This is the sign in a lift at an office block:

This lift can carry up to 14 people

In the morning rush, 269 people want to go up in this lift. How many times must it go up?

Example 3 (Silver, Shapiro and Deutsch, 1993): The Clearview Little League is going to a Pirates’ game. There are 540 people, including players, coaches and parents. They will travel by bus, and each bus holds 40 people. How many buses will they need to go to the game?

Terminology and the foundation of the framework

My point of departure is that the enterprise of developing tasks that emulate real-life situations is actually a matter of *simulation*. The framework is based on the assumption that

if a performance measure is to be interpreted as relevant to “real life” performance, it must be taken under conditions representative of the stimuli and responses that occur in real life”. (Fitzpatrick and Morrison, 1971, p. 239)

Comprehensiveness refers to “the range of different aspects of the situation that are simulated”.

Criterion situations are “those in which the learning is to be applied.”

Fidelity refers to the “degree to which each aspect approximates a fair representation of that aspect in the criterion situation” (Fitzpatrick and Morrison, 1971, p. 237, 240).

Representativeness refers to the combination of comprehensiveness and fidelity (Highland, 1955, cited in Fitzpatrick and Morrison, 1971, p. 240) and will be used as the technical term for the resemblance between a school task and a real-life situation.

The framework comprises a set of aspects of real-life situations that are reasoned to be important to consider in the simulation of real-life situations. A restriction of comprehensiveness is always necessary. It is not possible to simulate all aspects involved in a situation in real life and consequently it is not possible to simulate out-of-school situations in such a way that the conditions for the solving of the task will be exactly the same in the school situation. However, the characteristics of the school tasks and the conditions under which they are to be solved can affect the magnitude of this gap, and this gap can affect the similarities in the mathematics used. The proposed aspects were chosen on the basis that a strong argument can be made that the fidelity of the simulations of these aspects clearly can have an impact on the extent to which students, when dealing with school tasks, may engage in the mathematical activities

attributed to the real situations that are simulated. The match in mathematical activities here refers not only to methods and concepts used in manipulating mathematical objects within the ‘mathematical world’ in order to obtain ‘mathematical results’ but also to the competencies required in the process of creating a mathematical model based on a situation in the ‘real world’ as well as the competencies required for interpreting the obtained mathematical results in relation to the original situation. The framework also comprises a discussion of the affective domain and its influence on the students’ engagement in simulations.

The framework: aspects of importance

The aspects of real-life situations considered to be important in their simulation (see Figure 1) are:

Event

This aspect refers to the event described in the task. In a simulation of a real-life situation it is a prerequisite that the event described in the school task has taken place or has a fair chance of taking place. For example, picking marbles from an urn and noting their colours (a common event in probability word problems) is not something people do in out-of-school life and therefore does not have a corresponding real event. The events in the examples,

- *Example 1:* a person sees a Swiss roll in a bakery
- *Example 2:* a number of people wants to go up in a lift in the morning
- *Example 3:* a number of people involved in a team will travel by bus to a game,

can be considered to have a fair chance of happening.

Question

This aspect refers to the concordance between the assignment given in the school task and in a corresponding out-of-school situation. The question in the school task being one that actu-

A. Event	F. Circumstances
B. Question	F1. Availability of external tools
C. Information/data C1. Existence C2. Realism C3. Specificity	F2. Guidance F3. Consultation and collaboration F4. Discussion opportunities F5. Time
D. Presentation D1. Mode D2. Language	F6. Consequences G. Solution requirements H. Purpose
E. Solution strategies E1. Availability E2. Experienced plausibility	H1. Purpose in the figurative context H2. Purpose in the social context

Figure 1: The aspects of real-life situations considered to be important in their simulation.

ally might be posed in the real-life event described is a prerequisite for a corresponding real-life situation to exist. The question in Example 1a is a question that probably would not be asked in the described event, while the questions in the other word problems might be. The owner of the bakery may want to know that an adequate number of Swiss rolls are baked each day. The people in the lift queue may want to know when it may be their turn and the people in Example 3 may need to know how many buses to order.

Information/data

This aspect refers to the information and data in the task and includes values, models and given conditions. It concerns the following three subaspects:

Existence: refers to the match in existence between the information available in the school task and the information available in the simulated situation. In Example 1b, the mean and standard deviation are given, which is information that would not be available in the corresponding real situation. This results in a large discrepancy between the mathematics applicable in the school situation and the mathematics applicable in the corresponding out-of-school situation.

Realism: Since students’ solution strategies are partly based on judgements of the reasonableness of their answers, and an important reference is reality (Stillman, 1998), the realism of the values given in the school tasks (in the sense of identical or very close to values in the situation that is simulated) is an aspect of importance in simulations of real-life situations. The information given in Example 2 is not very realistic since it is not likely that a building would only provide one lift that can carry 14 people when 269 people need it in the morning rush every day.

Specificity: refers to the match in specificity of the information available in the school situation and the simulated situation. This match is sometimes important for the possibilities of the students reasoning to be similar in the in- and out-of-school situations since a lack of specificity can produce a slightly different context and since strategy choice and solution success is dependent on the specific context at hand (see Baranes, Perry and Stiegler, 1989; Taylor, 1989). For example, the difference between sharing a loaf of bread and sharing a cake can make students reason differently (Taylor, 1989). In addition, if the price of a specific sort of candy is the issue in the out-of-school situation and it is not known in the school situation that the price is about this object then the students will not have the same opportunities to judge the reasonableness of their answers. Examples 1-3 provide specifications of the situations to a reasonable extent compared to the corresponding out-of-school situations.

Presentation

The aspect of task presentation refers to the way the task is conveyed to the students. This aspect is divided into two subaspects:

Mode: The mode of the task conveyance refers to, for example, if the word problem is communicated orally or in written form to the students and if the information is presented in diagrams or tables. Since, for example, all students

do not cope equally well with written communication (e.g., Newmann, 1977), and mathematical competencies required to interpret diagrams are not the same as those required to interpret tables, the simulation of this aspect can influence the mathematics required or that which is possible to use.

Language use: Impeding impact of difficult terms (Foxman, 1987; Mousley, 1990) and sentence structure and amount of text (Mousley, 1990) have been reported. Thus, in simulations, it is of importance that the language used in the school task does not negatively affect the possibilities for the students to use the same mathematics as they would have used in the situation that is simulated. The term “dissection” in Example 1a is such a term.

Solution strategies

To be simulated, a task situation includes the role and purpose of someone solving the task. This aspect is divided into two subaspects:

Availability: The availability of solution strategies concerns the match in the relevant solution strategies available to the students solving school tasks and those available to the persons described in the tasks as solving the corresponding tasks in real life beyond school. If these strategies do not match, then obviously the students do not have the same possibilities to use the same mathematics that could have been used in the simulated situation. In many word problems, of which Example 3 is one, it is not known in what role the students are solving the task.

Experienced plausibility: refers to the match in the strategies experienced as plausible for solving the task in the school situation and those experienced as plausible in the simulated situation.

For example, when a textbook section starts with a description of a particular method for solving tasks, followed by a set of tasks, this may be experienced as a request to use this method and that other methods applicable in the out-of-school situation will not apply to these tasks.

Circumstances

The circumstances under which the task is to be solved are factors in the social context (Clarke and Helme, 1998), and are divided into the following subaspects:

Availability of external tools: External tools refer to concrete tools outside the mind such as a calculator, map or ruler. The significance of this aspect may be visualised by thinking about the difference between the mathematical abilities required to calculate the monthly cost of a house loan using specially designed software (which would be used at a brokers' office) and by doing so only having available a calculator.

Guidance: refers to guidance in the form of explicit or implicit hints. Hints in school tasks such as “You can start by calculating the maximum cost”, would clearly (if not also given in the simulated situation) cause a vast difference in what the students are expected to accomplish in the two situations.

Consultation and collaboration: Tasks in real life are solved solely by oneself, through collaboration within groups or with the possibility of assistance. In simulations,

those circumstances do also have to be considered since input from other people can affect which skills and competencies are required to solve a task.

Discussion opportunities: refers to the possibilities for the students to ask about and discuss the meaning and understanding of the task. A lack of concordance between in- and out-of-school situations in this aspect can cause differences in the mathematics used since this communication has been shown to have the power of affecting the experienced meaning of the task and the solution strategies applied (Christiansen, 1997).

Time: Time pressure is known to impede task-solving success. In simulations, it is therefore important that time restrictions are such that they do not cause significant differences in the possibilities of solving the school tasks compared with the situations that are simulated.

Consequences of task solving success (or failure): Different solutions to problems can have different consequences for solvers. Pressures on the solver and their motivations for the task affect the task-solving process – an aspect to consider in simulations. This may include efforts to promote motivation for word problem solving (people in situations encountered in life beyond school are often motivated to solve those problems). It could also mean putting the products into real use. This could, for example, be done by publishing the results of a statistical survey in the local newspaper or by confronting local politicians with the results. The students could also mark the reduced prices (when working with percentages) when selling self-made products for the purpose of collecting money for people in need (which many schools in Sweden and probably in other countries do).

Solution requirements

The notion of solution is to be interpreted broadly, meaning both solution method and the final answer to a task. Judgements on the validity of answers and discussion of solution methods can be requirements for the solutions to school tasks (in textbooks and marking schemes in assessments) or by phrases in the task text (such as the phrase “using derivatives solve the following task”). In a simulation, these requirements should be consistent with what is regarded as an appropriate solution in a corresponding simulated situation.

In theory, the minimum number of times the lift in Example 2 has to go up can be determined by dividing 269 by 14 and rounding off the answer upwards to 20. However, in a real situation, a realistic assumption would be that people arrive at different points of time or that some people working on the lower floors would take the stairs, resulting in the lift going up a different number of times than 20 (a similar argument about the links to reality in this task was made by Cooper, 1992). To prevent students from being forced to think differently than they would in corresponding out-of-school situations, calculations and answers based on such assumptions must also receive credit.

Purpose

This aspect is divided into two subaspects:

Purpose in the figurative context: The appropriateness of the answer to a task, and thus the necessary considerations to be made, sometimes depend on the purpose of finding the answer (for a discussion of the impact of the clarity of the purpose in Example 3 see the section The framework in use below). In other tasks, the whole solution method is dependent on the purpose (see Palm, 2002). Thus, in simulations it is sometimes essential that the purpose of the task in the figurative context is as clear to the students as it is for the solver in the simulated situation.

Purpose in the social context: Example 2 (the lift task) can be interpreted as a 'pure mathematics' task dressed up in a real life context. It can be solved by dividing 269 by 14. However, the task can also be interpreted as describing a real-life situation, which for its solution includes assigning to the situation all the properties that it possesses in real life. This requires a different reasoning and an inclusion of other competencies in the task solving as well (see the discussion above about the aspect Solution requirements). Thus, an unclear purpose in the social context of the school situation of which interpretation to make may impede the similarities in actions between the in- and out-of-school situations.

The affective domain

To attain the goals with simulations it is crucial that the students significantly engage in the figurative contexts. For this to happen, the affective domain will be of great significance in addition to the aspects proposed. The affective domain is here used in the sense of

a wide range of beliefs, feelings and modes that are generally regarded as going beyond the domain of cognition. (McLeod, 1992, p. 576)

The affective dimension must allow the students the freedom to act in school as if it was a real situation. To accomplish this experience for students, the rules of school word-problem solving would have to come closer to the rules of the simulated situations. The students must, for example, believe that their solutions are going to be judged according to the requirements of the real-life situation and not have to think about what different requirements the teacher might have (such beliefs are not consistent with the intentions carried by the generic form of the genre of traditional word problems described by Gerofsky, 1999). A repetitive encounter with simulations with a high degree of representativeness and figurative contexts that are experienced as familiar and meaningful could facilitate such beliefs and enhance the students' engagement in the figurative contexts.

The framework in use

Two research studies are presented which may serve as examples of the possible uses of the framework for research purposes.

The influence of the simulations' representativeness on sense making in word problem solving (Palm, 2002): There are many studies of students' sense making in word problem solving in which the students provide solutions that are inconsistent with the 'real' situations described in the tasks (for an overview of this research see (Verschaffel, Greer

and De Corte, 2000). What if the word problems were simulations with a higher representativeness? Would the students include more realistic considerations in their task solving?

The framework was used to revise word problems in the sense-making studies to attain versions of the same word problems with higher representativeness and to keep some control of the differences between the task versions. A significantly higher proportion of 'realistic' solutions were given to the versions of the word problems with higher representativeness than were given to the word problem versions with less representativeness.

The study included a variant of Example 3 (the buses task). The framework was first used to analyse aspects that were simulated with low fidelity in the word problems. In the next phase, the framework was used to modify the word problems to simulate these aspects with higher fidelity. In the case of Example 3, one of the modifications made was in regard to the aspect *Purpose in the figurative context*.

In the literature, answers to Example 3 that are not a whole number are regarded as 'unrealistic' – buses do not come in halves. But in this task it is not clear whether the answer is to be used directly as a number of buses to order or if the answer is to be used to form the basis of a decision about the number of buses to order, which may include a discussion of other factors such that if it is possible to have more than one child on a seat. In the latter case, an answer with 'half buses' may be informative not 'unrealistic'. Since this purpose would most likely be known in the real situation, a fictitious ordering sheet for buses was included in the revised-task version to clarify the purpose of the task in the figurative context.

Analyzing the representativeness of the applied tasks in the Finnish and Swedish national tests in mathematics (Palm and Burman, 2004): The following results indicate some of the information that can be acquired from an analysis using the framework. 50% of the applied tasks included both an event that could happen in real life and a question that was relevant in that situation. About 25% of the applied tasks fulfilled both these two conditions and the additional quality that the information/data given in these word problems were the same as in some corresponding real-life situation to such a degree that the word problems required the same mathematics for their solutions as would have been required to solve the tasks in the simulated situations. When the aspect of availability of external tools was also considered, the corresponding proportion was down to 20%. Such information, revealing not only the extent to which the applied tasks can be considered simulations with high representativeness (or not) but also in which ways, can assist the test developers in their development and assessment of the tests.

The framework could facilitate discussions amongst both teachers and researchers about the properties of school tasks that are intended to emulate out-of-school situations and be useful in the development and critical analysis of contextualised tasks intended for classroom instruction, textbooks and assessments as well as for research purposes. Furthermore, it may be used to guide further research about representativeness by suggesting aspects to be taken into consideration. These aspects can also help organise and syn-

these research findings in order to attain a coherent picture of the acquired knowledge of the influence of the representativeness of a simulation.

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