HOW DO STUDENTS UNDERSTAND MATHEMATICAL LECTURES? NOTE-TAKING AS RETELLING OF THE TEACHER'S STORY

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In this article, I examine the relationship between a teacher's lecture and the students' notes by viewing the lecture as a kind of story that the teacher performs. The students' notes can thus be thought of as re-tellings of the teacher's story. How do their stories compare to that of the teacher? How do teachers' styles of communication affect the stories students tell in their notes?

The use of a narrative lens to analyse a mathematical lecture opens a window on several issues. Firstly, it is necessary to distinguish between information-giving and narrative accounts, since the latter do not force an interpretation on the reader, and hence "the narrative achieves amplitude that information lacks" (Benjamin, 1968, p. 86). Therefore, one can distinguish between the *teacher's mathematical story,* and the *student's mathematical fabulae*—meant as students' interpretations and retellings of the teacher's lecture.

Can mathematics lessons be considered as forms of narrative? Answering this question points to a second issue: a debate about the distinct natures of narratives and mathematical formalism challenges us to take a position between those researchers who claim that mathematics cannot be narrative at all (Solomon & O'Neill, 1998), and those who draw on Bruner's (1986) findings about the power of narrative in cognitive and epistemological processes (and in this article I follow this position). In particular, several studies have shown that it is possible to exploit the epistemic power of narrative in mathematics learning processes: Burton (1996) argues for the positive role of discursive accounts of experience in the way in which students come to know mathematics; Radford (2002) introduced the concept of *symbolic narrative,* and argued that there can be forms of narrative (such as the symbolic one in the learning of algebra) that are central to the process of learning mathematics; O'Neill, Pearce and Pick (2004) found that there is a correlation between performance in generating narratives and mathematical ability. O'Neill *et al.'s* study suggests that the basis of mathematical thinking (*i.e.,* inferring relationships and logical chains) shares the same underlying skills as narrative comprehension. Radford's (2002) understanding of symbolic narrative explicitly points to the fundamental role of narratives in every moment of the mathematical activity the learners are involved in.

Thirdly, and consequently, one can consider taking a narrative lens to analyse a mathematics lesson, and ask which lens would be suitable. With Bal (2009), I consider five components of the story: characters, setting, action, plot, and moral; and I seek their counterparts in mathematical stories. With Stockwell (2007), I take a rhetorical approach, focusing on the "text", its internal patterning and performative effects, instead of paying attention to relationships with the biographies of the authors/teachers, or to issues related to philosophy, theory and meta-linguistic speculations. I am interested in the *texture* (Stockwell, 2007) of the mathematical lesson rather than in the social, cultural, and economic factors that may affect it, even without neglecting their role. The main focus of this article is on the structural components of a story, and the extent to which seeing a mathematics lesson as being made up of characters, setting, actions, plot, and moral, may shed light on the teaching process itself.

Finally, given the social nature of lecturing, it is possible to distinguish between its locutionary and non-locutionary aspects. The former refer to what is said explicitly, while the latter refer to attitudes, beliefs, values and actions of the speaker, expressed implicitly in the linguistic act (Bikner-Ahsbah, 2003). Furthermore, given the multimodal rather than modular character of teaching and learning processes, an approach that takes into account the role of both linguistic and extra-linguistic communication ought to be considered. Not everything is narrative, or can be analysed through a narrative lens. However, a narrative lens allows the examination of what is recorded by the students in their notes, in order to find possible *internal patterns* (Stockwell, 2007), which may shed light on the students' views of mathematics and its teaching.

Lectures as stories

My interest in students' lecture notes was provoked by the diversity I found upon first examining a range of students' notes that had been taken in the same lecture. The marked differences in the notes helped me appreciate that the students were not all attending to the same things during these lectures. The similarities were also surprising—why did all the notes contain certain words spoken aloud and certain symbols inscribed on the blackboard?

Inspired by Dietiker (2012), who analyses a mathematics textbook as a story, I have chosen to examine the teacher's lecture as a story that can be analysed in terms of the following components: characters, setting, action, plot and moral. The *setting* is the space where characters are placed.

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Sometimes the setting is not obvious, as it refers to underlying assumptions and/or axioms. The setting may also involve different semiotic registers (Duval, 1995), such as algebra or the Cartesian coordinate system. Mathematical objects are considered to be the *mathematical characters* of the story. They can play a central or a peripheral role, have multiple names, and have properties that can be introduced and developed gradually.

The role of *agents* (characters) and *place* (setting) has been pointed out by Bruner (2004) in research about autobiographies: following Rorty, he defines a path from the figure, the *dramatis persona* of the classical folktale which fulfils a function in the plot but does not own it, to the individual, which transcends society and is responsible for his future, in a sense creating a space where his rights are respected. Figure and individual are seen by Bruner as different characterizations of the relations between agent and place. In a mathematical story, too, it is possible to analyse the relation between the mathematical characters and the setting both in terms of the manner in which the setting (the semiotic register) shapes the character, and in terms of the distinction between active and passive transformations of the characters within the semiotic register(s).

The *action* is that which the actor performs. In mathematical stories, an action can be seen—according to Duval (1995)—as a transformation of one representation into another one. The result of an action can be a change in an object or in a setting, or both. Unlike in literary stories, mathematical actions can be changed into objects (through reification).

The *moral* is the intended message of the lesson. The *plot* is the sequence of actions and it involves the *shaping* of the story, which is linked to its *aesthetic effects.* A story might include elements of suspense, which, in a mathematics lecture, might involve revealing a main character only at the end of the story. Following Netz (2009), I classify a teacher either as Archimedean (when the elements of his lecture feature a mosaic structure and narrative surprise), or as being in accordance with contemporary mathematics, when the presentation is linear and the general structure of the argument is signposted so that the learner knows how different tools will be used.

Finally, the rhythm and the frequency of the story may affect how the students focus on different areas of emphasis in the story and foster their anticipative acts. Radford, Bardini and Sabena (2007) have shown that during a mathematical activity concerning early algebra, rhythm may help students to objectify a regularity at the sensorimotor level. This regularity turns out to be crucial in conveying a sensuous meaning of mathematical generality.

Using this framework, it is possible to investigate whether the components of the teacher's story are also present in the students' stories. The *text*, which is the manner in which the story is presented, will play a role in drawing students' attention—are the events presented as symbols on the blackboard or orally, in natural language? To describe the nature of the lecture as text, I developed Arzarello's (2006) notion of semiotic bundle, which refers to the range of semiotic tools used in the mathematics classroom. As I have shown, even within a somewhat narrow range of teaching modality, different teachers use these tools in very different ways (Andrà, 2010a). The text can be seen as the style the teacher uses to present the mathematical content.

Arzarello (2006) distinguishes two fundamental semiotic unities: the speech-gesture unity and the speech-written sign unity. I see these unities as corresponding to related teaching modalities: the body modality, when the communication takes place mainly through the teacher's gestures, and the blackboard modality, when the teacher communicates mainly through written signs. A teacher's style can be described in terms of the frequency of use of each modality and the rhythm of shifting between them. The notion of text can be further characterized according to Mason's (2010) discipline of noticing, by distinguishing between *accounting of* and *accounting for* elements of mathematical content in a lecture. The former refers to the act of describing a mathematical character, its properties, or the actions one can perform; the latter is related to the moral of the story.

When it comes to the students' fabulae, Fried and Amit (2003) investigated the private and public character of students' notebooks. Specifically, they consider two cases of teachers who actively control what the students write in their notebooks and when: for example, the teachers prompt their students to faithfully reproduce what is written on the blackboard, and this is also part of final examinations. In a sense, the teachers consider a good match between the signs on the blackboard and the ones on the notebooks as an indicator of students' understanding. This has a consequence: the focus is much more on performance, namely on the actions of the mathematical story, rather than on the moral. However, as for example Rodd has shown, in a geometric context, performance does not correspond to understanding [1].

My study is different from Fried's and Amit's: it entails a shift from the public sphere, in which the notebooks are susceptible to the teacher's inspection and judgement, to the private one, where mistakes, false starts, imprecise signs, and so on, are only under the consideration of the learner. Private writing allows freedom to explore, backtrack, and reflect, and is immune from expectations and the constraints of common practice (Fried & Amit, 2003). The presence of a private domain in learning may influence the students' ability to grasp mathematical ideas, requiring reflection (and not only communication, which is public). Fried and Amit's recommendation is taken as a starting hypothesis for my study, as well as a motivation to investigate the *mathematical fabulae* that are told in the students' notebooks.

A mathematical story and its re-tellings

In the rest of this article, I analyse the first 6 minutes of a one-hour university lecture on probability and statistics, concerning the Gaussian distribution. The lecture comes from a dataset of 18 lectures, which have been observed and videotaped in three subsequent academic years at the University of Torino, Italy (the lecture under consideration comes from the 2008-2009 academic year). The students are undergraduate students in mathematics. Lecturers were asked to be videotaped during a lecture that introduces a new topic, or a theorem and its proof. The original language of both the lectures and the notes is Italian, and they have been translated into English. The mistakes that have been made in

Italian are as faithfully as possible reported in the English translation. The choice of taking the first 6 minutes was driven by previous results concerning the intensity of use of semiotic resources at the beginning of the lecture (see Andrà, 2010a). An analysis of affect-related factors on the side of the students and of differences related to different teaching styles has been also taken into account (Andrà, 2010b). For the purpose of this analysis, at the beginning of the lecture the students were told that their notes would be collected and photocopied at the end of the lecture, but the lecturer would not have access to them. The video camera was focused only on the lecturer and did not videotape the students.

A lecture by Lorenzo has been chosen for its topic, which allows for examination of the relationship between gesture and diagram. Lorenzo took part in each year of the project. This lecture was the third time his teaching was videotaped. The four students' notes to be discussed were chosen because of their diversity, both with respect to the blackboard notes and to each other. The purpose of my analysis is not to identify "good" versus "poor" note-takers, but to analyse different students' stories as re-tellings of the teacher's lecture. As a consequence, the students' notes have not been chosen according to any criteria of "quality", but according to their differences in reporting the teacher's lecture. How do the students' notes focus on the characters? What if the students focus on the actions rather than the moral? Is the graphical register or the symbolic one the setting? Is there a relation with the students' notes and the teacher modality?

Lorenzo's mathematical story

At the beginning of his lecture, Lorenzo leans on the blackboard with his hands behind his back, talking slowly, and recalls the mathematical characters that were present in previous lectures. Then, he changes his posture (Figure 1, first image) and introduces the *crisis*—that is, the moment in the story that gives rise to actions:

1. L: Hence, you have [*draws a bell curve on the blackboard's top-left*] a certain distribution, which gives you a certain probability, say it is a curve such that the area under the curve tells you how much it is probable that a number would come out in that particular interval [*draws two points on the x-axis, and two vertical lines to outline the area—see Figure 1, first image*]. Ok? Then, the name of this curve is Gauss's bell [*on the side of the graph he writes N(x; ,)*]. And it has a well-defined distribution, which is [*writes the formula without saying anything*].

2. L: This one is called the normal distribution, or the Gaussian, and they are synonymous in quotation marks, obviously it is called the Gaussian by Gauss [*readjusts the brackets of the formula with the chalk*].

3. L: Well, you remember that you are in this situation and then, in order to solve problems of this kind, what is the probability of obtaining a given, a certain interval? Since you do not have a table for any mu and sigma.

4. L: You have a standard way of taking this problem and of transforming it into a problem with the zeta [*with his left hand, L covers an arch in the air, close to the blackboard—Figure 1, second image*], namely you make a variable transformation and you obtain a certain distribution that is called the standard Gaussian, the standard normal distribution [*draws the standard curve on the left of the other graph—Figure 1, third image*]

5. L: that, instead, has these features: it is centred on zero and it has standard deviation equal to 1. That is, it is like this [*he writes the formula without talking*].

6. L: Note that if you set mu equal to 0 and sigma equal to 1 here, you obtain that one [*he draws an arrow from the first to the second graph*]. And on this side you have all the tables you need to solve the problem. The passage from one to another is given in this way [*he draws arrows and conversion formulas—Figure 1, fourth image*]. Hence, note that these transformations are each other's inverse: if you know x, you compute zeta, if you know zeta, you compute x without problems.

In turn 1, Lorenzo, in blackboard modality, sets the story in the graphical register: the probability of an event is described as an area under the curve. At the end of turn 1, Lorenzo moves to the symbolic register. In turn 2, he describes the main character of the story, repeatedly naming "the Gaussian character". In turn 3, he repeats the problem, this time facing the students and introducing a part of the setting: the distribution table. Moreover, he changes the character that performs the action to "you". In turn 4, Lorenzo is in body modality: his metaphoric gesture (McNeill, 1992) anticipates the arrow (drawn in turn 7), which represents the transformation from one curve to another one—a "treatment" according to Duval (1995). In turn 5, there is a shift in the setting to the symbolic register, in blackboard modality. After the focus on characters (the "you" leaves the room to "the Gaussian"). Turn 6 features the action of moving from one distribution to another, and from one setting to another (from the algebraic register of mu and sigma, to the numeric one), and "you" is the agent again. There is a change in the setting, since the distribution table is part of the setting, involving the numerical register. This change of setting also changes the actions that can be taken, since one can now use the distribution tables to evaluate the formula.

7. L: Well, I have given you this as a matter of fact. The experiment of today is: I want to see if I am able to explain it, okay?

The moral is given in turn 7: the focus (at 06:19) is on "why" the transformation can be done. Turn 7 thus appears to be somewhat surprising, since everything before unfolded as a gradual elaboration of characters and actions during which

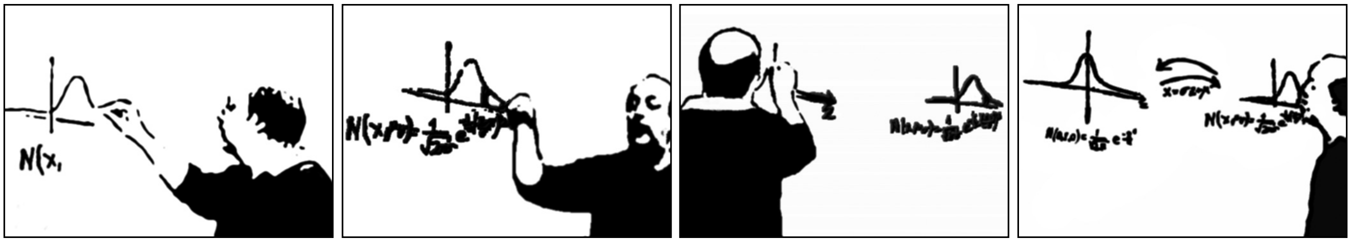

Figure 1. Lorenzo's teaching sequence. [2]

the need for and possibility of transforming becomes evident. Furthermore, there is a shift from *accounting of* in turns 1-6, to *accounting for* the actions, the changes in the setting, and the characters that are involved in the story in turn 7. This "account for" is given in body modality and features an Archimedean style of lecturing.

Students' mathematical fabulae

I now examine four students' notes (Figure 2), referring back to similarities and differences between their stories and that of Lorenzo. A first comment is that the first utterances, in body modality with a humble posture, have not been recorded by the students in their notes.

Barbara's first graph, representing the generic normal distribution, does not include the x_0 and x_1 symbols on the horizontal axis, as Lorenzo's did. However, although Lorenzo does not write the name of the curve on the board, Barbara includes it in her notes: "Campana di Gauss" (Gauss's bell). The formula is also recorded. Below the second graph, Barbara again records the name "Gaussiana standard" (Gauss's standard distribution), which Lorenzo did not write on the blackboard, and the value of the standard deviation ("dev. standard = 1"). However, Barbara does not record the value of the mean or the formula for the converse transformation (the standard normal distribution into the generic one, written in Lorenzo's turn 6). Barbara includes no arrows.

Barbara makes several choices in terms of what to record in her notes: we can infer something about her mathematical story, which focuses mainly on characters and very little on action. Although the formula enabling the transformation is recorded, there is no evidence that it accounts for any action in her story. This point is particularly interesting, since Lorenzo draws attention to this action both through speech ("transforming", "shift", talking about the areas being conserved), through his gesture (Figure 1, second image), and through the arrows on the blackboard.

Alex's notes (Figure 3) are strikingly different in that they include no graphs. As with Barbara, Alex writes the name of the distribution "La campana di Gauss." Unlike Barbara, she also writes the problem "dato – cambiamento di variabili distribuzione standard" (given – change in variables standard distribution). However, she omits information that is present in the graphs: there is no reference to the intervals $[x_0, x_1]$ and $[z_0, z_1]$, nor to the areas under the two curves respectively. Nonetheless, she writes down the formula for the transformation and adds "conservazione delle aree" (conservation of the areas), below it. She then writes

$z = \frac{x-\mu}{\sigma}$ è una retta" (it's a straight line).

Alex uses segments and arrows to capture actions and links between the characters of the story. This approach allows her to create a new structure for the recording of speech.

While Alex's story features characters, it also involves action: given some data, there is a transformation that conserves area. The action can be read in the area of her notes that links the formula to the text "conservazione delle aree". In Alex's notes, there is a complete absence of the graphical setting, which was strongly featured in Barbara's notes and in Lorenzo's lecture. We might infer that Alex finds the graphs unnecessary, perhaps mere illustrating devices, and so chooses to not record them. In this sense, the only setting of the story for Alex is the symbolic one. But from the point of view of probabilistic thinking, Alex is missing a crucial part of the story.

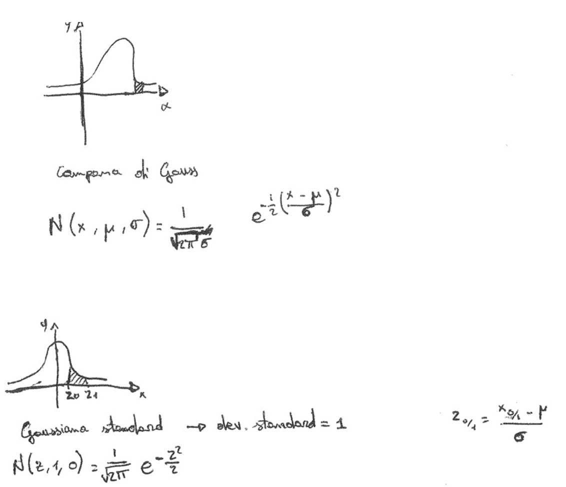

Figure 2. Barbara's written notes.

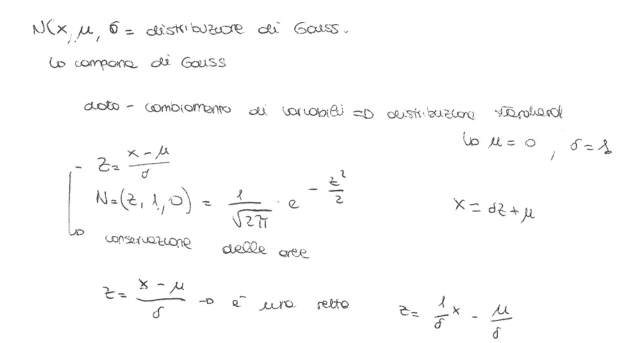

Figure 3. Alex's written notes.

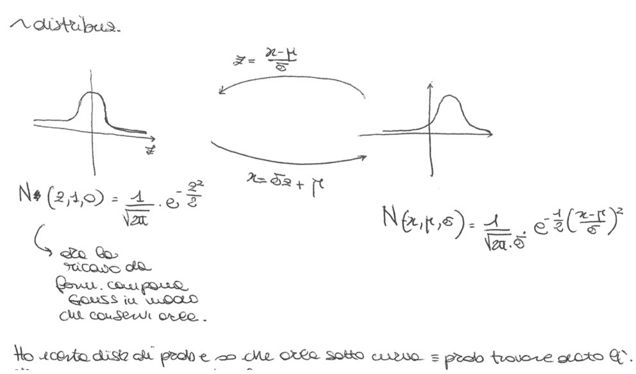

Figure 4. Diana's written notes.

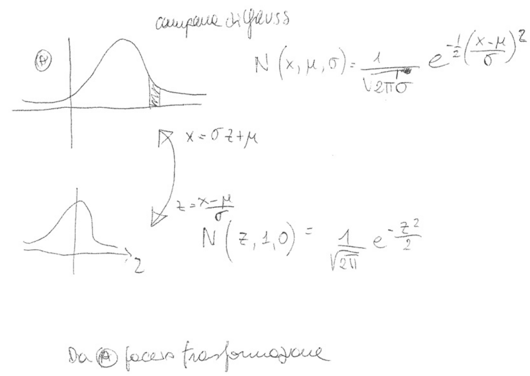

Figure 5. Carlo's written notes.

Diana records the graphical features included on Lorenzo's blackboard, except for the axis labels and the area under the curve between the intervals (Figure 4). She also faithfully reproduces the two arrows going from one graph to the other, labelled with the respective value of the variable *x*. The layout of the notes shows which graph the formulas act on. Diana additionally writes "Ora la ricavo da form. campana Gauss in modo che conservi aree" (Now I obtain it from the shape of Gauss's bell so that it conserves the areas), under the formula on the left. Finally, she writes "Ho una certa distribuz. di probl. e so che aree sotto curve = prob trovare dato li" (I have a certain probability distribution and I know that the areas under the curves = probability of finding the data there), at the bottom of the page.

In her notes, Diana includes the oral story told by Lorenzo. In fact, the last two pieces of text she writes are very similar to what Lorenzo says, to the point of using the first person voice in the last statement (Lorenzo uses "you"). Diana seems much less focused on the characters of the story (she chooses to not write down the names of the curves, even though she does write down other things that Lorenzo says orally). But she is very attentive to the action, as can be seen in her use of the arrows (relating one graph to the other) and in her words (which describe the action of conserving the areas and of computing the probability). While she includes the graphs, she ignores the more detailed parts of the setting, like the intervals and the shaded areas under the curve. Finally, Diana, much more than Alex or Barbara, attends to the moral of the story, which is expressed in "ora la ricavo da form. campana Gauss in modo che conservi aree" (now I obtain it from the shape of Gauss's bell so that it conserves the areas). This line actually comes from a statement that is said by Lorenzo later in the lesson, in an excerpt that has not been reported here. Diana was thus able to go back in the flux of her written notes to find a point in which it makes sense to insert the moral of the story. Also, the action is important for Diana: the verbs "obtain" and "conserve" express the operational aspect of the moral of the story.

Carlo records the name of the distribution ("campana di Gauss"), but he transposes the horizontal structure of Lorenzo's blackboard into a vertical one and replaces the two arrows used by Lorenzo with a single bi-directional one (Figure 5). Both formulas are recorded, but there are elements of the graphs that are omitted, such as the intervals, the labels on the axes and the area under the curve in the second graph. Referring to the upper graphs, which he has labelled A (this label does not appear on the blackboard), Carlo writes "Da A faccio trasformazione" (from A I make a transformation). He does not record anything related to the areas under the curve, nor to the conservation of the area under the transformation.

I am particularly struck by the fact that Carlo chose to use one bidirectional arrow, suggesting that the main features of the story for him is the transformation back and forth from one curve to the other. He also explicitly writes about the transformation of A. Action is thus central. Carlo's story is less focused on the characters of the story (what is being transformed) and the moral (why make the transformation?). As with the other students, the setting is rather vague, relating a graphical register to a symbolic one, without much attention to the details of the graphical one.

Evidently, the students do not just tell the story that Lorenzo writes on the blackboard. Nor do they just tell the story that Lorenzo says aloud. They pick and choose. It does appear, however, that the blackboard is a more privileged locus of the students' story-telling in that the bulk of their notes are made up of the symbolic and graphical elements recorded by Lorenzo. We might hypothesise that the students attend more to the blackboard because they have learned that what is written there is deemed important by the teacher. But we might also hypothesise that the students choose to record the non-linguistic elements of the story because these are the ones they feel they will not remember, or will not be able to explain on their own, in contrast with the oral part of Lorenzo's story, which they might find more transparent. This brings me to consider the goal of the students' notes, which may not be so much to tell a story, but to record the elements of a story whose plot is understood.

Discussion

Looking at the students' stories, I can outline some common features and some differences. Why are characters attended to so much more than actions, especially in Barbara's notes? Why are there so few verbs in the students' re-tellings? Might this be related to the relative absence of active verbs in mathematical writing more generally (see Morgan, 1998)? Alternatively, paying attention to the characters' names recalls a common practice: when we meet a new person, we record her name.

An important action, in mathematical terms, is missed by some students: the transformation from one curve to another one. It is anticipated by a metaphoric gesture (Figure 1, second image), it is said orally and it is written on the blackboard (Figure 1, fourth image). In Fried's and Amit's (2003) research, these mistakes in the students' notes would have been noticed by the teacher, given the public nature of notebooks in that study. This absence is related to Lorenzo's gestures. Melinger and Levelt (2004) argue that deictic gestures, which identify real or abstract entities or locations in space (McNeil, 1992), are often intended to communicate. Deictic gestures, produced in lieu of speech, or with deictic expressions such as "here" or "there", are especially uncontroversial. On the other hand, iconic gestures crucially share a transparent relationship with some semantic aspect of the concurrent speech. In a sense, iconic (and metaphoric) gestures may be seen as private gestures. Hence, it is possible to see Lorenzo's gesture (Figure 1, second image) not as a tool to communicate, but as a tool to think and organize his speech (Goldin-Meadow, 2003).

There are many aspects of both the teacher's and the students' stories that I was not able to attend to in this article. However, I hope to have pointed out some fruitful lines of inquiry that might provide insight into teachers' communication and students' note-taking. Other lectures videotaped in my study tell much more linear stories, and they are more faithfully re-told by students. While this faithfulness might help students better use their notes to study, it may not encourage the kind of reflection and critical reading that seems to be prompted by Lorenzo's non-linear style (see also Fried & Amit, 2003).

Notes

[1] "Teaching geometry interactively: communication, affect and visualisation." Presentation by M. Rodd at the MAVI-18 conference, September 12-15, 2012, Helsinki, Finland.

[2] Thanks to Jean-François Maheux for preparing these drawings.

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