TEACHING WITH CONVERSATIONS: BEGINNINGS AND ENDINGS

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A key focus for developing teaching approaches in many countries is supporting learners' conversations about their mathematical thinking. Conversations allow learners and the teacher to consider, question and add to each other's thinking and to co-produce generative mathematical ideas and connections. Conversations require genuine communication among classroom participants. Communicating mathematics is an important mathematical practice, which supports other mathematical practices such as connecting, generalizing and justifying ideas.

Research in mathematics classrooms where conversations take place suggests that developing and sustaining these conversations is extremely challenging and demanding on both teachers and learners (Chazan and Ball, 1999; Lampert, 2001). Guidance to teachers on how to work with mathematical conversations often does not go "beyond telling teachers not to tell" (Chazan and Ball, 1999). I suggest that we will make progress in understanding mathematical conversations if we consider that they have different phases, which require different kinds of work from teachers. I focus on two important phases of mathematical conversations: beginnings and endings. Drawing on a recent research study [1] with high school mathematics teachers in Johannesburg, South Africa, I address the questions: How do successful conversations begin in mathematics classrooms; and, once they are in progress, how are they successfully concluded?

Mathematical conversations

Classroom conversations have a number of features that distinguish them from other classroom discourse. There is genuine student involvement, which means that students articulate their own ideas rather than produce what the teacher wants (Pirie and Schwarzenberger, 1988). Teachers take learners' ideas seriously and engage with them on own their own terms, rather than in relation to what the teacher wants to hear (Davis, 1997; Nystrand, Gamoran, Kachur and Prendergast, 1997). Mason (1998) and Lampert (2001) argue that teacher and learners need to see all contributions to the conversation as conjectures, which are open to investigation, discussion, critique and revision. Genuine mathematical conversations require all the participants to listen carefully to each other, to appreciate the import of others' contributions, to build on or critique others' ideas in sensitive and helpful ways, and to express their own ideas so that others can engage with them (Mason, 1998). Cobb (1998) argues that genuine conversations include reasoning about mathematical ideas beyond the procedures that draw on those ideas. All of the above authors agree that conversations are not ends in themselves but are useful only if they support substantive learner engagement with important mathematical concepts.

Research in South Africa shows that there are few, if any, mathematical conversations in mathematics classrooms and that teachers engage with learners' ideas in superficial ways, if at all (Brodie, 1999; Chisholm *et al.*, 2000; Taylor and Vinjevold, 1999). Although many teachers are enthusiastic about and express support for the new curriculum [2], they struggle to enact many of the ideas in their classrooms. The social conditions of poorly resourced schools and teachers; large classes; the challenges of multilingual education; learners' weak mathematical backgrounds; inadequate training on the new curriculum; and lack of appropriate materials have all been offered as explanations for South African teachers' difficulties with the new curriculum (Chisholm *et al.*, 2000; Taylor and Vinjevold, 1999). These are undoubtedly part of the explanation, however international research suggests that there may be other reasons.

Studies conducted outside of South Africa also suggest that mathematical conversations that support learner engagement and thinking are rare, even though such reforms have been in place for much longer. Among the 18 teachers in their study, Fraivilling, Murphy and Fuson (1999) considered only six teachers to be "skilful" in eliciting and supporting learner thinking, while only one was successful in eliciting, supporting and extending learner thinking. Huffered-Ackles, Fuson and Sherin (2004) worked with four teachers, and generated a framework that describes the development of what they call the "math-talk learning community". The framework has four levels and only one teacher's trajectory took her and her learners through all four levels. These studies were conducted in well-resourced classrooms and suggest that teaching approaches are central in enabling mathematical conversations.

Studies of successful mathematical conversations identify a number of challenges for teachers. These include: supporting learners to make contributions that are productive of further conversation (Heaton, 2000; Staples, 2004); respecting and valuing all learners' thinking while working with the diversity of their mathematical ideas (Lampert, 2001); respecting the integrity of learners' errors while trying to transform them and teach the appropriate mathematics (Chazan and Ball, 1999); seeing beyond one's own long-held and taken-for-granted mathematical assumptions in order to hear and work with learners' ideas (Chazan, 2000; Heaton, 2000); maintaining a "common ground" which enables all learners to follow the conversation and its mathematical purpose and to contribute appropriately (Staples, in press, 2007); and generating mathematical practices such as making connections, generalizing and justifying (Boaler and Humphreys, 2005).

The above research shows that the pedagogical demands of mathematical conversations can be daunting and that we need to understand more about the practices involved in generating and sustaining these conversations. My research suggests that it is useful to think about different phases of conversations, particularly beginnings and endings. An important place to start developing teachers' skills in conversation might be at the beginning: How might teachers begin useful conversations?

Contexts of the conversations

In my recent research study, two teachers, one in Grade 10 [3] and one in Grade 11, were successful in beginning conversations but less successful in ending them. The Grade 10 teacher's school is in a poor socio-economic area, is under-resourced, and serves black learners whose parents work in menial jobs or are unemployed. There were 45 learners in the class, and through learner interviews and classroom observations, the learners' mathematical knowledge was established to be at least two years below grade level. The Grade 11 teacher's school is in a lower-middle class area, with adequate resources and with a racially diverse learner profile. Most of the learners' parents are employed, some in middle management or the professions. There were 35 learners in the class and the learners' knowledge was established to be at or near grade level. The two teachers were enrolled in an in-service degree program at the time of the research. They were thus better informed than most teachers about new curriculum developments, particularly about the possibilities for creating mathematical conversations in classrooms.

Each teacher's lessons were observed for one week, between 150 and 250 minutes of class time for each teacher. During this time, I was able to see the learners work in groups and in whole class sessions on at least two fairly extensive tasks in each classroom. My focus, in this article, is on extended mathematical conversations within the whole-class sessions. There was one of these in each class during the week of data-collection. I considered an extended conversation to be one that continued for at least 50 turns of talk; that had contributions from learners that engaged with significant mathematical ideas; and where learners and the teacher responded to each other's ideas in ways that took the ideas forward. The teacher worked to elicit and engage learner contributions and also made significant contributions to the conversation, but these were always in response to the thinking that was evident in the learners' contributions. Each conversation could be assigned a clear beginning and end, which separated it from other talk in the classroom.

The conversations were analyzed in relation to how they began, progressed and ended, the extent to which learners' ideas entered into and steered the conversations and the extent to which learners reorganized their thinking through the conversations. Some of this analysis is presented here to illuminate the practices of beginning and ending conversations. Each of the two extended conversations that I observed began with a learner's question. In the next two sections, I describe the work that preceded the learner's question and how the question became the focus of an extended conversation.

Beginning: Can zero be negative?

During a previous task in the Grade 10 class, the teacher became concerned that many learners were thinking that if an expression is preceded by a negative sign (*e.g.*, $-2x$) then the expression is negative. The teacher's interpretation of the learners' thinking was that they focused on superficial characteristics of the expression (the negative sign), rather than its deeper meaning as a negative number multiplied by another number, which could be either negative, positive or zero. The teacher was also concerned that learners were not substituting numbers into such expressions appropriately, nor were they multiplying correctly. In order to address this, the teacher deviated from his original plan and asked learners to say whether the expressions: x, $-2x$, x^2, $3x^2$, $-x^2$, $(x + 1)^2$, $-(x + 2)^2$, $2(x - 3)^2$ were always positive, always negative, or sometimes positive, sometimes zero, sometimes negative.

As the class was discussing $-2x$, the following interaction occurred (all learners' names are pseudonyms):

Teacher:	And what is negative two times zero, Fred?
Learners:	Zero
Teacher:	Zero, what is this value going to be [points to the column with 1]?
Lebo:	Sir, can I please ask, why do you say
Teacher:	Shhh quiet, yes? [He points to Lebo.]
Lebo:	Why do you say, uh, positive zero because a negative times a positive it's going to give you a negative?
Teacher:	Very nice, a negative times a positive, is equal to a
Learners:	Negative [chorus].
Teacher:	A negative

In the interaction we see the first expression of a learner's question, asked by Lebo (rewritten here): "Why is $-2 \times 0 = +0$ and not -0, given that multiplying a negative by a positive should give a negative?" The teacher had previously emphasized the rule. This question became the starting point for subsequent discussion. However, this discussion did not happen immediately. Aside from Lebo's question, the above interchange might be considered to be constraining of learners' mathematical meanings. The teacher asked two very simple questions (rewritten here): "What is -2 times 0?", and "What is $2x$ if $x = 1$?".

In the first case, learners produced the correct answer, which the teacher affirmed by repeating it. In the second case, Lebo interrupted with her question. The teacher responded to Lebo's justification for her question, that a negative times a positive gives a negative. He affirmed it, saying, "very nice" and then repeated it, waiting for a chorused answer from learners, which he then repeated. The only aspect of the above interaction, which may allow for some engagement with a learner's meaning, is that the teacher allowed Lebo's interruption and gave her a chance to ask her question. However, he did not immediately recognize the significance of her question, but rather used her justification to make a point that he wanted to teach. This suggests that, at this point in the lesson, the teacher was intent on teaching certain concepts and used learners' contributions to help make his teaching points, rather than considering learners' ideas in their own right, and developing conversations.

However, immediately after the teacher affirmed the chorused "negative" he asked a question of Lebo, which suggests that he reconsidered her question:

> Teacher: So, what do you want us to write, negative.
>
> Lebo: Negative zero.
>
> Teacher: Negative zero.
>
> Learners: No.

The teacher gave Lebo a chance to clarify her claim; she argued that -2×0 should give -0. Other learners responded "no" indicating that they did not agree and many hands went up.

The conversation then continued for about 70 turns. During this time, six learners, including Lebo, made individual contributions, taking between one and twelve turns each. Some examples of contributions (rewritten here) were:

- zero is neutral
- zero is neither negative nor positive because of where it is on the numberline
- +0 and −0 is the same
- zero is like x because sometimes its positive and sometimes its negative
- zero is nothing
- zero is not nothing because we get problems with zero in like $0 + 1$ and 0×0
- zero is a number because its on the numberline

The teacher responded after each learner turn in a number of different ways. Sometimes he pressed the learner to elaborate her/his idea, sometimes he called on another learner to contribute, sometimes he repeated a contribution, and sometimes he inserted his own contributions. He often refocused learners on the question in order to keep the point of the discussion clear, for example he asked:

> Does it make a difference if we write positive zero or negative zero?
>
> Why do we write it just as zero, why don't I write negative zero?

A detailed analysis of the conversation shows that there were many places where learners built on and challenged each other's contributions and reflected on their own contributions. It also shows how difficult it was for the teacher to keep track of all the contributions and respond to them in ways that took the conversation forward. This is discussed elsewhere (Brodie, 2005). What I have shown above is how a teacher allowed a learner question into the discussion and then shifted his teaching style in order to allow the question to become the focus of a mathematical conversation, which then introduced a range of ideas related to the question. The teacher did not immediately recognize the significance of the question, nor its potential for discussion, but as he responded to it, he came to recognize its value and opened it to the class for discussion. This allowed a shift from a constrained question-answer session, to a more open conversation

Beginning: graphs and equations

In the grade 11 classroom, learners were working on the question:

> What changes as the graph of $y = x^2$ shifts 3 units to the right to become $y = (x - 3)^2$, and four units to the left to become $y = (x + 4)^2$?

A learner, Winile, was reporting back on some of her group's observations. As Winile finished her report back, Michelle put up her hand to ask a question:

> Michelle: Okay, can I ask a question?
>
> Teacher: Okay.
>
> Michelle: Okay, look on Task 1 right. You said that if it is a positive, you move to the right and if it is a negative, you move to the left. So now, can you please tell me why on your second drawing, where it says $y = (x - 3)^2$, [looks at Winile] can you see that? Say, yes, Winile if you understand
>
> Winile: Yes, I can see it.
>
> Michelle: Alright, so now how come in the bracket there's a negative but where the turning point is, is a positive. That's what I would like to know
>
> Learner: [Inaudible.]
>
> Teacher: Okay, Lorrayne.
>
> Lorrayne: [Inaudible.]
>
> Teacher: Okay now. Just one moment, just one moment. Sorry to break your word, Lorrayne. Guys listen, it is essential that you pay attention – you become part of the discussion. Otherwise, learning is not going to happen. Carry on Lorrayne
>
> Lorrayne: Sir, you have a negative three in the bracket and it's a square, when you square something, remember Sir said when you square it, it becomes positive.

Learner:	If it's a negative.
Lorrayne:	ja [yes] [4]
Michelle:	And then if you look at $y = (x + 4)^2$, why is it that the turning point is a negative
Learner:	But the equation is positive
Michelle:	And the drawing is positive.

In the above extract, Michelle asked her question directly of Winile and not of the teacher Following this, Michelle and Lorrayne co-produced the question, "Why does a negative sign in the brackets correspond to a shift to the right and a positive turning point; and a positive sign in the brackets correspond to a shift to the left and a negative turning point?" Michelle asked the first part of the question and as Lorrayne tried to answer it, Michelle came back with the second part, which could not be dealt with by Lorrayne's response [5]

After this conversation, a number of other learners confirmed that they were grappling with the same question. In the next extract the teacher takes the opportunity to ask for clarification as to exactly what the question was In doing this, he was establishing a common ground for further conversation, a mechanism that Staples (in press, 2007) has identified as crucial to sustaining high-level conversations in mathematics classrooms.

Learner:	I asked that too [Some learners laugh.]
Learner:	I'm also asking the same question.
Teacher:	What question are you asking?
Michelle:	The question ....
Teacher:	Yes.
Michelle:	Look at our drawing where .
Teacher:	Okay Where's my drawings? [Finds drawings]
Michelle:	Where it says $y = (x + 4)^2$ on the left hand side.
Teacher:	Right
Michelle:	Our turning point is a negative four.
Teacher:	Okay
Michelle:	Then Lorrayne that said with the one on the right, where it says y equals x negative three squared, and the turning point is a positive. Because you squaring it, it will become a positive. But what happens with um, the one on the left?
Lorrayne:	The negative one. The equation is positive but the graph is on the negative side.
Teacher:	The equation is positive but the graph is on the
Learners:	Negative side.

Again, Michelle and Lorrayne co-produced the question. Once they had done this, the teacher asked for responses to the question and conversation ensued.

The conversation continued for the rest of the lesson, for almost 400 turns, and may have continued further if there had been more time. A number of learners made extensive contributions as they grappled with the idea of how the sign in the equation influences the graph David focused on the fact that 4 is a constant in the equation $y = (x + 4)^2$ but could not elaborate how this affected the horizontal movement of the graph. Candy asked, "Can't it just be like that", suggesting that they just accept it as a rule without justification, to which Michelle responded that she could not accept that, and the teacher affirmed her demand for a justification. Michael suggested, as Lorrayne had previously, that in the case of $y = (x - 3)^2$, multiplying the - 3 would give a positive number, but when challenged by Michelle, acknowledged that his explanation did not work for the case of $y = (x + 4)^2$.

These learners were dealing with superficial aspects of the equations; what they look like Winile made a breakthrough when she began to argue that they could not make a direct link from the equation to the graph, but that they had to take account of the underlying relationships between the variables in the equation that gave rise to the graph [6] Although somewhat inarticulate, her next two contributions show that she is thinking that the x-value in combination with 4 produces a y-value, rather than that the x-value and the 4 are somehow directly related as the others were suggesting

Winile:	The positive four is not like the x, um, the x, like, the number, you know the x [showing x-axis with hand], it's not the x, it's another number ... you substitute this with a number, isn't it, like you go, whatever, then it gives you an answer.

and later

Winile:	We're not supposed to get what x is equal to, we getting what y is equal to, so we supposed to, supposed to substitute x to get y.

Although this conversation began, like the previous one, with a learner question, the way in which the Grade 11 teacher enabled and maintained this conversation differed from the Grade 10 teacher. The Grade 11 teacher was immediately alert to the possibilities in the question He also took time to allow learners to clarify the question together. This teacher had an explicitly stated goal of supporting learner conversation and this was the first time in the three lessons that I observed in which he had succeeded to begin a conversation. A reason for the success in this case is most likely because a number of learners had raised the same question in their groups and had struggled to make sense of Winile's report back. So they had a clear focus, in which they were interested. It may also be the case that the teacher recognized the significance of the question immediately, because he had seen it arise in the groups and because it is a predictable question in this area of curriculum. The Grade 10 teacher was faced with a question that did not obviously come from the task and was surprised by it. It therefore took some time for him to recognize its significance, although once he did, a productive conversation ensued.

Each beginning led to extended, substantial conversation, with learners responding to each other and shifting their ideas through the conversation. I now move on to the question, how did these conversations end?

Ending: Zero is neutral

In ending conversations, the challenge for the teacher is how to remain open to a range of ideas, while trying to draw them together, relate them to each other and build towards some resolution in the conversation. A second important issue is when to conclude a conversation. Staying with the same idea for too long can lead to boredom or frustration for the learners, while not sustaining the conversation long enough can mean that ideas are not well developed. Teachers are always aware of the next problem or task that they need to move to as part of the ever-present constraints of examinations and an overcrowded curriculum.

In the discussion about whether -2×0 should be written as -0, a number of ideas, mentioned above, were expressed by learners. Some of these built on or challenged each other. For example, when Fred argued that zero is nothing and therefore can't be positive or negative, Lebo argued that zero is not nothing; it's a number that appears in expressions like $0 + 1$ and so can be operated on. Other learner contributions introduced new possibilities, such as "zero is neutral", or "zero is between the negative and positive numbers so it has no sign". The teacher had to decide how to draw together the range of contributions in order to conclude the conversation. In this case, the teacher had diverted from his lesson plan to include the conversation on an unexpected learner's question. The question was relevant to the task and engaged the learners in thinking about an important issue in mathematics: the meaning of zero. However, there were another six expressions to deal with, many of which would be more difficult than this one, and he still needed to teach the important conceptual points that he planned for this lesson. [7] The following extract shows how the teacher ended this conversation.

Teacher: They gave you nothing, a value. But, so, so, nothing has a value, which is zero. So, zero's got a value, its nothing

Learner: It's a negative or positive.

Teacher: But is it important to say, is it negative. Or someone was saying here ... zero is a neutral, ne, [right] that's how we will pretend to see it, ne [right]? So, we can never write negative zero. Zero stays, that's how we write it. It's never negative nor.

Learners: Positive

The teacher confirmed Lebo's point that zero "has a value" or is a number, and its value is nothing and moved quickly to a conclusion that zero is neither negative nor positive, and they should see it as "neutral". The point that zero could be seen as neutral was made earlier by a learner, so in making this final decision, the teacher took two learner contributions, brought them together and closed the discussion. As he did this however, Lebo shook her head indicating disagreement. Earlier on in the conversation, she had agreed with the argument of some of her peers that if zero is written without a sign, then it can be thought of as positive zero, but her claim, that $-2 \times 0 = -0$ was still on the table and had not been adequately challenged.

In the next extract we see that the teacher acknowledged Lebo's dissatisfaction but signaled that the conversation must end although it might continue among the learners during break. Nevertheless, Lebo made a final point, where she related the idea of nothing to her everyday experience of playground talk (that she had clearly been a victim of) where learners call each other "nothing". The teacher responded to this briefly, by drawing on her analogy to further make his point that zero is to be thought of as neither positive nor negative. He also acknowledged her, in the terms that she introduced, as a "positive 2", and therefore not "nothing".

Teacher: I know Lebo's not happy.

Lebo: I'm not

Teacher: Yes. [Learners laughing.]

Learner: Never.

Teacher: You will have to convince her during break

Lebo: Sir. Can I please ask you, if they say you are nothing, then, and, you have something that they, that they don't have, Sir?

Teacher: Ja, ja [Yes, yes] they can't. If someone says you are nothing. They can't say, you are say, you a positive nothing

Lebo: Yeah, right.

Teacher: They can't say, you are a negative nothing. They can't say so. But, if he says, hey you a two, hey, you a positive two, can you see, he's putting value there, you see, you see, but if he says you are a negative two, then it means that something's bad. You see, that's in the connotation of negative. But if he says you are nothing, are you positive nothing. There's no difference if I say you are positive nothing or you are nothing. It's exactly the same. Lebo, you are a positive two

It is clear from the above that the teacher wanted to end this conversation and move on with his teaching agenda. Even so, he allowed one last point from the learner whose question began the conversation. He summarized and resolved the issue by stating what they would agree on (how they would 'pretend' to see it). This resolution took account of some learners' contributions but did not adequately answer the question on the table.

Not ending: graphs and equations

The conversation in the Grade 11 classroom went on for much longer, with many twists and turns that are difficult to recount in the short space of an article. After about 300 turns of talk the teacher summarized the progress that had been made so far as follows:

> Teacher: What we saying is that to get the y to be zero on the turning point, okay, what did she say is that this x must be negative four (pointing to the x in $y = (x + 4)^2$) and that is why your turning point is negative four and zero, because for the y, remember this here (pointing to $(x + 4)^2$), represents what is y equal to. This value, this, when you work this out, like you said earlier on, is that it gives you the values for y when you substitute the x, so in other words, on your turning point, you're going to have negative four there, because your x-value there is negative four and the y-value is zero, okay.

The teacher was pointing to the substitution of –4 for x in the equation $y = (x + 4)^2$) which explains why the x-coordinate of the turning point is -4 and why the graph shifts to the left. His summary built on Winile's points discussed above and some responses from other learners. After this some learners still had questions and the teacher allowed learners to raise them. Some of these questions were poorly expressed and some were not relevant to the points that had been made about the relationships between the graph and the equation. However, the teacher continued to hear these and to allow others to discuss them. The class continued to engage with each other's ideas, but from this point on, the discussion lost focus and it is not clear that anything substantial was added to the key conceptual point that had been made about the relationship between the equation and the graph. An exchange occurred when a number of learners, including the learners who asked the question, openly expressed their frustration at not being able to follow each other's ideas (for more detail see Brodie, 2006)

Beginning and ending conversations

In response to my first question: "How might teachers begin a useful conversation?", one answer suggested here is that teachers allow a learner question to form the focus of the discussion. In both examples, the conversations began with a learner question, one expected and one unexpected. I am not suggesting that this is the only way in which conversations might start. However, it is one way, which is likely to engage learners' interest, particularly if they share the question. This suggests that teachers might be on the lookout for learner questions and work at recognizing their significance for learners' mathematics learning. Even if the possibilities in a learner's question are not immediately apparent, giving the learner a chance to articulate the question and making time for others to help, might give the teacher and learners time to come to see the value of the question.

It is likely that not all learner questions will allow for productive conversations. The two questions in this paper shared certain characteristics: they were provocative, they suggested underlying intuitive notions of number and graph which run counter to the appropriate mathematical conceptions, and they allowed for a range of contributions which might help resolve the questions.

Once conversations have started, there are more challenges for teachers and learners. I have dealt with some of the challenges of sustaining conversations elsewhere (Brodie, 2006), as have others (Chazan and Ball, 1999; Staples, in press). The final challenge comes in ending the conversation at the appropriate time and in appropriate ways, so that learners do not get bored or frustrated, and that mathematical learning is optimised.

In both of the above examples, learners did get frustrated. In the Grade 10 lesson, the learner who asked the question was not satisfied with the way it was resolved. In the Grade 11 lesson, a number of learners became frustrated during the discussion, even when the teacher gave a clear summary of what had been established during the conversation. The Grade 10 learner needed more discussion and an answer to her question, the Grade 11 learners needed less discussion and possibly a clearer explanation of what had been resolved.

My analysis suggests that ending conversations may be more difficult than beginning them. Two teachers who successfully began and maintained conversations struggled to find the right moment and ways in which to end them. This suggests that teacher-educators might want to work with teachers on how to pull the threads of a conversation together to make for successful resolution. Pulling a conversation together entails taking account of the diversity of ideas that have been expressed, putting them into a relationship with each other and bringing some resolution. This suggests a more active role for the teacher than "not telling" (Chazan and Ball, 1999). The teacher will clearly have to listen carefully in ways that hear what the learners are trying to say rather than evaluating their contributions (Davis, 1997) but also in ways that will allow her/him to relate the contributions to each other and to the question being discussed. It is also likely that the teacher will have to provide additional information, missing links and an overall frame for the conversation, so that, on ending, learners have a sense of what they could have learned. [8]

As has been pointed out in the literature, conversations in mathematics classrooms are rare. During one week of observations in two teachers' classrooms, I was privileged to see two very interesting conversations, which hold much potential for other teachers and teacher-educators to learn from. It may be that in specifying the pedagogical challenges involved in different phases or moments of conversations, these will become easier for teachers to begin, maintain and end.

Notes

[1] An earlier version of this paper was presented at the 14th Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE) and published in the proceedings

[2] The new curriculum in South Africa encourages teachers to allow more learner participation in lessons and to work with learners' contributions

[3] There is a wide age range In South Africa, Grade 10 is the tenth year of schooling

[4] Translations appear in brackets

[5] Lorayne's response also did not deal adequately with the first part of Michelle's question, because $(-3)^2$ is 9 and not 3

[6] A more detailed analysis of Winile's learning through this conversation is available in Coetzee (2003, 2004)

[7] Moreover, this task had been planned in response to learner difficulties with an initial task, so the teacher needed to get back to that as well

[8] I acknowledge here a point made by one of the anonymous reviewers that the beginnings and endings of conversations do not need to occur immediately before and after the conversation Conversations can be returned to and continued at various times This would allow for an ending at a later time influenced by subsequent work, where important points are drawn together and new insights offered.

References

Boaler, J. and Humphreys, C (2005) *Connecting mathematical ideas: middle school cases of teaching and learning*, Portsmouth, NH, Heinemann.

Brodie, K (1999) 'Working with pupils' meanings: changing practices among teachers enrolled on an in-service course in South Africa', in Zaslavsky, O. (ed), *Proceedings of the twenty-third annual conference of the International Group for the Psychology of Mathematics Education*, Haifa, Israel, Technion, Israel Institute of Technology, **2**, pp 145-152.

Brodie, K (2005) *Textures of talking and thinking in secondary mathematics classrooms*, unpublished Phd Dissertation, Stanford, CA, Stanford University.

Brodie, K (2006) 'Resisting reform pedagogy: teacher and learner contributions', in Novotná, J , Moraová, H , Krátká, M and Stehliková, N. (eds), *Proceedings of the thirtieth annual conference of the International Group for the Psychology of Mathematics Education*, Prague, Czech Republic, Charles University in Prague, **2**, pp. 233-240

Chazan, D. (2000) *Beyond formulas in mathematics and teaching: dynamics of the high school algebra classroom*, New York, NY, Teachers' College Press.

Chazan, D and Ball, D (1999) 'Beyond being told not to tell', *For the Learning of Mathematics* **19**(2), 2-10.

Chisholm, L., Volmink, J , Ndhlovu, T, Potenza, E , Mahomed, H., Muller, J. *et al.* (2000) *A South African curriculum for the twenty first century Report of the review committee on Curriculum 2005*, Pretoria, South Africa, Department of Education, http://www.education.gov.za/dynamic/dynamic.aspx?pageid=329&catid=10&category=Reports&legtype=null

Cobb, P. (1998) 'Theorizing about mathematical conversation and learning from practice', a contribution to, 'Learning mathematics through conversation: is it as good as they say?', *For the Learning of Mathematics* **18**(1), 41-51

Coetzee, K (2003) *Learning to teach mathematical reasoning in a collaborative whole classroom discussion*, Unpublished Research Project, University of the Witwatersrand, Johannesburg, South Africa

Coetzee, K (2004) 'Mathematical reasoning in a collaborative whole class discussion', in Buffler, A and Laugksch, R (eds), *Proceedings of the twelfthth annual meeting of the Southern African Asociation for Research in Mathematics, Science and Technology Education*, Cape Town, South Africa, University of Cape Town, pp 134-139

Davis, B (1997) 'Listening for differences: an evolving conception of mathematics teaching', *Journal for Research in Mathematics Education* **28**(3), 355-376.

Fraivillig, J., Murphy, L and Fuson, K (1999) 'Advancing children's mathematical thinking in everyday mathematics classrooms', *Journal for Research in Mathematics Education* **2**, 148 -170

Heaton, R (2000) *Teaching mathematics to the new standards: relearning the dance*, New York, NY, Teachers' College Press.

Hufferd-Ackles, K , Fuson, K and Sherin, M (2004) 'Describing levels and components of a math-talk learning community', *Journal for Research in Mathematics Education* **35**(2), 81-116

Lampert, M. (2001) *Teaching problems and the problems of teaching*, New Haven, CT, Yale University Press

Mason, J (1998) 'Talking while learning mathematics versus learning the art of mathematical conversation', a contribution to, 'Learning mathematics through conversation: is it as good as they say?', *For the Learning of Mathematics* **18**(1), 41-51

Nystrand, M , Gamoran, A , Kachur, R and Prendergast, C (1997) *Opening dialogue*, New York, NY, Teachers' College Press

Pirie, S and Schwarzenberger, R (1988) 'Mathematical discussion and mathematical understanding', *Educational Studies in Mathematics* **19**(4), 459-470

Staples, M (in press, 2007) 'Supporting whole-class collaborative inquiry in a secondary mathematics classroom', *Cognition and Instruction* 25(2)

Taylor, N and Vinjevold, P (eds) (1999) *Getting learning right report of the President's Education Initiative Research Project*, Johannesburg, South Africa, Joint Education Trust

See? Seeing is not always used to mean looking with our eyes. The double meaning reminds us that when we use our eyes we often see what we have already foreseen. We look at objects and see them in a perspective that we have learned to impose on our view. It has taken the experiments of modern artists, and the study by psychologists of the child's view of the world, to remind us that it is possible to see in many other ways .... I find my memories of dialogue with Geoff Sillitto useful when trying to think of seeing as a dynamic interchange between the perceiver and the perceived.

(Dick Tahta (1970) 'Idoneities', in Members of the Association of Teachers of Mathematics (eds), *Mathematical reflections Contributions to mathematical thought and teaching, written in memory of A. G. Sillitto*, Cambridge, UK, Cambridge at the University Press, pp 27-28.)