How to Recognize Hidden Geometrical Thinking: a Contribution to the Development of Anthropological Mathematics*

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Confrontation

You are mathematics educators, are you not?
So let us see if you are good at mathematics

— Do you know how to construct a circle given its circumference?
— Do you know how to construct angles that measure 90°, 60° or 45°, using only the strips of paper I have distributed to you?
— What is the minimum number of strips of paper you need in order to be able to plait a broader strip?
— Can you fold an equilateral triangle out of a square of paper?
— Do you know how to construct a regular hexagon out of paper strips?

I gave you five minutes. Who solved all the problems? Nobody?
How is that possible?
Who solved four problems? Nobody? Three of them? ....
You failed?
Do you not have the necessary mathematical abilities?
No, that is not the reason; you need more time, don't you?
But you are mathematicians, are you not?
You need more time to analyze these non-standard problems. All right. But let me say to you that many of our (illiterate) mozambican artisans know how to solve these problems ... (obviously "formulated" in another way)

Introduction

The President of the Interamerican Committee on Mathematical Education, Ubiratan D'Ambrosio, has stressed the need for the recognition, incorporation and compatibilization of *ethnomathematics* into the curriculum [e.g., D'Ambrosio, 1984, p. 10]. This integration of mathematical traditions

> requires the development of quite difficult anthropological research methods relating to mathematics [D'Ambrosio, 1985, p. 47];

> Anthropological mathematics ... constitutes an essential research theme in Third World countries ... as the *underlying ground upon which we can develop curriculum in a relevant way*". [D'Ambrosio, 1985, p. 47]

In order to be able to incorporate popular (mathematical) practices into the curriculum, it is first of all necessary to *recognize their mathematical character*. Traditional counting methods, e.g. by means of knots in strings, and counting systems are relatively easily recognized as mathematics. But what about geometrical thinking?

Traditional mozambican houses have conical roofs and circular or rectangular bases. Rectangular mats are rolled up into cylinders. Baskets possess circular borders. Fishtraps display hexagonal holes. Could these examples figure in the mathematics lesson as illustrations of geometrical notions?

Only as illustrations?

This is a rather fundamental question that has recently also been posed by Howson, Nebres and Wilson in their discussion paper on *School mathematics in the 1990s*:

> There has been increasing talk, particularly with respect to developing countries, of "ethnomathematics", i.e. mathematical activities identified within the everyday life of societies. Thus, for example, a variety of types of symmetries are used for decoration in all cultures, numerous constructions are erected which illustrate mathematical laws. To what extent are these activities really "mathematical"? What is it which makes the activities "mathematical" rather than, say, "capable of mathematical elaboration or legitimisation"?" [Howson et al, 1985, 15]

In order to answer this question, let us analyse some examples.

First example

Take two strips of paper in your hands. How do you have to fold them around another in order to be able to weave them further (see Figures 1 and 2)? What has to be the initial angle between the two strips? Vary the angle. What do you discover?

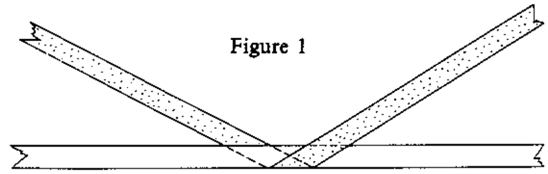

Figure 1

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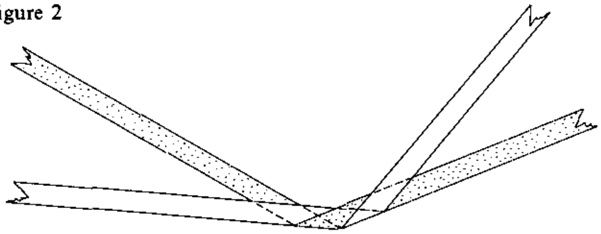
Figure 2



Only one special angle makes further plaiting possible (see Figures 3, 4 and 5). Two types of strips can be woven in this way (see Figures 6 and 7). The strip pattern in Figure 7 admits changes in direction, like the "circling around" in Figure 8. It is exactly this possibility that makes this strip weaving process very useful. For example, mozambican artisans use this method for making their straw hats by knitting together the successive wings of a plaited spiral.

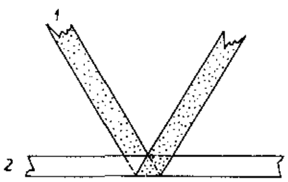


Figure 3

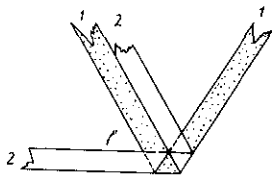


Figure 4

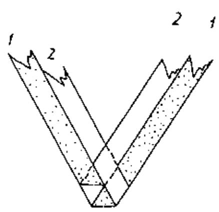


Figure 5

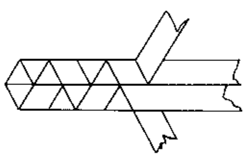


Figure 6

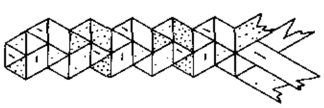


Figure 7

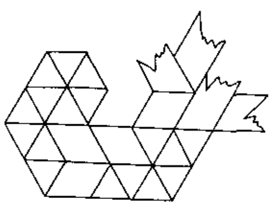


Figure 8

Now I repeat the question. Can this result only be used in the mathematics lesson as an illustration of geometrical notions? What is your answer?

When discovering the strip weaving method, did you do mathematics? Did you analyze the effects of the variation of the angle between the two initial strips of paper?

Let's go further. What can be said about that particular necessary angle between the two strips? Observe the resulting strip. That particular angle goes three times into a straight angle (see Figure 9). The little triangles possess three of those angles, therefore ...

What other geometrical knowledge can be obtained? (see e.g. Figure 10).

Figure 9

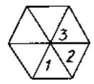
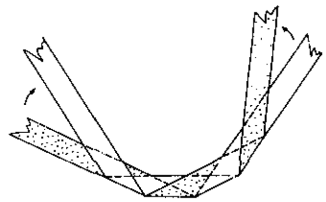


Figure 10



Second example

Consider the following practical problem. In many situations it is disadvantageous to have a densely woven basket, e.g. when transporting small birds in a basket, they must be able to breathe. So it is useful to have a basket with holes. A basket with holes will also be less heavy. Can you weave a basket with holes?

Like in Figure 11?

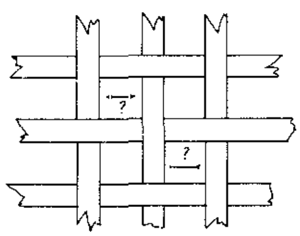


Figure 11

Are the holes fixed? More or less flexible? This may be permitted? Why not? How may the problem be solved?

Maybe by weaving in more than two directions? What happens when you introduce supporting strands? e.g. in a diagonal way? (see Figure 12). How do you have to introduce them so that the holes become fixed? Is it possible to adapt the three directions in such a way that they become more "equal"?

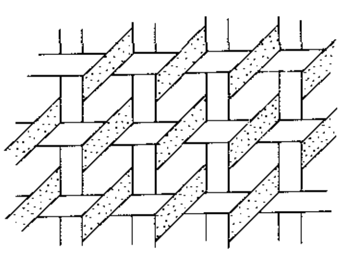


Figure 12

The resultant regular hexagonal pattern is exactly the one mozambican peasants use for their light transportation baskets and fishermen for their fishtraps.

Do we do mathematics?

You are still doubting? Please suspend your judgement for a while longer.

Let us solve together another practical production problem.

Third example

How can you fasten a border to the walls of a basket when both border and walls are made out of the same material? Try to solve this problem for yourself. Take two equal strips of paper in your hands, and consider one of them as part of the border, the other as belonging to the wall. How should one join them together?

Should we join the border- and wall-strips as in Figure 13? No ... It is necessary to wrap the wall-strip once more around the border-strip. In this way (Figure 14)? No? How then? As in Figure 15?

Figure 13

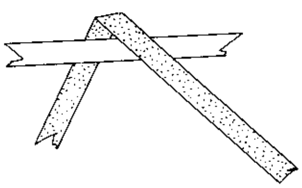


Figure 14

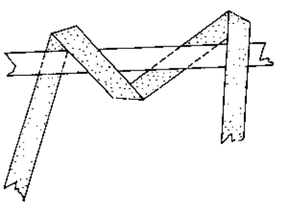
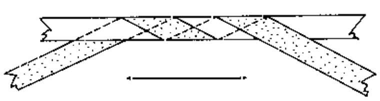


Figure 15



But what happens when you flatten the wall-strip (see Figure 15)? How can we avoid the problem? What has to be the initial angle between the border- and the wall-strip (Figure 16)? Let us complete the border and the wall. What happens? See Figure 17. Are there any other possibilities? Introducing more horizontal strips ... what now? Once again a hexagonal pattern appears (Figure 18). What other geometrical knowledge can be obtained? Possibility of a hexagonal tiling pattern (Figure 19), etc.

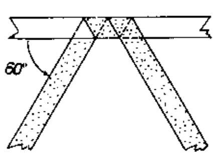


Figure 16

Figure 17

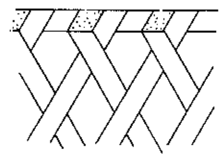


Figure 18

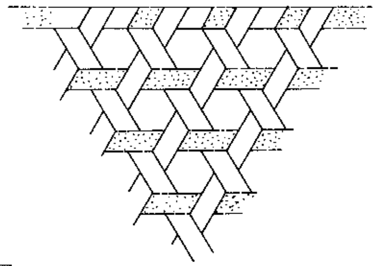
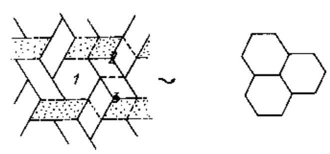


Figure 19



Did you do mathematics?

Let us try to draw some conclusions from these few examples [many other examples can be given, see Gerdes, 1985b].

A method for recognizing hidden geometrical thinking

In our analysis of the geometrical forms of traditional—mozambican—objects, like baskets, mats, pots, houses, fishtraps, etc., we posed the question: *why* do these material products possess the forms they have? In order to answer this question, we learned the usual production techniques and tried to vary the forms. It emerged that the forms of these objects are almost never arbitrary, but generally possess many practical advantages, and are, a lot of the time, the only possible or the optimal solutions of specific production problems, as in the examples we have given. The traditional forms reflect accumulated experience and wisdom. They constitute an expression not only of biological and physical knowledge about the materials that are used, but also of mathematical knowledge. [The first results from this research are summarized in Gerdes, 1985b.]

Cultural and pedagogical value

There exists "hidden" or "frozen" mathematics. The artisan who imitates a known production technique is—generally—not doing mathematics. But the artisan(s) who discovered the technique, *did* mathematics, *developed* mathematics, was (were) thinking mathematically.

By unfreezing this frozen mathematics, by rediscovering hidden mathematics in our mozambican culture, we show indeed that our people, like every other people, did mathematics. After so many years of colonial repression of our culture we encourage, by defrosting our frozen mathematics, an understanding that our people—and other formerly

Continued on page 17

Then she gave up and went for a "back and forth" method. This intuition was interesting. A procedure that uses this algorithm to fill in a square (the turtle always goes back to the bottom left corner) involves a manipulation of trigonometric functions that is beyond the reach of 9 year old children but could be a good exercise for high school children

In conclusion, most of the problems that children are confronted with in Logo can be solved in direct mode by successive approximations, with no real need for more sophisticated programming techniques. The refinement of technique or "style consciousness" [Howe, 1978] will come from the environment, that is the choice of working situations, and from the interventions of the teachers much more than it comes from the child's spontaneous interest. Teaching strategies and pedagogical progressions are not coming from the magic of Logo but from the exploration and the understanding of potential situations that Logo can provide.

A similar question arises about what mathematical knowledge is acquired by children using Logo It is sometimes difficult to evaluate what children are learning while doing Logo in terms of classical mathematical formalism. If one can understand mathematical formalism as a language to talk about mathematics, then Turtle geometry should be understood as a means of learning how to realize things that have a mathematical structure. It is, in this sense, a primitive way of doing math.

The relationship between formal "process-oriented" and informal "product-oriented" knowledge can be seen as conflictual but it can also be seen as complementary. When children are producing sophisticated drawings without any concern for the way these drawings are done, they might be concerned essentially with feeling able to produce such drawings With the help of teachers, this know-how might later become a core for the development of a more elaborate mathematical language.

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Continued from page 12

colonized peoples — was capable of developing mathematics in the past, and therefore — regaining cultural confidence [Gerdes, 1982, 1985a] — will be capable, now and in the future, of developing and using mathematics creatively

Defrosting frozen mathematics can serve as a starting point for doing and elaborating mathematics in the classroom, as we showed in the geometrically-related examples we gave.

At the same time "unfreezing frozen mathematics" forces mathematicians and philosophers to reflect on the relationship between geometrical thinking and material production, between doing mathematics and technology Where do (early) geometrical ideas come from? [Gerdes, 1985b]

Editor's note

The Proceedings of the conference at which Professor Gerdes delivered a fuller version of the above paper are published under the title, "Mathematics and Culture: a Seminar Report," edited by M. Johnsen Hoines and S. Mellin-Olsen. The report can be ordered from Caspar Forlag, 5046 Radal, Norway.

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