# "Mathematising the Hard Stuff"

**DAVID BLUNDELL, ROSALINDE SCOTT-HODGETTS, STEPHEN LERMAN**

The research reported here focuses on the feelings and beliefs of a group of mature women who are undertaking their first year of a mathematical elective which forms part of their BEd (Hons) Degree course. They are all intending to become primary school teachers.

The course is designed in two parts. For the first two years the emphasis is on the unifying concepts and underlying strategies which provide the basis for the development of mathematics over the whole spectrum of mathematical content areas. So, instead of studying blocks labelled, for example, "Abstract Algebra" or "Graph Theory", the students work on different themes: "Notions of Infinity", "Axiomatisation and Abstraction", "Expressing Generality". Material to aid much of this study has been written (for a different purpose) by members of the Open University Mathematics Education Team, who have close links with the mathematics education team at South Bank Polytechnic.

Only the third and fourth years of the Elective are organised in the "traditional" way—that is to say, by organising the curriculum under "content" headings—e.g. Group Theory, Operational Research, Statistics.

The fundamental belief which underlies this course structure is that a recognition and mastery of these general concepts and strategies will enable students to approach *any* new content area with confidence and competence, and that the efficient acquisition of *meaningful* mathematical knowledge is thus facilitated. Furthermore, this acquisition is not dependent upon the skills of the "teacher", but rather upon those of the "learner"—leading to much greater independence and self sufficiency.

Examples within each theme are taken from a wide range of mathematical fields. At the time of writing, the students have spent several weeks investigating the theme "Doing and Undoing", incorporating examples drawing upon Number Theory, Algebraic Functions, Conjugacy (geometric transformations), notions of closure, graphical representation, algebraic structure, number systems and much, much more.

After five weeks of the course the students were asked to write a record of their feelings about mathematics, to encompass *their* early learning experiences, their subsequent development, and their reactions to the mathematics elective work. The selection of extracts from these records, given below, is not intended to suggest that all their experiences before this course were negative. In a number of instances, it was the occurrence of some positive mathematical work that brought them to choose the mathematics elective. Rather, the selection was made on the basis that firstly these students all refer to a significant other person in their early learning, all with powerful but destructive effects, and also because they all emphasised the difference between their expectation and the realisation of the course. For this reason we have called the extracts "contrasts".

## Biographical contrasts

ANNE

"The only maths teacher I remember, although only vaguely, is the one who kindly allowed me to spend most of my time outside the classroom door."

"The maths elective is mostly interesting and occasionally boring, although only because we need to spend longer on some topics than I would like to. This is not a complaint, I wish there had been more people around who made certain I fully understood one step before progressing to the next . . . the approach to the subject is totally different to any I have used before, and maybe (no promises, mind) it will unlock channels I didn't even know I had."

LESLEY

"September 1965. I hate maths . . . I recall heading a page of my maths book "funny sums". For this I was sent to stand outside the classroom, told I was a "bloody imbecile" and that I was the worst pupil he had ever had the misfortune to teach. Just my way, as an 11-year-old, of asking for help . . . End of term report grade C, position in class 30th, teacher's comment "could do better". I never believed I could do better, nobody showed me how."

"The course is not entirely how I expected it to be, it's much better. I enjoy having to think about what I am doing and discovering things for myself. I've realised it's the best way that I can understand. It does sometimes feel like we're being broken in gently before the "hard stuff", whatever that might be. Anyway, I've never been convinced that 5 + 2 = 7! November 1988. I love maths."

CATHY

"My only clear memory of a maths lesson occurred during my third year at senior school when the following dialogue took place:

Teacher: You really are hopeless. Your sister could do better than this.

Me: Well let her do it then! Whereupon I left the room—very bold for a 14-year-old in the sixties."

"I must admit I blanched somewhat when told that the elective was to degree level and for a while I wondered if I had chosen the right subject . . . After the first couple of days I began to wonder if I had stumbled into the wrong room by mistake. Here we were doing sums where numbers had been replaced by letters. No irritating formulae, just challenging fun! From the daft to the ridiculous, I

thought, until one day I realised where we were headed—backwards! Most of the work we have done so far is not so much arriving at a right answer, more finding out where the question has come from and why . . . The rules hadn't been rewritten, we just weren't allowed to see them. It really has been an eye-opener. I have spent years doing sums by rote, now I am being shown how they wrote the rote . . Fun is not usually a word that is associated with maths but that's what it has been up until now Fingers crossed it will continue to be fun."

BRIAR

"Once there was a little girl called Briar. Daddy was a school teacher, he taught top class juniors His main interests, or curriculum strengths, were science and mathematics. Unfortunately, Briar hated her Daddy . . I suppose if one were to attempt to be analytical about this period in Briar's life—she probably rejected maths as a protest against her father. The interest and will to succeed was lost."

"The maths elective is not at all as I had expected, so far it has all been investigative work, it makes your brain work overtime . . All of the maths classes have so far been very enjoyable—so much so, in fact, that I am tensed and ready for the time when the party is over!"

SANDRA

"My secondary school education got off to a very bad start, because, for a variety of reasons, I missed a whole year of schooling Unfortunately the teachers were never sympathetic and failed to recognise my previous academic abilities, and as a consequence I was placed in the lowest group for all subjects"

"I wanted to take this opportunity to really stretch myself in maths and combat any uncertainty I feel. I definitely feel that I will get the help and support I need to accomplish a good standard in my maths from the staff in the maths department. I find this new approach to the subject makes me feel more relaxed about it, although my confidence is still low I feel that there is a mental block that is disabling my ability to think mathematically . . . From my first class in the maths elective, a few weeks ago, I have enjoyed every minute of it and feel that this is half the battle."

## Biographical extracts of the course designer

The principal course designer is very aware of the influence of her own previous mathematical/educational experiences on the development of her own mathematical beliefs. Some significant factors are outlined here:

1955 We are doing workcards. I can do them really fast. Miss Cummings is *so* pleased. I've done adding, taking away, times and division. There are about thirty sums on each card I love pleasing Miss Cummings

1959 The whole class had a test in the hall Now Jimmy Baker, Andrew Black and I have to take more tests. A man is coming specially. Jimmy and Andrew can not read Am I like them? Why me?

1959-1963 I can't succeed. I'm really trying. If I get them all right, that is OK If I don't, that is failure I don't get praise. I *often* get told off. I'm very untidy. I *can't* spell (I *do* try, really I do.) When I passed my 11+ my teacher said that it showed I *could* do things if I try I cried (I *do* try, really I do.) I can do maths, though. I know it's right—all those ticks—and I'm *so* fast.

1963 They explained about the tests. I'm not stupid. The problem is quite the reverse. I *should* be doing better. I should always get top marks. Oh, I see (I *do* try, really I do.) I can do maths. I can do physics This year I can do almost everything—but I can't be neat and tidy I *can't* spell.

1963-1971 I can do maths: I can do algebra, I can do geometry, I can do calculus I almost always get good marks *But* I know that I can't be excellent—I can't live up to expectations. Why do people expect so much of me? Why can't I win approval? (I do try, really I do.) I'm very scared I jump through all the hoops. I work fast No praise, but *lots* of ticks over the years.

1979 I love teaching maths. I want my pupils to love it too I don't want them to jump through hoops. Now I understand, some mathematics Nobody every asked me to do that before—they wanted those right answers But I'm doing maths for *me* now, and for my pupils, I hope. I am full of wonder, as I discover the secrets, the power of mathematics I want my pupils to know this. Occasionally they do!

1986 I teach teachers now. I've been asked to write a new maths course for people training to be primary teachers I don't want them to get ticks. I don't want them just to "know" mathematics I want them to be mathematicians—to recognise the unifying concepts and underlying strategies which pervade mathematics So many people have helped me so much since 1979 I go to talk to John Mason at the Open University, and explain what I want to do. He has been thinking about similar things. This is no surprise—the Open University maths team have greatly influenced my own thinking. They are bringing out material which is just what I need I'm really excited.

## Articulating the contrasts

Early in the second term the students were asked to expand upon their earlier comments and talk further at length about their attitudes and feelings towards mathematics:

"Maths was either right or wrong . . and you didn't know why . . . you had to show your working so

you could see where your dreaded mistake was "

"We haven't (with the elective) found one dreaded mistake yet . . there's not one set way of doing it . . . there's easy ways . . . we're never afraid of anything now, are we. We will say that we can't do this . . . somebody come and help us—we know it's OK to do that. Working in a group helps—someone will trigger off a new idea, then we can start working again. That's completely different from school. We were regimented. Sat in rows. You couldn't turn round and say, "Hang on, I can't do this—give me a hand." That was copying. Yet I think that that is what can progress you onto the next stage. Oh yeah, 'cause we don't copy off one another, we just bounce around ideas. You still do it yourself, don't you. And, if you explain it you understand it as well, because it clarifies it for you, explaining it.

As the students continue with their elective studies they articulate this contrast as a clear contradiction which we broadly describe as that between the *inside* and the *outside* representations of their mathematical self: it becomes clear that the struggle to resolve this conflict delineates the very essence of their activity within the elective and is the root of their confidence and sense of achievement.

The inside view entails their sense that the maths is challenging and appropriate, further that it is "real" maths so that their achievements inspire confidence. At a meta-level they have a sense that they are "learning how they wrote the rote" which is implicitly empowering. They make a nice distinction between being wrong and failing, also they note that they are permitted time in which to evaluate their mistakes and foster understanding.

There is a clear contradiction, however, between this sense of themselves as able to undertake real mathematics, and their internalised sense of what others expect from mathematicians. They want labels to append to the mathematics which they do: big, prestigious (in their eyes) labels such as algebra, calculus, etc. Clearly they were satisfied that they had recognisably done some "probability" in a particular session, not only do they now know that they can do it, but they can also satisfy what they felt the outside requires of them. It's interesting that Linda now discounts the need to be good at mental arithmetic, as a prerequisite for "outside" accreditation; but for the group as a whole, examinations and A grades are still seen as lending credence to their confident grasp of the "biggies"—that is, the prestigious topics which are invariably viewed as really "hard".

This inside/outside dichotomy comes out in the things which they say at different times in the conversation. For example, that "yes this is real maths," yet they'd "feel cheated" if this was all they did for their undergraduate course.

They don't seem to be aware of the nature of the "doing/undoing" material itself, with its process perspective. It's working well, in that inside they feel much better about themselves as mathematicians, yet it doesn't fit their recalcitrant content-based model desired from the "outside."

INSIDE

We're never afraid of anything now, are we? Whatever you give us we'll jump in and have a go—or we'd sit back and scratch our heads (laughter).

The idea is that you'll become so confident and happy with maths that you can advise other people. If it carries on (being as it is) I think it will achieve that aim. We'll be equipped to be advisers. I feel so much more confident in maths now—because I don't feel that I can fail; because there isn't a right way to do it and a wrong way to do it. I know that whatever I do—even if I get it wrong—just having tried it . . . there's no shame in getting it wrong, is there. You think "Oh, I've got that wrong; I'll have another go." I don't think you are wrong—it's just that you may be looking at it a different way. But it's not failing, is it? You're not looking for an answer all the time. Sometimes you're just fiddling around seeing what will happen if you do something.

(*Prompt.* How do you come to know that you've achieved something? How is it you come to feel successful?)

Because you've understood what you are doing. Failure is when you haven't understood what's going on. It's like sitting in front of a blackboard at school and you think, yes, I can do that. Then at home you get out your pen and it's gone.

I really love doing maths. I really look forward to Tuesdays and Fridays. All this time I've been going on thinking I'm no good at maths—and now I'm doing it.

(*Prompt.* How confident are you that this mathematics, which isn't about failing, and isn't about right and wrong, is *real* mathematics?)

Oh, *it's real*. Yeah, oh yes. It certainly *is*. We're convinced (laughter). The "hard stuff" is here but we're coping.

OUTSIDE

I want to do the hard stuff. Because that's what is expected, isn't it. Like calculus and algebra.

The thing that worries me is that you're suddenly going to pop up with an exam, and I won't be able to do it. I can merrily investigate—but if there was an exam paper, then there has got to be an answer.

To me, maths isn't maths without an exam. I mean, that's the way we were brought up with maths, you had a test every Friday. If you didn't pass the test then you didn't know it. If you haven't been tested you haven't had two years of degree work. You're not going to have had what you expected. I'd feel almost cheated. It's not really being tested, but it's having something really hard—challenging.

(*Prompt.* So do you feel at the moment that the work is too easy—not sufficiently challenging?)

(Raucous laughter) No! No, it's challenging, but I don't think you could convince me now that it's a degree course. 'Cause all through maths, right, everything's got a name—this hasn't got a name.

(*Prompt.* It has got a different kind of name—it's called "doing and undoing." I keep giving you the name.)

(Laughter) Yeah, right But then that covers everything. That's not the actual—like—what you're doing You don't stand up and say "Right, we're going to do algebra, or geometry or ." it just doesn't exist anymore

*As the discussion progressed, it seemed that there were attempts made to resolve this conflict:*

(*Prompt*: So is there something about the maths we do which you can't quite value in the same way? You won't feel that you've "done" mathematics until we've given you the right sort of labels—the labels which you were expecting?)

Well if we were to come across it in the future, would we recognise it—would we know what to do Yeah. But *would* you, or would you look at it . I feel confident to look at in in a different way. You'd probably just look at what was written down and ignore the label Yeah, and think ah! If you do that, and that, and that . .

There's still a feeling that something is going to creep round the corner. You're preparing us—you're keeping quiet so that you don't frighten us off. If you'd started off with the hard stuff and stayed with notions of right and wrong, I think people would have switched off. I think most people would have left, but if you prepare for it, and problem solve, and think your way round it—look at it from different angles Maybe talk about it, yeah, then you can sneak the hard stuff in, and maybe we won't realise it, because we'd just get on

But we're going to come across labels. I can't think that we're going to have two years of degree level work without having the labels put on

Like the week, with that probability. That was the first time we really had a label. Oh, this is probability. I think it's done on purpose Yes, in case we think, like, "is this vectors—'cause I can't do vectors" (laughter). Perhaps they leave the labels off on purpose. Maybe it's because you know that we'd switch off, like you said, "now we're going to do matrices" we'd say we might as well go home (laughter).

(*Ros* Actually we're "doing" those this afternoon).

Whereas if you've got a name like doing and undoing, you have a go, because you're not put off by things you've done before That's the conclusion I've come to anyway. Then afterwards you can look at it and see that "oh yeah," it's like something I've done before Why on earth was I afraid of these

But everything is labelled and you know they are (say) vectors, and you can't do them

(*Prompt*: How hard do you think a degree in maths is?)

Oh—*really tough.* (Laughter) You've got to be really clever to do that I don't think I could do that

But I don't think I could have applied a lot of maths I learned earlier. Like probability. I could *never never* have done that. But we've got increased confidence Yeah. And we're more likely to go home, and look up something we haven't done. We *could* be taken on to a degree But it's just that fear . . .

## Discussion

*Language*

It is when considering mathematical language that we begin to see a shift towards resolution of the "inside/outside" conflict with its commensurate confidence/achievement goal structure. For the emergent confidence acquired via their "inside" perspective authorises them to distinguish what they see as content from form, the mathematics from the "technical terms" Since, whilst they persist in their desire for prestigious labels which they can append to their activities so as to legitimise their achievements, they are increasingly aware of the particular game which they are playing, and can subsume the "outside" voice to "inside" control. The students are happy with the material because it doesn't intimidate them, yet they want the labels which do, so their best scenario is to do the work, be told what it is they've done and then feel in control of the label.

INSIDE

If someone comes up with some brilliant new word, you're completely stumped—but it may be something you can do. Like those questions you gave us last week, which were from the BSC(Hons) Mathematical Studies assignment. When you wrote them up in technical language, mathematical language, we didn't know where to start Like that probability—but we did it on the train going home. We really understand it It is really the language

But you have an "A" level, and you know the language, I don't think that you're actually . . . you don't give the impression of being like 100 miles in front of everybody else, it's only that you know the language, just a couple of miles. You don't feel that you are, so that shows that it must be working

*Confidence and achievement*

Clearly the students have gained in confidence by this stage of the course, but it is not yet an unequivocal confidence. It seems as though they are still ambivalent, feeling confident when they focus on their process abilities, but doubting when concentrating on content in mathematics. And yet they are aware of making significant progress:

> "I feel now that every week I'm achieving something I'm being stretched but I'm not being broken. And every week I come out and think, "I did that" and I feel it's a continuing thing "

However, they do not overtly acknowledge and value the acquisition of process skills to the extent which we might have hoped.

From our own perspective, we see amazing changes—specialising, conjecturing, symbolising, generalising and formalising are part of their "standard" mathematical behaviour now. They are also keen to extend—challenging each other (and us) with "what if?" questions. They have been offered the "process vocabulary" whenever it seemed appropriate—but so far don't seem ready to analyse their "doing and undoing" in these terms. The ability to extract the essence of novel mathematical situations has also developed significantly They "know some mathematics" now. They "know" about identity elements and inverses, they "know" about closure and about restricting domains Their failure to talk about these indicate that,

again, these are not labels which they recognise as categorising mathematics.

## Conclusion

The students' expressed views indicate something fundamental about the perceptions which we hold of ourselves: that ambivalence does not necessarily resolve into compromised unitary conception of self. Rather that the self is so often dualistic, and marked by a shifting conflict between fundamental oppositions, in this case their undoubted confidence in their achievements and hesitance about what might lie beyond.

For our part we hold that realisation is half way to resolution, that the students' awareness of this dualistic conflict will mobilise their confidence and affirm their sense of what is now possible. Further, that we must continue to build on the foundations for a meta-discourse which we have laid, that is to offer process labels which are authenticated through the student's sense of empowerment.

# A FOUCAULDIAN ANALYSIS OF MATHEMATICAL DISCOURSE

Apologies are due to the author, Maggie McBride, of the above-named article in our last issue, and to our readers, for the omission of the last seven notes at the end of her article. The complete set of notes is printed below.

## Notes


[1] Michel Foucault, *Power/knowledge: selected interviews and other writings 1972-1977* trans. Colin Gordon et al., ed. Colin Gordon (New York: Pantheon 1980), 131

[2] Irene Diamond and Lee Quinby, eds., *Feminism and Foucault* (Boston: Northeastern University, 1988)

[3] Hubert Dreyfus and Paul Rabinow, *Michel Foucault: beyond structuralism and hermeneutics* (Chicago: University of Chicago, 1983), 217

[4] Michel Foucault *Power/knowledge* 119

[5] Michel Foucault *Power/knowledge*, 198

[6] Hubert Dreyfus *Michel Foucault* 220

[7] Hubert Dreyfus, *Michel Foucault* 221

[8] Michel Foucault, *The archaeology of knowledge* trans. A.M. Sheridan Smith (New York: Pantheon, 1970), 191

[9] Michel Foucault *The archaeology of knowledge* 117

[10] Michel Foucault, *The archaeology of knowledge* 183

[11] Michael Sullivan, *College algebra and trigonometry* (San Francisco: Dellen, 1987); and Rick Billstein, et al., *A problem solving approach to mathematics for elementary school teachers* (Menlo Park: Benjamin/Cummings, 1984)

[12] Michel Foucault and Gillies Deleuze, Intellectuals and Power," *L'Arc* (1972): 3-10

[13] Louise Wilkinsin and Cora Marrett, *Gender influences in classroom interaction* (Orlando: Academic, 1985), 17-55; and J.R. Becker, "Differential Treatment of Females and Males in Mathematics Classes," *Journal for Research in Mathematics Education* 12 (1981): 40-53

[14] T. Good et al., "Effects of Teacher Sex and Student Sex on Classroom Interaction," *Journal of Educational Psychology* 65 (1973): 74-87

[15] C.P. Benbow and J.C. Stanley, "Differential Course-Taking Hypothesis Revisited," *American Educational Research Journal*, 20 (1983): 469-473

[16] Michel Foucault, *Discipline and punish: the birth of the prison* trans. Alan Sheridan (New York: Pantheon, 1977), 184-5

[17] Paul Cobb, "Contexts, Goals, Beliefs, and Learning Mathematics," *For the Learning of Mathematics*, vol 6, no 2 (June 1986): 6-8

[18] Michel Foucault *Archaeology of knowledge*, 117

[19] Michel Foucault, *The history of sexuality: Volume I: an introduction*, trans. Robert Hurley (1978; rpt. New York: Vintage, 1980), 3

[20] Michel Foucault, *Archaeology of knowledge*, 57