FORMATION OF MATHEMATICS GRADUATE STUDENTS’ MATHEMATICIAN-AS-TEACHER IDENTITY

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Graduate studies in mathematics are the means by which scholars of mathematics are, if we are cynical, produced or, if we are hopeful, educated. It is reasonable to assume that doctoral students emerge from their studies prepared both to do mathematics research and to teach mathematics. Expressed differently, it is expected that new mathematics PhDs have become “stewards of the discipline”—people who generate, critique, transmit and transform the knowledge and values of the field (Shulman, cited in Golde, 2006, p. 3). However, there is a view that graduate students are not prepared sufficiently for their day-to-day university work (Walker et al., 2008), let alone prepared to be fully-fledged stewards of a discipline.

The process of graduate study and its impact on the preparation of future scholars has been a topic of interest for many years. Some researchers have studied the induction processes of graduate studies (e.g., Clark, 1987). Others have focused on the socialization process that takes place during graduate programs, where “persons internalize behavioral norms and standards and form a sense of identity” (Weidman, Twale & Stein, 2001, p. 6). Still others have concluded that doctoral education is “a complex process of formation,” where what is shaped in that process is “the scholar’s professional identity” (Walker et al., 2008, p. 8). Moreover, Becher and Trowler (2001) have noted that “being a member of a disciplinary community involves a sense of identity” and that the process of taking on that identity is most intense at the end of graduate school (p. 47). Colbeck (2008) notes that “developing an identity as a professional scholar is an essential task for a doctoral student” and that it is during transitions to new roles that “adaptations to one’s sense of self are more likely to occur” (p. 9). These ideas about identity formation point us, in particular, to the possible meanings that graduate school experiences have for how students develop their identities as mathematics teachers, within the process of becoming scholars.

In this article, we reflect on the experiences of six graduate students who studied in the mathematics department of a comprehensive, research-intensive university in Canada (Beisiegel, 2009). We use their experiences to theorize how the identity of mathematician as teacher is shaped. By mathematician as teacher as an identity, we mean the persona that emerges when the individual encounters the teaching situation; that is, when the mathematician is related to the learner by virtue of teaching mathematics and all that that entails. We propose that through the process of graduate studies, the voice of a mathematician emerges while the voice of a mathematics teacher is silenced. Stewardship of the discipline (Golde & Walker, 2006), rather than being the renewal or transformation of teaching practices, is the replication of traditional teaching methods and of the canon of Western mathematics. However, in spite of the replication, of carrying on tradition, for human beings the repetition of day to day living brings about what Caputo (1987) calls a remainder, something left that comes from the “power of the individual to forge his personality” (p. 21). The remainder represents the transformative potential of the mathematician as teacher.

With notions of identity in mind, we are drawn to von Foerster’s (2003) uncanny ability to offer mathematical analogies that offer insight into human and social phenomena. He observed that much of what is done in education is an attempt to turn complex beings (what he calls non-trivial machines) into predictable beings (or trivial machines). A trivial machine, he said, is characterized by a one-to-one relationship between its “input” (stimulus, cause) and its “output” (response, effect) (p. 208). He suggests that our education system is designed to transform children from non-trivial to trivial machines because then they will become predictable adults. In the non-trivial machine (the learner), there is a different relationship between input and output: “Their input-output relationship is not invariant, but is determined by the machine’s previous output. In other words, its previous steps determine its present reactions” (p. 208). Hence, the learner as a non-trivial machine is impacted by its experiences, not because the experiences are direct inputs processed into predictable output, but, instead, because these experiences alter the machine itself at the same time as the unique, and hence unpredictable, output is being generated. Using von Foerster’s analogy, we argue that research on university mathematics teaching suggests that trivialization is at play in graduate programs in mathematics. In particular, we demonstrate how graduate study within one department of mathematics works on graduate students’ identities to trivialize the mathematician as teacher.

Unlike previous studies, this article is not about how to “fix” mathematics teaching at the university level. Rather, it is based on a study that had the objective of comprehending how it is that university mathematics teaching, in spite of various curricular-reform movements, critiques, and initiatives
to enhance the teaching of mathematics at university level, remains seemingly impervious to such suggestions. In particular, this article focuses on understanding the experiences of graduate students in mathematics, a topic that has only recently emerged in conversations about university mathematics teaching (Border, Speer & Murphy, 2009; Golde & Dore, 2004; Wulff, Austin, Nyquist & Sprague, 2004).

**Studying the development of the mathematician as teacher**

In order to better understand the development of mathematicians as teachers it is important to learn about what graduate students experience in their programs. After all, it is through the formative experiences of graduate school that mathematics graduate students’ identities are shaped as mathematicians, researchers, teachers, or more generally as professors of mathematics. [1] Through multiple semi-structured (hermeneutical) conversations (Carson, 1986) with six graduate students conducted individually and in two focus groups over the course of an academic year, we learnt about their experiences in the mathematics department in which they all studied.

Emily, Sarah and Steven were master’s students, with Emily just beginning her graduate studies, and Sarah and Steven in their second year. Chris, Robert, and John were doctoral students, with Chris completing his last few courses and thinking about dissertation research, Robert in the midst of his dissertation research, and John close to his defence and applying for academic positions. Their ages ranged from 22 to 33 years. While each of their paths to graduate study in mathematics was unique, all but one of the participants expected to work in a university or college once they completed their degrees. During their graduate programs, each of the participants had been assigned some form of teaching assistant work. This work included a combination of helping one-on-one with homework exercises in tutoring centres, grading homework assignments and exams, or leading one-hour tutorial sessions.

Analyzing transcripts of our conversations with the participants, a number of themes emerged related to: teaching assistantships, standard mathematics textbooks and model lessons, undergraduate content, attitudes of undergraduates, stereotypes of mathematicians, coursework and research. These themes were observed as forces that acted on the development of graduate students’ identities. In this article, we discuss only two of these forces: teaching assistantships and mathematical texts. We focus on these themes because they represent important features of the teaching experiences that mathematics graduate students have; in particular, their teaching assistantship work represents their first experiences in a teaching role, while texts represent the content they rely on when they teach.

**Structures of teaching assistant work**

Graduate students’ first exposure to teaching undergraduates is most often through a teaching assistantship, for which they are expected to grade papers, tutor students, or teach courses. These activities have strong constraints and bring constant demands that structure students’ work. These structures format graduate students’ ideas of their work as teachers. In reflecting on this study, we propose that the structures of their work as teaching assistants [TAs] prevented the graduate students from engaging in meaningful experiences with undergraduates. In particular, their work in a tutoring centre consisted of helping seemingly countless undergraduates, repeatedly explaining how to solve the same assignment questions. Emily noted the toll of this work on her desire to ensure that students understood the mathematics:

Emily: So often when they [undergraduate students] come to you … it’s pretty demanding … there’s so many of them … I try so hard not to give the answers away, but so often you’re basically one step away. And it’s nice to see them do a few steps on their own and to be able to see that process. I don’t find the [tutoring centre] is very conducive to that because the second they’re able to be independent, they move away because there’s someone else in line. Right? Like you don’t get to see that [learning process].

Robert’s experience of the large number of help seekers was impacted by having to address the same problem, over and over again:

Robert: I don’t like to teach the tutoring centre in the sense that you don’t really teach things. You’re just kind of being a problem solver. You know, people have a particular problem on the assignment. And so I find that, a lot of TAs at the end of the day, because they’re being asked the same question for the thirtieth or the fortieth time that week, that at the end of the day, they are just so tired of the question that they would just tell anyone who comes in and asks that question basically how to do it. And I kind of found that because it happened to me, too.

Steven spoke of the exhaustion that he felt in helping students one-on-one for many hours. John said that his teaching assistant work was not helping him to learn how to teach because it was “just a tutoring exercise.” Sarah spoke of how she “was not excited to tell [undergraduates] how to do” problems, but it just happened “so many times.”

These experiences are distressing given that the graduate students had initially been interested in connecting with others about mathematics and in helping students understand mathematics, a feeling that has been found to be common among graduate students (Golde & Dore, 2004). However, the structure of their assistantship work meant that the graduate students did not have opportunities to observe undergraduate students’ growth of understanding of concepts over time. Consequently, the act of tutoring became an unrewarding and tiring experience, with Emily describing how her work became “how fast can you turn them over.” When there were many students waiting for help, rather than provide the undergraduate students with a more in-depth learning experience, it became “a lot faster to plug and chug.”

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The task of grading papers also had a significant impact on how the graduate students felt about their interactions with undergraduates and mathematics. The directions from the course administrator were to mark only one or two specific questions on a homework assignment, even though the undergraduates were required to do more problems. Chris described his disappointment in this structure and the time constraints that limited what he was able to do:

Chris: It’s very frustrating when I’m marking one question and I can see all the other questions they’ve done and it’s just complete crap. I’m only supposed to mark one question. So it’s hard. I just don’t have time to go through everything and make comments.

To Chris, it seemed neglectful not to correct more problems, especially when he could see that students were doing other problems incorrectly. Like Emily, Chris’s expressed desire to go about things in a particular way resided in his concern for the students. He wanted students to understand the mathematics and felt that they would not learn if most of their homework problems went uncorrected. During examinations, the graduate students were called upon to work on a grading assembly line, each assigned to marking one problem on students’ exams. These periods often required up to ten extra hours of work in one day. Most often, each graduate student was assigned one problem to mark on each of the exams. Steven expressed the heavy load and exhaustion this work brought: “It’s just tough when you’re on your two thousandth paper of the day. I just want to get the hell out of there.”

Other than the directions the department provided for marking papers, there was little, if any, guidance or support for teaching—no mentoring, no dialogue, no venue for asking questions or for questioning how things worked, for expressing their frustrations or making suggestions of how they might like to work or what they might find most valuable in their interactions with undergraduates. The participants’ interests in, and even passion for, helping students learn were frequently silenced by the structures of their teaching assistant work. John’s statement below indicates the impact of this silencing:

John: A student [an undergraduate] said, “I just don’t understand how it works. I do see why some things are a certain way” or they’re [the undergraduate] kind of fed up with the course a little bit and I might have the same complaints. I mean, but there’s nothing I can do about it, so it’s, you have no control. You can’t really work outside of a certain box.

The graduate students had ideas of how things might be improved, not only for their own work, but also for the ways in which they helped the undergraduates. Yet, it became clear that the graduate students neither had a voice, nor felt that they could express their own ideas within the structures for working with undergraduates. The structures of their tutoring and grading work trivialized who they were as teachers. By answering the same question or grading the same problem repeatedly, the workload and the strain caused them to become trivial machines, preventing the level of engagement with learners that they desired. They surrendered their ideas and hopes about the ways in which they wanted to interact with the undergraduate students they were charged with helping.

**Structured by the text**

Mathematical texts and the structures of the content within them work as powerful forces on learners and teachers (e.g., Nicol & Crespo, 2006). Davis and Hersh (1981) have commented on how the human is often written out of mathematical text: the mathematician’s writing “follows an unbreakable convention: to conceal any sign that the author or intended reader is a human being. It gives the impression that, from the stated definitions, the desired results follow infallibly by a purely mechanical procedure” (p. 36). Morgan (1998) has argued that presenting “processes as objects […] is part of the strength of mathematics, but at the same time, it increases the impersonal effect, strengthening the impression that it is these process-objects that are the active participants in mathematics rather than the human mathematicians” (p. 15). Such impressions from texts can influence the teaching of mathematics, since “[Mathematics] is considered a serious and exact science, a strict discipline, and such images of seriousness, exactness and strictness often inform how it is taught and how it is understood” (Jardine, 1998, p. 53).

We propose that one mechanism for the trivialization of identity is the mathematical texts [2] that participants were exposed to through their years of study and the new experience of relying on texts in presenting mathematics to others. The graduate students’ understanding of mathematics through texts, through the presentation of mathematics in classrooms, and through lectures and seminars, all influenced their views of mathematics and how mathematics should be taught. The texts, and the content within them, spoke to their mathematician-as-teacher identity of what was salient, the source of authority in mathematics, and the identification of what was imperative and central in a university mathematics classroom.

The graduate students understood mathematics to have a particular form that consisted of axioms, definitions, theorems and their proofs. They seemed to view mathematics as set in stone, inflexible, unbending. This fixedness was reflected in their discussions about mathematics. For example, Steven and John saw mathematics as embodying an objective system of truth:

Steven: Within whatever our truth system is, I think that’s as close to absolute truth that you could probably get [with] mathematics. In no other subject can you make a statement and say, “Well, this is right.” Right? You can’t have an opinion. You can’t have anything except the fact that “Yeah, this is true.”

John: Yeah. True. It doesn’t have the subjectivity that most other things would have.
Steven: Well, most every other thing has subjectivity. Mathematics would be the only subject where you can take out subjectivity. Right?

Their comments went further than Bass’s (2006) assertion that mathematics “is the only science that thus pretends to claims of absolute certainty” (p. 104) and claim that mathematics is, indeed, absolutely certain; that it is what Burton (2004) refers to as “The Truth” (p. 21).

How might a mathematics professor have a voice deeper than and outside of a text that utters the “truth” and demonstrates what is the “best case scenario”? Within the experience of these graduate students, the authority of the text played a part in the trivialization of their mathematician-as-teacher identity. In their experiences, taking on some of the teaching work of professors, the participants felt the weight of mathematics and the authority of the curriculum. This was particularly relevant when talking about calculus, the course in which the graduate students most often gained their first experiences as teachers:

John: It’s easy to keep teaching calculus like this. We’ve done it forever. We know exactly what we have to do. Almost everyone does it the same way. I mean even by the time you have your PhD, you’ve probably been teaching calculus three or four times. You’ve taken it. You’ve TA’d for it. I mean, you know the problems. You know the classic examples. You almost don’t even need a book. You can just walk up there and start teaching.

There was the sense in what John said that there is nothing left for the professor to understand, know, or learn about teaching calculus. It seems that the years of seeing calculus presented from the text have left an indelible imprint that there is but one way to offer calculus to students. When talking about the classic problems and examples and doing things in the same way as everyone else, we are reminded of Davis and Hersh’s (1981) observation that in mathematics, “the main thing was what you wrote down. As to spoken words, either from the class or from the teacher, they were important insofar as they helped to communicate the import of what was written” (p. 3). When graduate students have only their own understandings of mathematics and the texts of their past to rely on, they feel as though they have an inconsequential role to play in shaping how mathematics is taught and learned at university level. For example, in Robert’s description of how he might prepare a lecture, he first described a process of looking at different texts and creating a combination of resources, saying that he would “take the ingredients a little bit from here, a little bit from there… so that it suits what the class is doing.” However, he followed this idea with: “The teaching that comes out at the end of the day is not much different from if you just take your supervisor’s notes and just write them in class. I don’t think it’s really that different.” A similar sense is apparent in Steven’s description of what it is like to be a learner in a mathematics classroom:

Steven: I don’t know. It’s like we’re both [the professor and the students] going through the motions. Maybe it’s like the students are going through the motions by coming to class, the professor’s going through motions of teaching it. It’s like we’re all going through the motions. And maybe that’s the point, then, of why be enthusiastic because we’re just going through the motions.

Sarah shared a similar picture of the mathematics classroom, saying “It’s the prof talking at the board for 45 minutes and you’re just sitting there.”

Hearing these graduate students refer to mathematics and the authority of texts, the notion that mathematics teaching must take on a certain form, we began to wonder—does it matter who the mathematics professor is. After all, “personal flavours are entirely lost in the objective mathematics they, as teachers, thrust towards reluctant learners” (Burton, 1998, p. 140). Robert explained it this way:

Robert: There’s a certain rigor involved that how, you could teach a little bit better, but I don’t know how much variety you can actually put in. How much different with professor A, different from professor B, …

In his statement, the image that Robert conveyed was one where the professor did not have much bearing on what might occur in the classroom. In the language of professor A and professor B, there is an interchangeability between professors, as though their identities might be so alike or the differences so insignificant that it would not matter who was in the classroom. As Jardine (2006) observed, in mathematics there exists a “mood of detached inevitability: anyone could be here in my place and things would proceed identically” (p. 187). Chris, too, commented, “I don’t really care who teaches calculus and who doesn’t,” and so to him the teacher did not seem to be of importance—calculus was the important presence. We might think there is hope for graduate classes, but not in Robert’s view: “I mean for a graduate course, you can use a different textbook, a different approach, but then even in that case, I think the difference [in how one teaches] is subtle.”

Said (1983) has written that “texts are a system of forces institutionalized by the reigning culture at some human costs to its various components” (p. 53). The significance of mathematical texts, the images of the university mathematics classroom, and perceptions of mathematics as embodying truth were seen in the participants’ statements, and in how they viewed the relationship of professor and mathematics in the classroom. The authority of mathematics and of mathematical texts had influences on classrooms and identity implying that the mathematician’s role becomes subordinate to that of the mathematics itself” (Burton & Morgan, 2000, p. 435).

When graduate students face learners for the first time in the mathematics classroom they bring to that encounter their interpretations of the mathematics to be taught, what it looks like and how it should be presented. Although these inter-
interpretations are personal, they are superseded by the view that the mathematics they will teach is *anything but personal*. The participants in our study believed that the mathematics to be taught can be found in the textbook. They expressed views of the mathematics they would teach as absolute and well-structured. They agreed that there is a need for an instructor in undergraduate mathematics, but the role of the instructor is solely to deliver the content to the learner. That content includes the examples and explanations, theorems and proofs, and exercises and problem sets found in any good textbook. In the textbook, because the content is organized for the learner, there is no need for the professor to do anything with it other than to pass it on to the next generation.

Britzman (2003) has written about how inflexible perceptions of curricula can influence teachers: “When knowledge is reduced to rigid directives that demand little else from the knower than acquiescence, knowers are bereft of their capacity to intervene in the world” (p. 46). In the context of this study, it seems that the teaching of mathematics was tied to rigid directives and that the graduate students were unable to see any other way of teaching mathematics. In Robert’s admission that “There’s no freedom in talking about it,” we understand that he has lost his capacity to intervene in his world of teaching mathematics. Indeed, if mathematics is absolute truth, if it is fixed, if the curriculum represents what is best, then what imprint can a professor give to the classroom or leave with students? What influence can he or she have over what is and how it is presented to students? What place can their identities have in this situation?

**Repetition and the remainder**

The doctoral students represent the participants that were closest to being mathematicians and university mathematics teachers. As they are “almost” professors of mathematics, it is important to look at what they have to say about who they can or will be as mathematics teachers. Chris was just beginning the second year of his program. When he spoke of his future role as a mathematics professor, he expressed a desire to be different from his own teachers, but then said, “I don’t think I will be.” Robert, who had completed his coursework and was well into his doctoral research, spoke of the mathematics teacher in terms of professor A and professor B, as though the identities of university mathematics professors are interchangeable and, thus, rendered irrelevant. Finally, John, the most experienced of the doctoral students, had begun to look for a job as a mathematics professor. He said teaching would not be important to his work and so he did not think about it anymore. On the continuum of their experiences, then, it seems that there is a subtle but profound transition in the mathematician–as–teacher identity, from “wanting to be different” from one’s professors, to “who I am won’t make any difference”, and finally to “I know it won’t matter.” Is it that the education of the mathematician is complete when the trivialization of the teacher in the mathematician’s identity has been achieved?

Although the study points us to this conclusion, we refuse to end here. The examples of exemplary university-level mathematics professors, those whose teaching practices are unique and engaging, make us realize that complete trivialization is *not* inevitable. In thinking of how trivialization might occur, Caputo (1987) reminds us that individuals hold on to their uniqueness within the power of structures and experiences: “By repetition the individual becomes himself, circling back on the being which he has been all along” (p. 12) because, he suggests, “There is always a ‘remainder’ no matter how much is subtracted from the individual by the taxing business of everyday existence” (p. 21). In the context of the study explored here, it became clear that despite the pressures, time constraints and expectations the participants were under, there were “remainder” for each of them, indications that they were different from those they were supposed to repeat, where it seemed the participants were subtly and gradually “carving out an identity for themselves” (Caputo, 1987, p. 30). For example, Steven was going to keep pressing for a new hybrid of mathematics and computer science because he felt it was important for the evolution of the discipline; Sarah said that she would hold onto and share her love for mathematics whether it was in a department of mathematics or elsewhere; and Chris was going to continue making more problems than he was instructed to because it represented not only a learning experience for understanding his students’ thinking, but also because it would help him be a better teacher. In such examples, we see that complete trivialization is not taking place in spite of the formatting power of the forces within the graduate student experience. We suggest that it is in the remainder that the possibility lies for the emergence of new mathematician–as–teacher identities in the young mathematics professor.

What possibilities are there for transforming the graduate student experience in mathematics departments to encourage the identity of teacher to remain present and alongside that of mathematician? Is there a collective remainder that has been activated or that could be activated with the growing concern for the scholarship of teaching and learning in tertiary education? We believe that the possibility for changing practices in graduate school begins with raising awareness of the experiences of learners in those settings. Within the experiences of these mathematics graduate students and their responses to those experiences, there is confirmation of Colbeck’s (2008) conclusion: “when two identities with contrasting meanings and expectations are activated at the same time, an individual is likely to encounter stress” (p. 10). However, with Caputo’s remainder, we see a possibility for “simultaneous activation of two identities with similar levels of salience and commitment” if “the identities involve shared meaning” (Colbeck, 2008, p. 11). How, in graduate studies, can structures be modified and activities incorporated so that graduate students of mathematics can learn about their multiple identities as mathematician, researcher, professor, teacher and faculty member? What changes can be made to graduate studies so that mathematics graduate students complete their programs to find themselves as stewards of their disciplines, who live with these various identities not in conflict, not understood as mutually exclusive or prioritized over another, but where their multiple identities can be brought to the surface and supported?

We see valuable suggestions in Colbeck’s (2008) work on identity development in doctoral education. She recommends creating “shared meanings across different identities”
in order to “help doctoral students craft professional identities that integrate their identities as researcher, teacher, and service provider” (p. 12). She notes that such role integration will increase graduate students’ well-being and energy, allowing them to more fully ‘apply research skills to improve [their] teaching and [their] students’ learning,” allowing them to have “thoughtful communication with students” (p. 13). Our work suggests that mathematics doctoral programs might begin this work by focusing critical discussions on teaching assistantships and texts to create a culture that uncovers the shared meanings between mathematician and mathematics teacher.

Notes
[1] In North America and some other parts of the world, academics of any level are referred to as professors.
[2] The word “text” has double meaning here. It is both the text of the textbook and lectures and it is the text of the context, the unspoken that is written into our phenomenological selves.

References