# Theories for the Classroom: Connections between Research and Practice

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In this brief essay I am trying to build a case for, or lay out the grounds for, possible connections between the "discipline" of mathematics education and the practice of mathematics education in the classroom. To do this I will be paying special attention to the recent constructivist, interactionist, and enactivist theorizing and research in mathematics education. I will argue that this research, both in its practice and in its "products", offers occasions for teachers to think differently about their practices, about their students' practices, and about the curriculum. Because such inquiries are aimed at explaining and interpreting student mathematical-activity-in-a-setting and, in more recent developments, researcher-student-teacher mathematical-activity-in-a-setting, teachers are in a position to derive practical ideas from the work which they can then bring to their observation and thinking about student mathematical actions, as well as about the ways in which they can provide occasions for those actions. But before turning to the details of the discussion I would like to consider an example. Imagine the following classroom scenario.

> A teacher of grade 2 children has heard from the mathematics coordinator that using a "100 frame", a $10 \times 10$ grid of 10 rows of numerals from 1 to 100, is a good way to move children from solving addition and subtraction problems through counting by ones to more powerful tens-based or numeration-based strategies. Consequently the teacher shows an example or two on a large demonstration "100 frame". She dutifully and clearly shows that in adding 23 to 14, for example, one can "jump up" the frame from 14 to 34 by adding ten twice (or twenty ). From 34 one simply counts on 3 more to 37 to complete the process. Following this demonstration, and perhaps a few more involving interactions with some children through leading questions, the teacher now provides similar tasks for the children.
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> As she goes around the teacher notices that Gregory is simply using the frame as a counting space; Natalia is "solving" the problems slowly and somewhat erratically counting on by ones, sometimes losing count and starting over; Alexandro is excited by what he perceives as a new way to add and does the tasks more or less as anticipated; and Tanya has breezed through several exercises and now is doing the "additions" without reference to the frame at all

What conclusions might a teacher draw from this experience or from a broader one where some children seem to "get it"; many others use the "100 frame" but still count on by ones; and some children can only use the frame as a sequence of number words or perhaps ignore it altogether? If she draws any conclusions at all—and certainly none are necessary if she just returns to her previous practice (whatever that was)—she might say, "Well, the 100 frame works for some children but not for most".

The question I want to raise is whether such a teacher might expect any help from the "discipline" of mathematics education or from mathematics education research in dealing with this or any other uncertainties regarding classroom practices? Of course an example this vague and yet complex, as well as a question this vague and yet complex, suggest there is no simple answer. There are many practices of mathematics education research and many related practices in contemporary cognitive psychology or cognitive science. There are, for example, empirically-based research practices which might try to see if this teacher's experience is typical. It certainly would not have been surprising back in the sixties to find studies which would compare classes using 100 frames with those not using them. Perhaps a consequence of such a study would be "there is no significant difference"; or if students were blocked on intelligence there might have been an interaction effect. A more contemporary study might use discriminant analysis to identify groups of children (based on a variety of personal, family, and school variables) who would gain from using the "100 frame" or search for patterns in pretest or demographic variables which would "predict" success. Such research might, if communicated properly, inform our teacher. But I would claim that such mathematics education research is far from the practices of the teacher regardless of its claim to being informative or highly generalizable to any teacher, classroom, or group of children. Such findings would be useful if the teacher believes that her classroom represents a "simple" environment with straightforward causal relations between her practice and particular student actions. But if the teacher observes her classroom as a complex environment in which she needs in her practice to explain many actions, each having implications for others, then such empirical studies of disembodied variables are of limited use

As you might expect from my example, I wish to consider other mathematics education research practices and their connection to teaching practice. My imaginary teacher might have found a recent research report by Paul Cobb [1995] revealing. In it he traces the work of a few children using the "100 frame" to do addition. In reading Cobb the teacher would find an affirmation of her observation of differences in her students' actions and of her observation of the general variation in performance of her students as they worked with the 100 frame. But such a reading would offer more than that. Rather than thinking that her teaching the students to add with the 100 frame would in some way provoke them into matching some predetermined process or performance, she would be given the opportunity to think differently. As Cobb suggests, neither the "100 frame" itself nor good clear instruction in its use is a sufficient condition for the child's mathematical behaviour. It is the schemes, the lived history, the structure of the child which determine what he or she does with the 100 frame. Because Cobb's research arises from an interaction with children it is likely that our imaginary teacher and her practice will be informed by the notion that children who can think of a number like 23 as a number which can be composed in many ways and is a unity in its own right will use the "100 frame" differently (and in the eyes of the teacher more effectively) than a child who can only see 23 as the result of a count. The concepts and theories drawn from research like Cobb's or Confrey's or my own in a constructivist or an enactivist framework can be thought of as "theories for" rather than "theories of" In other words, ideas drawn by the teacher from Cobb will not tell her how her students must behave but will provide her with insights that she can use in observing and listening differently to the mathematical actions and languaging of her students, and in entering into a different form of conversation with them (one which will focus less on "right answers" *per se* than on her students' mathematical actions, re-presentations and explanations)

Of course some might question whether the mathematical education ideas drawn from such interactive research with children and teachers in classrooms are well based or have much power, conceptually or in practice. I would claim that they have and just because they have they at least potentially have much to offer to the contemporary practice of mathematics teaching and learning. In his observations of children Cobb makes use of concepts drawn from many years of constructivist research Such research in which the researcher both interacts with and observes (through the use of video) children working in carefully designed settings has three goals. First, it allows the researcher to observe the child's constructive mathematical activity. Then it takes this activity as a serious primitive source of mathematical ideas and enables the researcher and the children to co-construct "children's mathematics", the concepts of which can provide the teacher with a learner's-eye-view of mathematics [Steffe and Wiegel, 1994] Such a "learner's-eye-view" is known to inform teachers in their own development of effective practices, not by in any way prescribing such practices but by showing them how to better use their observations of their students' actions in their own thinking about their practices (See, for example, the work of Cobb, Yackel, and Wood [1992], or the work on Cognitively-Guided Instruction in, e g., [Fennema and Franke, 1992].) Finally, such "children's mathematics" can become the source of a "mathematics for children". Such programs of practice which go well beyond the admonition to "use manipulatives" have been extensively developed with teachers, both for children and for older students, including university students. (Work by Dubinsky, Schoenfeld, Sfard, and Sierpinska comes to mind.)

In our example Cobb was making use of a distinction made by Steffe and Cobb [1988] between students using numbers as *unifying compositions* (essentially, as results of some counting scheme) or using numbers as *composite unities* (as units in their own right that can be decomposed in many ways). Such mechanisms identified by research are useful because of their explanatory power in both the research community and the community of practice [Maturana, 1991]. Thus a teacher reading Cobb is connected to a tradition of research-based concepts This tradition finds support in and is in rapport with constructivist and enactivist philosophies of cognition which have been developed over the past 30 years by, for example, Piaget [1980]; Bateson [1979]; Maturana and Varela [1980, 1987]; von Glasersfeld [1995]; Varela, Thompson and Rosch [1991] While some researchers such as Geary [1994] find that conclusions from such "biological" non-representationist theories can only apply to early number learning, both the philosophies and their consequences have been used in working with persons at many age levels and in mathematics classrooms from pre-school to university level

The argument above has been developed to show that mathematics education research has developed ideas in such a way as to be useful to teachers. Further it is argued that such ideas which provide a teacher with different tools *for developing* her practice and *for observing* her students (rather than providing her a picture of the reality of her classroom or of some ideal one), derive from a careful research practice and a consistent philosophy of mathematics and its cognition (see Ernest [1991]).

Is there anything in this community of theorizing and research practice which suggests that it might well be related to the community of mathematical teaching practice? Over the years Steffe (for example, in a presentation to ICME-7 in 1992) has maintained that a researcher in the constructivist tradition must be a teacher This is an ethical stance which suggests that in doing research the researcher is not simply using the child and his or her actions as a subject of study but is engaged with the child in helping them both learn. In such research, where the researcher is trying to construct the mechanisms of the student while the student constructs the mathematics, the unit of analysis and interpretation is "the- student-in-a-setting" In earlier constructivist research both the conditioning of the environment and the actions of the researcher faded into the background and the emphasis was on the mind of the student. But even that older model of constructivist research provided in its actions a different frame for the behaviour of a teacher. Because the researcher is *not listening for* a student to match a pre-conceived answer or process but is *listening to* the student to understand his or her mathematical activity in its own right, the process of the researcher-as-teacher provides a different model of listening and a different curricular goal for the teacher [Davis, 1996].

In more recent work the practice of mathematics education research has evolved to take into consideration the impact of the environment and of interactions in it on the mathematical thinking of the individual Because much of such research now takes place *in vivo* in classrooms or other educational environments instead of *in vitro* (under specially controlled conditions) its potential direct relevance to practice is raised. The unit of analysis now becomes the teacher/student/environment and the actions and thinking of the student at once act to bring forth a world of mathematical significance and are occasioned by the possibilities and interactions in that world. Thus this research necessarily takes the teacher and her world to be fully implicated in their research and its findings. Practice drawn from the research is thus "practice drawn from practice". Further, reports from such research have an interpretive narrative character rather than a prescriptive theoretical character (consider, for example, Cobb, Yackel, Wood [1992]; Cobb [995]; Confrey [1995, 1996]; Lampert [1991]; Lampert *et al.* [1996]; Pirie and Kieren [1994].

These research practices illuminate how research might relate to the practices of teaching and learning. Such practices point to alternative effective teaching practices and to new emphases:

- on listening to rather than simply listening for;
- on acting with students in doing mathematics rather than simply showing students how to do mathematics;
- on establishing effective discourses of mathematical argument or mathematical conversation rather than simply the discourse of telling, interrogating, and evaluating;
- on the mechanisms of students' mathematical thinking rather than simply on students' answers;
- on the teacher and students as fully implicated by their actions each in the learning of the other; and
- on the •teacher as co-developer of a lived mathematics curriculum not just a recipient of or a conduit for a predecided curriclum.

It has been said that mathematics education research especially of the constructivist variety has had little to say to practicing teachers and has even taken away from teachers' sense of power and efficacy [Smith, 1996]. In showing how the teacher imagined above might read contemporary mathematics education research, or more importantly in her practice engage with it, I have tried to illustrate how current mathematics education research could be fully implicated in the effective practice of mathematics teaching and learning.

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