# Unlearning to Teach Mathematics [1, 2]

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A constructivist perspective holds that children's learning of subject matter is the product of an interaction between what they are taught and what they bring to any learning situation. This view is based on increasing evidence from cognitive science research that pupils' prior knowledge and beliefs powerfully affect the way they make sense of new ideas [see, for example, Anderson, 1984; Davis, 1983; diSessa, 1982; Posner, Strike, Hewson, & Gertzog, 1983; Schoenfeld, 1983] Although children are usually the focus of these studies, this article is based on the premise that teacher education could be improved by taking this perspective on *teacher* learning

*Where do prospective teachers come from?*

Like pupils, prospective teachers come to formal teacher preparation with ideas and ways of thinking that influence what they learn from courses and field experiences [e.g., Feiman-Nemser & Buchmann, 1986]. Long before they first enrol in their first education course, they have developed a web of interconnected ideas about subject matter, about teaching and learning, and about schools.

Consider the case of mathematics: Before prospective teachers enter their first professional course, they have already spent years in math classes as students The experiences they have had with mathematics shape their feelings about the subject and about themselves in relation to it Moreover, the time spent in mathematics classrooms as students gives prospective teachers a specialized "apprenticeship of observation" [Lortie, 1975]. Watching teachers and paying attention to their own experiences, they develop ideas about the teacher's role, form beliefs about "what works" in teaching math, and acquire a repertoire of strategies and scripts for teaching specific content. Their experiences do more than develop their feelings about mathematics and images of math teaching, however These experiences also affect the way in which prospective teachers understand the subject — particular concepts and procedures as well as the nature of mathematics itself. In short, prospective teachers do not arrive at formal teacher education "empty-headed"; instead they bring with them a host of ideas and ways of thinking and feeling related to math and the teaching of math, drawn largely from their personal experiences of schooling.[3]

*Where does teacher education meet prospective teachers?*

In recent years, cognitive science research has increasingly filtered into the syllabi of teacher education courses. Prospective teachers are taught about schema theory and children's misconceptions and told that comprehension is a process of interaction between reader and text. Ironically, however, this perspective on learning does not typically influence what teacher educators do with *their* students Rarely are teacher education students treated as learners who actively construct understandings about specific subject matter and its pedagogy. Often they are viewed as simply *lacking* particular knowledge or skills without taking into account what they already know and believe.

This lack of attention to what teachers bring with them to learning to teach mathematics may help to account for why teacher education is often such a weak intervention — why teachers, in spite of courses and workshops, are most likely to teach math just as they were taught Mathematics teacher educators must find ways to address this conservative cycle if they hope to change the nature of mathematics teaching and learning in schools. Changes in requirements or improvements in curriculum alone are unlikely to alter this pattern. To effectively base teacher education on a conception of the "teacher as learner" [Feiman-Nemser, 1983], we need to know more about these learners and develop strategies for working with what they bring with them.

This article demonstrates that these two issues — learning about prospective teachers and developing strategies for working with them — can be pursued in tandem Below I describe the permutations project, a unit developed for the introductory elementary teacher education course at Michigan State University which is designed to surface and challenge what entering elementary teacher candidates know about mathematics and how it is taught and learned. Drawing on this project and students' responses to it, the second part of the article focuses on three dimensions of a framework for understanding what prospective elementary teachers bring with them to teacher education in mathematics. The paper concludes with a discussion of the implications of this perspective for research and practice in teacher education

## The permutations project

> Permutations was a new concept for me before this. When I found out it involved a mathematical concept, I was overcome with fear. Doing this exercise, however, has turned out to be very valuable for me. It seemed too fun and entertaining to be math. Consequently, I was able to see math in a new light, which was very surprising to me Christy[4]

*The context: exploring teaching*

Exploring teaching is the introductory course in the Department of Teacher Education at Michigan State University. Required of all prospective elementary teachers, it is taken prior to enrolling in one of the five alternative undergraduate teacher education programs Students are told that this is not a course in "how to teach," but an opportunity to think critically about teaching and *preparing* to teach It is intended to raise far more questions than it answers, to encourage prospective teachers to examine their commitment to teaching and to surface, challenge, and extend their current ideas about teaching, learning, and learning to teach The questions opened up in the course are supposed to help them focus on what they will need to know more about — or even care more about — in order to become effective teachers.

The permutations project is a curriculum unit that I developed for this course with many of the above purposes in mind; I have taught it three times and other course instructors have also taught it.[5] The unit is intended to help prospective teachers examine the assumptions about teaching, learning, and learning to teach, that they bring to teacher preparation by having them focus very closely on a "case of teaching and learning." What is being "taught and learned" is the concept of permutations. I chose mathematics as the subject matter because I thought that prospective teachers were likely to have brought considerable "baggage" with them about mathematics, and about how it is taught and learned, based on their past experiences with it As such, I thought it would provide a fertile ground for our inquiry.

*Orienting assumptions*

My hunches about what the prospective teachers might already know, think, believe, and feel, guided the development of the unit. These hunches grew out of the literature on pre-college and college mathematics teaching and learning, as well as out of my own experience as a mathematics student, teacher, and staff developer.

Permutations was selected as the mathematical topic because it was a concept that many of the intending teachers might never have studied formally; they could therefore engage in this activity truly as learners. If they had previously encountered the concept in a mathematics class, their understanding of it was likely to be procedural [Hiebert, 1986], i.e., they would know to use factorials.

Other hunches about what the prospective teachers might believe included the following:

- Mathematics does not have much relationship to the real world and most mathematical ideas cannot be represented any way other than abstractly, with symbols.
- Knowing mathematics means "knowing how to do it."
- Teaching mathematics involves telling (or showing) the students how to do different kinds of problems
- Teachers ask questions to elicit right answers; if a teacher questions your answer, it means you have made a mistake.
- Learning mathematics is scary
- Good teachers make mathematics *fun* for students.
- Elementary school mathematics teaching does not require much knowledge of math — anyone who can add, subtract, multiply, and divide knows enough mathematics to teach little kids. Learning to teach, therefore, is mainly a matter of acquiring techniques.
- Love of children, not knowledge of subject matter, is the basis of elementary school teaching
- Young children are eager to learn and trusting, but are not yet capable of thinking about complicated mathematical ideas or solving real problems

The unit's content, activities, and approaches, described below, are designed to raise these (and other) ideas to the surface for examination and analysis

*Content and activities*

The unit lasts for two weeks Over the course of the project, the prospective teachers first learn about permutations themselves, then watch a teacher helping a young child explore the concept, and finally try their hand at helping someone else (child or adult) learn about permutations. During each of the phases of the permutations project, they are encouraged to pay close attention to what they are thinking, doing, and feeling

Three readings accompany the unit The first, by David Hawkins [1967/1974], entitled "I, Thou, and It," is a lovely essay highlighting the special kind of relationship that teachers have with students around subject matter. According to Hawkins, the teacher's role is to respect the students' thinking and to encourage their growth by providing "the kinds of environments which elicit their interests and talents and which deepen their engagement in practice and thought" [p 48]. The prospective teachers also read Herbert Kohl's book, *Growing minds: on becoming a teacher* [1984] and an excerpt from Vivian Paley's [1981] book, *Wally's stories*. Both Paley's and Kohl's accounts of their teaching illustrate the centrality of subject matter knowledge in teaching and the importance of respect for children's thinking. Both also portray how, when one sees the world through children's eyes, one's adult grasp of the subject matter often proves insufficient [Hawkins, 1967/1974]. No reading about mathematics has been assigned, primarily because the goal has been to challenge their thinking about the subject matter through their experience with it in the class.

*Phase one: learning.*[6] For the first two class periods, the students participate as learners of mathematics. I am the teacher I try to pique their curiosity by challenging them to make sense of the fact that the 25 students in the class could sit in 1,551,121,000,000,000,000,000,000 *different* seating arrangements — and that, furthermore, if they switched seats every 10 seconds, trying to make all these arrangements, it would take almost five quintillion years. These numbers are incomprehensible. The class backs up to 15, or even 10, people, hoping that smaller numbers will result in a more manageable number of seating arrangements But of course these numbers still yield an amazing number of possibilities (10! = 3,682,000), so the concept is explored from the simplest cases, starting with just 2 people, and then 3 and 4 Some terms the class acts out the problem; other terms I introduce objects such as Cuisenaire rods to represent the situation. The discussion is lively and students propose a variety of explanations and ideas. I solicit alternative approaches and ask questions such as, "Do you see a pattern here?" or, "Why are you multiplying those numbers?" My purpose is to encourage them to talk about what they think is going on as a means of getting them to figure out the idea of permutations for themselves.

For homework, I distribute a varied set of problems and ask the students to try the problems with two goals in mind: (a) to extend their own understanding of permutations and (b) to pay attention to the role the homework plays in their learning — how they tackle it, how they feel about it, and why. We discuss the problems and the students' thoughts about them in class. I encourage them to explain and justify their solutions, and alternative approaches always emerge, to many students' complete astonishment.

Gradually, over two class periods, most students figure out the pattern by induction and develop some understanding of the concept of permutations. Generally the students work collaboratively. Still, some students feel very tense, even though how well they "get it" has no bearing on their course grade.

Every term, two or three students who have taken more mathematics than the rest think they understand the concept and proceed to try to explain it to their classmates. With great surprise, they discover that this is much more difficult to do than they realized. As one student wrote later, "I tried to present my theory in hopes of helping the rest of the class, but as my mouth opened, I found it very difficult to put into words what came easily to mind."

*Phase two: observing learning and teaching.* The next phase of the project offers the students another view of learning and teaching mathematics. During the third class period, they observe me to try to help a young child (age 6, 7, or 8) explore the concept of permutations. I use some different tasks with the children (e.g., lining people up forming two, three, and four digit numerals; distributing candy to family members) but a similar kind of teaching occurs in which the teacher asks questions aimed at helping the child develop his or her ideas about possible choices and arrangement of objects. As they watch, the prospective teachers have several purposes — to pay attention to (a) how I interact with the child, (b) what kinds of tasks I select and how these are structured and why, and (c) how the child is thinking, what he or she is doing and saying. In addition, many of the students find themselves thinking further about the subject matter, and several have reflected afterwards that it is during this period that they learned the most about permutations. Although it would seem to be an intimidating setting, all the children have been remarkably relaxed and have talked aloud freely about their theories and ways of thinking about the problems.

Before the class ends, the prospective teachers have an opportunity to ask the child questions. Some ask about particular things the child said or did, trying to understand more clearly what he or she might have been thinking. Others try to ask about the concept in some other way to see how the child is understanding it. Still others ask about how the child felt about the experience.[7]

During the fourth class period, the students discuss this observation, exploring both the teaching and the learning that occurred. They talk about what they saw as they watched the child and what they thought about it. They ask me questions about things I did or said, and together we analyze the choices I made.

*Phase three: teaching.* In the final stage of the project, the students take on the role of teacher, and try to help someone else explore the concept of permutations. Some choose children, others work with their roommates or parents. We spend some time in class, usually in small groups, discussing preparations for this. Many try to model their approach after what they have seen me do with the child; others draw on what helped *them* understand the concept.

Afterwards, as teachers, they discuss what they learned about their learner, about the subject matter, about teaching. Many report how strong their inclination was to *tell* their learners "the answer" instead of helping them construct their own understandings. Others confront the limits of their own knowledge and the effect of the lack of their effectiveness. To gain these kinds of insights is the point of this phase of the project, not to perform a model lesson on permutations.

To conclude the project, students write a paper, a "case study of teaching and learning," in which they integrate what they have learned across these different experiences. While they can construct this case study in whatever manner they choose, they are asked to include their experiences both as learner and teacher and to draw some tentative conclusions about mathematics, about the teaching and learning of mathematics, and about learning to teach math.

**What do prospective elementary teachers "bring" to teacher education in mathematics?**

Students' responses to the permutations project provide valuable glimpses of what prospective elementary teachers know and believe when they enter teacher education. In this section, I focus on three strands: knowledge of and about mathematics, ideas about mathematics teaching and learning, and feelings about oneself in relation to mathematics. These strands form part of a preliminary framework for understanding what prospective teachers bring with them to teacher education in mathematics [Ball, in preparation].

**Knowledge of and about mathematics**

Teacher educators tend to take prospective teachers' subject matter knowledge for granted, focusing instead on pedagogical knowledge and skills [Ball & Feiman-Nemser, in press]. Similarly, many prospective elementary teachers often assume that "common sense and memories of their own schooling will supply the subject matter needed to teach young children" [Feiman-Nemser & Buchmann, 1986, p. 245]. Yet recent research highlights the critical influence of teachers' knowledge of and ideas about mathematics on their pedagogical orientations and decisions [e.g., Ball & Feiman-Nemser, in press; Kuhs, 1980; Lampert, 1986a; Leinhardt & Smith, 1985; Shroyer, 1981; Steinberg, Haymoe & Marks, 1985; Thompson, 1984] The question for teacher education should be: What *do* prospective teachers know and how is that knowledge understood and organized? What matters about the mathematics they know?

*Knowledge of mathematics.* In the permutations project, many of the teacher education students were surprised to discover how crucial subject matter knowledge was when they tried to teach the concept of permutations to another person. For example, Sam wrote,

> When I decided to be a teacher, I knew there were a lot of things I had to learn about teaching, but I felt I knew everything there was to teach my students, until we began our permutations project. During the permutations activities, I found I was as much a learner of subject matter as I was a learner of the art of teaching . . . I found that my education in the future will not be limited to "how to teach," but what it is I'm teaching. My knowledge of math must improve drastically if I'm to teach effectively.

It is obvious that knowledge of mathematics is basic to being able to help someone else learn it Teachers must understand concepts and procedures themselves in order to select and construct fruitful tasks and activities for their pupils, as well as to flexibly interpret and appraise pupils' ideas. Buchmann [1982] explains:

> A mathematics algorithm invented by a student, for example, must not be diagnosed as a mistake simply because it deviates from the teacher's way of arriving at a solution On the other hand, content knowledge helps the teacher recognize the source of mistakes that learners make and to unravel the patterns of misunderstanding. [p. 65]

Yet specifying and justifying exactly what mathematics prospective elementary teachers need to know is a difficult issue Some people address it by making lists of courses or topics that teachers should have (e.g., one course in algebra, one in geometry). A recent NCTM commission report entitled *Guidelines for the preparation of teachers of mathematics* [1981] offers the most specific treatment; however, this document is long on lists and lean on argument to support or qualify those lists. For primary grade teachers, why no course in number theory? And what kind of course in algebra is needed by junior high teachers?

Other people describe subject matter knowledge in terms of qualitative standards such as "flexibility" and "depth" According to Hawkins [1972/1974], for example, a teacher's "own mathematical domain must be ample enough . . to match the full range of a child's wonder and curiosity, his unexpected ways of gaining insight" [p 118] so that the teacher is able to sense when a child's explorations are taking him or her to "mathematically sacred ground." Although it presents a lovely image, the problem with this approach is that it skirts the issue of *what* should be known

Equating a particular number of courses or a major in mathematics with a thorough understanding of the subject is of course simplistic There is a need to conceptualize, however, what it means for teachers to know mathematics "flexibly" or "in depth." Some current research projects are using interviews and structured tasks to explore how teachers think about their mathematical knowledge and how they understand (or *mis*understand) specific ideas [e.g., Leinhardt & Smith, 1985; McDiarmid & Ball, 1987; Steinberg, Haymore, & Marks, 1985]. What counts, according to these researchers, is the way teachers organize the field and how they understand and think about concepts (as opposed to just whether they can give "right" answers) This work promises to help combine the "list" approach (which focuses on what teachers should know) with the qualitative approach (which focuses on how they should know it) as well as offering better ways of assessing what prospective teachers know

Beyond this, researchers are also working on what "subject matter knowledge for teaching" might entail and how that might differ from other kinds of subject matter understandings. Shulman and his colleagues have been modeling this in terms of what they call "pedagogical content knowledge," which includes

> the most regularly taught topics in one's own subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations — in a word, the ways of representing and formulating the subject that make it comprehensible to others [Shulman, 1986, p 6]

In addition, understanding mathematics in order to teach it means being able to *think pedagogically* about the subject [Feiman-Nemser & Buchmann, 1986]. Teachers must view mathematics through the eyes of their pupils [Dewey, 1916/1964a; Dewey 1916b/1964b; Highet, 1966]. For instance, they must be able not only to perform multi-digit multiplication computation accurately but also to know how to use pictures, stories, and objects to model the underlying concepts as well as invent appropriate alternative representations [see Lampert, 1986b]. Teachers also need to be able to appraise curricular materials and instructional activities, assess what their students understand, and plan ways to help them learn

Although the permutations project did not provide a broad view of what the prospective teachers understood in math and how they organized that knowledge, it did reveal the importance of another dimension: the ideas they had *about* mathematics. Participating in the project surfaced their assumptions about the nature of mathematics and what it means to "know" something in mathematics.

*What is mathematics — what does it mean to "know" it?* For many of the prospective teachers, knowing math had always meant being able to produce the answer the teacher wanted, but with little attention to *why* the algorithms worked. Ellen reflected:

> I'm learning about mathematics in this class . . . math isn't just memorizing formulas — it is knowing *why* a

problem is done the way it is . . . In high school, [it was] memorizing formulas, theorems, and definitions

Confronting this new way of knowing was not always easy for the students, especially for those who had been successful in school For example, Cindy wrote:

> I have always been a good math student so not understanding this concept was very frustrating to me. One thing I realized was that in high school we never learned the theories behind our arithmetic. We just used the formulas and carried out the problem solving For instance, the way I learned permutations was just to use the factorial of the number and carry out the multiplication . . We never had to learn the concepts, we just did the problems with a formula. If you are only multiplying to get the answer every time, permutations could appear to be very easy. If you ask yourself *why* do we multiply and really try to understand the concept, then it may be very confusing as it was to me

And Tami reflected that although she had always enjoyed math, she now realized that,

> I have learned to understand mathematics by memorizing formulas . . . [I am] conditioned to looking for formulas instead of the processes to obtain the answers.

These comments suggest that, besides knowledge *of* the subject, prospective teachers also bring ideas of what mathematics *is* — what it's about, what it's good for, where it comes from, and how right answers are established, that shape their understanding of and approach to the subject.

*What does it mean to know mathematics for teaching.* For some students, the experience of learning permutations challenged their prior ideas about knowledge. But others, who felt comfortable with their understanding during the learner phase, became unsettled when they tried to teach the concept of permutations to another person. For example, Laura, who had been very active in class during the learner phase, analyzed the difficulty she had encountered:

> I was trying to explain my understanding of [permutations] to those who did not yet understand. I had to keep rearranging my perspective, that is, approach the idea from different angles in order to try to present the concept in a way that would help others understand it The problem was that I could not see what link was missing . . My understanding of it was so straightforward and simple that I didn't know how else to approach it I was not able to articulate the concept in a way that increased their understanding. . . *I* understood the material and found myself searching for the phrase or diagram that would make it as self-evident for them as it was for me.

During this teaching phase, many of the prospective teachers realized that subject matter understanding for teaching might be different from that needed for personal functioning. They characterized it as the difference between knowing permutations "for yourself" and knowing permutations in order to be able to help someone else learn it, but this distinction also entailed for them some new ideas about mathematics itself. Jan explained her new insight:

> There isn't a universal explanation [for permutations] We needed many different versions; different people understood different examples . . . You really have to know your subject matter well enough to be able to play around with it If you can only give one explanation, many of your students won't understand. . . . If you know your subject matter well (inside and out), it is easier to find different explanations and examples You can't be tied down to just one way of doing a problem

*Ideas about mathematics teaching and learning.* As the prospective teachers' comments about mathematics indicate, they also come to teacher education with ideas about mathematics teaching and learning. For example, Alison wrote:

> I was trying to teach my mother permutations. But it turned out to be a disaster. I understood permutations enough for *myself,* but when it came time to teach it, I realized that I didn't understand it as well as I thought I did. Mom asked me questions I couldn't answer. Like the question about there being four times and four positions and why it wouldn't be $4 \times 4 = 16$. She threw me with that one and I think we lost it for good there.

Her account shows that, while she realized that her understanding of permutations was shaky, she also assumed that her role as a teacher should be to answer the learner's questions.

*Formative experience in learning to teach mathematics.* The years spent in math classes, watching teachers and being pupils contribute to prospective teachers' assumptions about teaching mathematics What have they seen in those years? In all too many mathematics classrooms, the teacher (or the textbook) is the authority, theorems are proved by coercion — not reason, and confusions are addressed by repeating the steps in "excruciatingly fine detail" [Davis & Hersh, 1981, p 279]. Describing mathematicians as "promoting not the emancipation but the enslavement of minds," Kline [1977], a mathematician himself, characterizes mathematics teaching, saying:

> Mathematicians have a naive idea of pedagogy They believe that if they state a series of concepts, theorems, and proofs correctly and clearly, with plenty of symbols, they must necessarily be understood. This is like an American speaking English loudly to a Russian who does not know English. [p. 117]

Davis and Hersh [1981] describe an "ordinary mathematics class" as follows:

> The program is fairly clearcut. We have problems to solve, or a method of calculation to explain, or a

> theorem to probe. The main work will be done in writing, usually on the blackboard If the problems are solved, the theorems proved, or the calculations completed, then the teacher and the class know they have completed the daily task [p 3]

This pedagogy, this "ordinary" class, while perhaps all too typical, is not the sort of class mathematics teacher educators would want beginning teachers to construct Yet prospective teachers may have never seen a teacher teach mathematics in a way that focuses on student thinking and on mathematical activity.

Whatever their particular experiences, budding teachers develop ideas about how to teach mathematics and about what the roles of students and teacher in a mathematics classroom are. If they were successful in mathematics, prospective teachers are likely to approve of the patterns they saw, and thus be uninterested in alternative ways of teaching. If they struggled, they may aspire to teach differently. But even if they are critical of their own past teachers for teaching badly and for making them feel stupid, they may lack alternative models. Learning to teach mathematics, therefore, requires overcoming the limits of their firsthand experiences as students [Buchmann & Schwille, 1983] [8]

*Teachers: how should they help students learn mathematics?* For several of the teacher education students, the permutation project challenged their conception of what it means to *teach* as well as what it means to *learn* Many brought an image of math teaching in which the teacher "tells knowledge" and asks questions to check up on students Commenting on my teaching, Jan observed, "It seemed strange to me that you asked us *why* we multiply. Whenever I have been taught mathematics, I was never asked why I was always just told to multiply."

The prospective teachers started to see that questions could be a valuable tool if learners were to discover or create understanding for themselves Maureen wrote about her teaching experience with this insight about questioning:

> By the end of our time together, I had learned not only how valuable questions were to teaching, but I realized that how I asked them and when I asked them made a big difference. I started to get a feel for when to let Joni talk herself into a circle, and when frustration would back her into a corner and I should help her. I could steer her with the word "Why?" and although it was very subtle, it made her look deeper How exciting! All I did was help Joni to find a few doors, and *she* could do the opening all by herself.

*Learners: who can learn mathematics?* In addition to developing ideas about how mathematics is taught and learned, experience in mathematics classrooms may affect prospective teachers' beliefs about learners, or what it takes to learn mathematics. Research suggests that teachers' beliefs about learners also influence their teaching: what they teach, in what ways, to whom, and how they think about their students' success or failure in learning mathematics [Anyon, 1981; Brophy, 1983; Steinberg et al., 1985]. For example, three of the four teachers studied by Steinberg, et al. [1985], believed that ability to learn mathematics was an innate human characteristic. One explained that "some people have mathematical minds" and another believed that individuals are either able to think in a "humanities-type mode" or a "hard science mode, where you are thinking in right and wrong answers" [pp. 18-19).

In the permutations project, the prospective teachers are often astounded when they observe the young child in class Invariably, they want to know if the child is "gifted" because they cannot believe that a six- or an eight-year-old can think or reason in this way. Their surprise reveals their assumptions about young children's capacity to reason and make sense; the experience of observing a young child provokes some of them to revise their assumptions For example, after observing one eight-year-old, Karen wote,

> One thing I noticed from watching Janna is that children are good thinkers. Teaching isn't just to tell children what you know and expect them to learn it. We tend to think of children as a clean slate in which the teacher's role is to fill up this so-called slate.

The permutations project also seemed to challenge some students to reexamine their assumptions about themselves as learners. For example, Christy remarked about herself, "Most of all, I realized that I *do* have the ability to learn mathematics when it is taught in a thoughtful way."

*Feelings about oneself in relation to mathematics.* Finally, while there are of course prospective elementary teachers who enjoy and feel competent in math, many have had negative experiences with mathematics and they do not feel successful as learners of math. These feelings about themselves in relation to mathematics are part of what they bring with them to teacher education Molly reported that she felt "very insecure" at the beginning of the permutations project, explaining, "Math, in whatever form, has always come in a painstakingly slow manner and scenes from Math 108 and Accounting 201 seem always to haunt me." Similarly, Terri told me later, "When you told the class that we were going to be using math for the next project, I froze — my palms got sweaty, and I didn't hear anything you said for the rest of that hour "

These feelings are important to acknowledge in exploring what prospective teachers bring with them to their formal preparation to teach math, for they are likely to affect what they learn from teacher education in mathematics. For example, Mandy said afterwards that she "got lost," "got nowhere" and "did not enjoy the permutations activities because I was transported in time back to junior high school, where I remember mathematics as confusing and aggravating. Then, as now, the explanations seemed to fly by me in a whirl of disassociated numbers and words "

Prospective elementary teachers are typically more apprehensive about teaching mathematics than any other subject [Ball, in preparation; Burton, 1979; Smith, 1964]. They worry that they will not be able to answer students' questions and that they will not be able to explain procedures adequately. They hope that if they teach a lower grade, their lack of subject matter knowledge will not matter and they are therefore unlikely to take more mathematics courses than the number minimally required for teacher certification. Consequently, what they have learned in elementary and high school math classes often comprises almost all of their subject matter preparation for teaching. Their feelings and opinions about math may thus affect their approach to learning to teach it, and ultimately the way they as teachers teach math.

**Conclusion**

The permutations project illustrates a strategy for helping prospective mathematics teachers surface and examine the knowledge, beliefs, and attitudes they bring with them to learning to teach mathematics. In order to help prospective teachers learn to teach mathematics effectively, mathematics teacher preparation must be oriented toward a view of teachers as learners [Feiman-Nemser, 1983]. This requires taking into account what teacher candidates bring and developing ways of challenging, changing, and extending what they know, believe, and care about. This is not a simple agenda.

*What do prospective teachers bring?*

Learning more about what prospective teachers bring with them to their professional education entails empirical and conceptual work. Prospective mathematics teachers bring lots of ideas with them — but which of these seem important to explore in relation to learning to teach mathematics? This depends on the view of mathematics teaching one holds.[9] Different views of teaching imply different ideas about what teachers may need to know, to be able to do, and to care about. As such, what one seeks to learn about prospective teachers as well as one's assessment of the appropriateness of their current ideas will depend on one's view of what they need to know or care about.

*What are the implications for the practice of teacher education?*

Becoming clearer about prospective teachers' ideas will not lead to obvious implications for teacher education and curriculum. First, some convictions may be resistant to change and may even interfere with what teacher educators try to teach. If, for instance, prospective teachers think of word problems as "problem solving" — and find them threatening besides — it may be difficult to help them move beyond an exclusive focus on computational skills in thinking about math.

Still there is no reason to assume that all ideas that seem inappropriate will be difficult to change; some may be readily exchanged based on new evidence or experience. The prospective teachers' experience with the permutations project led several of them to change their minds about some things they had firmly believed — that subject matter knowledge for elementary teaching is unproblematic, or that mathematical concepts could be explained in only one way, for example.

Second, other ideas that prospective teachers hold may be firmly rooted in tendencies or habits. For example, during the teaching phase of the permutations project, some of the prospective teachers became aware of their strong inclination to *tell*. Although they wanted to let their learners figure things out for themselves, many of them found it difficult not to jump in and do it for them. Changing tendencies is not the same as changing one's mind and presents a different set of challenges for teacher educators.

Finally, prospective teachers bring many appropriate ideas which do not need to be challenged or altered, but which teacher educators must extend. For example, many prospective teachers are convinced that they should help students *understand* math and not just tell them, "This is the way to do it." Unfortunately, they probably did not learn mathematics this way when they were in school. So when they try to help a second grader understand subtraction with regrouping, they fall back on the rules and algorithm rhymes[10] they were given, teaching pupils to recite, "Three take away six, can't do it, cross out the two, put a dash in front of the three, thirteen take away six is seven" or "You can't subtract *up*." Prospective teachers may already think that they should teach for "understanding," but they may have a limited notion of what "understanding" something in mathematics means; moreover, their own mathematical background may leave them substantively unprepared. Teacher educators need to prepare prospective teachers to act on the appropriate inclinations they bring with them.

*What effect do different approaches to mathematics teacher education have?*

At the conclusion of the permutations project, one student wrote, "One thing I learned from this experience is that I am not qualified to teach math yet. Once I have the math methods class, then I will be much more ready." Some people would smile cynically at this student's expectations for his methods class. Yet we actually know very little about what prospective teachers encounter or learn in teacher education.

In order for teacher education to become a more effective intervention in preparing a elementary teachers to teach mathematics, we need to examine the influence of different kinds of teacher education experiences on teacher candidates' knowledge about and orientations toward mathematics and math teaching and learning, as well as on what they actually do in their classrooms. For example, what is taught in different math methods courses, and what do prospective teachers learn? What goes on in the mathematics courses that teachers take at the college level and how do those experiences fit with students' prior experiences in math?[11]

How can teacher educators productively challenge, change, and extend what teacher education students bring? Knowing more about what teachers bring and what they learn from different components of and approaches to professional preparation is one more critical piece to the puzzle of improving the impact of mathematics teacher education on what goes on in elementary mathematics classrooms.

**Notes**

[1] An earlier version of this article was presented at the eighth annual meeting of the North American Chapter of the International

Group for the Psychology of Mathematics Education East Lansing, Michigan; September 1986.

[2] Preparation of this article was supported by the National Center for Research on Teacher Education, Michigan State University. The NCRTE is funded primarily by the Office of Educational Research and Improvement, United States Department of Education The opinions expressed herein are those of the author, and do not necessarily reflect the position, policy, or endorsement of the OERI/ED (Grant No OERI-G-86-0001)

[3] In addition to the influences of schooling, culturally shared views (e g., that mathematics is not essential for everyday life, or that the world is divided into mathematical and non-mathematical "types") probably also influence prospective mathematics teachers' conceptions of the subject of themselves, and of their future pupils.

[4] Quotes in this paper are drawn from papers written by elementary teacher education students at the conclusion of the permutations project over three terms All names used are pseudonyms

[5] Susan Florio-Ruane provided the initial spark for this unit when, upon her return from a sabbatical year at Harvard University (1984-85), she was inspired to try some of the things she had seen Eleanor Duckworth doing with her students. Conversations with Susan stimulated my work on the permutations project Sharon Feiman-Nemser, who first helped me to learn to think about teachers as learners, also provided many thoughtful suggestions and comments Florio-Ruane and Feimn-Nemser are both faculty members in the Department of Teacher Education at Michigan State University

[6] I refer to "phases" only in order to call the reader's attention to the students' primary activity at different points during the unit. The unit itself is actually an integrated set of experiences, not a series of discrete phases. Students learn about teaching during the "learning" phase and about subject matter during the "observing phase" — this is inherent in the integrated nature of teaching and learning As one student noted, "I have come to the conclusion that teaching, subject matter, and learning are all interwoven together and they all have an effect on how the other turns out"

[7] Many of the students find it very difficult to frame their ideas as questions; instead, they lead For instance, trying to get a sense of how the child reacted to different kinds of tasks, one student asked six-year-old Sarah, "Don't you think it was easier to do it with candy bars than with blocks?"

[8] An interesting question to pursue is whether there are differences in the views of secondary teacher candidates (who have taken a lot of mathematics at the college level) and elementary teacher candidates like these students (whose mathematics experience is usually primarily at the high school level)

[9] A "view of teaching," as I am using the phrase, includes conceptions of mathematics as a discipline and as a school subject, ideas about what is worth knowing in mathematics and what it means to know something, and ideas about how people learn mathematics the teacher's role, and the purposes of instruction.

[10] I borrow the term "algorithm rhyme" from Blake & Verhille [1985]

[11] The National Center for Research on Teacher Education at Michigan State University is currently pursuing such questions with a longitudinal study of eleven elementary and secondary teacher education teacher programs and their participants The study focuses on the subjects of mathematics and writing and includes teacher education at the preservice, induction, inservice levels, as well as alternate routes to teacher certification

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