

# MATHEMATICS TEACHER EDUCATORS' USE OF NARRATIVE IN RESEARCH, LEARNING AND TEACHING

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The symposium on learning in honour of Laurinda Brown reminded me of her contributions to the field and influence on my interest in mathematics teacher educator knowledge and learning. Through her research, her practice, her editorships, her co-leading of discussion groups and working groups at the Psychology of Mathematics Education conferences, and her publications, Laurinda has played an important role in supporting mathematics teacher educators in researching mathematics teachers and/or researching their own knowledge development. This includes supporting mathematics teacher educators' use of biographical or self-based methods that use narrative as a way of learning from, and reporting on, experience as reflected in her work. For example, in Brown (2015), she offers narrative accounts of aspects of her journey in developing as a researcher and her engagement with enactivist ideas and the development of enactivist research projects. This article, then, addresses two of the themes I associate with Laurinda: mathematics teacher educators and narrative. These themes also emerged during the symposium that inspired me to consider the ways in which mathematics teacher educators use narrative in their work. Thus, the intent of this article is to highlight examples of ways in which mathematics teacher educators have used narrative as a way of knowing in their research, learning and teaching. Through these examples, I challenge the claim that narrative is subjective and unscientific, and hence not appropriate for research, and highlight the insights narrative can bring that might not be available to other approaches. In particular, given the under-representation of research on mathematics teacher educators (Beswick & Goos, 2018), I highlight examples of *self-based research methodologies*, with a focus on narrative, as appropriate and meaningful ways for investigating teacher educators' professional knowledge and learning, which according to Beswick and Goos, need more attention given the growing emphases in many countries on teacher quality. I first address the perspective of narrative as a way of knowing, which I then use to discuss the examples to provide insights of what is possible and future direction in using narrative in teachers' and mathematics teacher educators' learning.

## Narrative as a way of knowing

The importance of *narrative* in understanding self is well established. In addition to specifying experience, narrative has been conceptualised as a way of making sense of human

actions. Ricoeur (*e.g.*, 1988) acknowledged the narrative ways in which we understand our self, others and the world we live in. Polkinghorne (1995) described narrative as “a cognitive scheme; a scheme by means of which human beings give meaning to their experience [...] a framework for understanding the past events of one's life and for planning future actions” (p. 11). Bruner (1986) explained, “narrative is concerned with the explication of human intentions in the context of action” (p. 100). Bruner (2004) also considered narrative to be a major way in which people make sense of experience, construct the self, and create and communicate meaning. Other researchers have focused on narrative as the stories people tell—the way they organise their experiences into temporally meaningful episodes (Ellis, 2004); a way of knowing and discovering new aspects of the topic and one's relationship to it (Richardson, 1994); and the way of “shaping or ordering experience, a way of understanding one's own or others' actions, of organizing events and objects into a meaningful whole, of connecting and seeing the consequences of actions and events over time” (Chase 2011, p. 421).

An underlying theme to these related views of narrative that I consider here is narrative *as a way of knowing*. In particular, it is a way of knowing about oneself, others, or one's world; a way of learning through one's own experiences and/or the experiences of others. As a way of knowing, the power of narrative comes from the relationship it affords between author/teller and self, author/teller and reader/listener, and reader/listener and self. It is within these relationships that new understandings, meanings and experiences emerge. Here I use narrative as a way of knowing as the basis for considering examples of ways mathematics teacher educators have used or could use narrative. I do not frame this article in the form of a narrative (*i.e.*, a temporal sequence of events that are connected to a theme, internally consistent, and present a coherent whole), but present the examples as a collection of snapshots of my engagement with narrative as a mathematics teacher educator and the engagement of other mathematics teacher educators with narrative based on illustrative examples of published research in international mathematics education journals. These examples are ones in which the researchers describe their work as involving narrative in some way. In the following sections, I present four categories of examples consisting of narrative: at the symposium, in researching teachers, in researching oneself, and in

supporting teacher learning. These categories were selected based on my experience with narrative in my research and teaching (e.g., Chapman, 2008). While not intended to be exhaustive of what is possible, these categories provide a landscape of central aspects of the use of narrative in mathematics teacher educators' work beyond my own.

### **Narrative at the symposium**

I include the symposium on learning as a separate category because, unlike the other categories of examples, it is not about a research or classroom context but a situation where a group of mathematics teacher educators shared and discussed ideas. The symposium also had a unique format in which I considered narrative to play a significant role as a way of knowing. For example, many of the presenters used stories of personal professional experiences to engage participants; to share ideas and stimulate reflection and discussion of issues or specific situations in the teaching of mathematics or in mathematics teacher education. The sharing of stories created a space for participants to engage with the stories in different ways to construct meaning from them, which could be about the author/teller, themselves, and/or the ideas presented. As an example, I provide snapshots of one way in which I engaged with specific aspects of some of the stories.

These snapshots focus on how I resonated in the stories regarding my own experiences or thinking that resulted in further reflection following the symposium. Alf Coles' stories of noticing 'nots' (i.e., something students have not noticed or not done) prompted me to think of the activities in which I engage prospective teachers [PTs] to enhance their ability to notice and the importance for them to make noticing both 'nots' and 'knows' (e.g., what students know and do not know, do and do not do, see and do not see) an important pedagogical strategy in working with their students. Tracy Helliwell's stories that included moving from classroom teacher to teacher educator, that is, transitioning from teaching school children to teaching adults (i.e., PTs), made me wonder about opportunities and challenges mathematics teacher educators who lacked classroom experience at school level (as I do) encountered as they transitioned from teaching postsecondary mathematics to working with PTs in mathematics education (as I did). Elaine Simmt's story of a teaching experience with grade 7 students in teaching a mathematics lesson as part of a research project prompted me to think about what counts as mathematics in the classroom for me from the perspective of engaging students in working as a mathematician. Finally, Richard Barwell's stories of learning to teach mathematics and teachers of mathematics prompted me to think about the use of empathic listening in working with teachers to support mutual understanding and the importance for me to understand more from the perspective of the teachers and understand myself as a mathematics teacher educator in relation to them. The stories also spoke to me of similar experiences in my early years of working with PTs that led to shifts in my way of working with them to an inquiry-based perspective as co-inquirers and co-learners.

The implication from these examples is that how we choose to interact with a story could tell us something meaningful about ourselves and/or the author/teller of the story.

These examples illustrate the focus on oneself; that is, how the stories offered me a way of knowing something meaningful about me as a mathematics teacher educator. I consider this way of knowing to be, for the reader/listener, a process of resonance between the reader/listener and the author/teller of a story as opposed to one of analysis of the story. The resonance approach is important to narrative inquiry (Connelly & Clandinin, 2006) as a methodology, discussed later, that involves studying and understanding self in relation to others. Stories that are descriptive of an experience or event without analysis or judgment offer opportunities for the listeners/readers to resonate with the lived experience in an empathetic or self-reflective way with their own stories.

### **Narrative in researching mathematics teachers**

Mathematics teacher educators have used narrative in a variety of studies to understand prospective and practising mathematics teachers. The focus tends to be on using stories, usually collected through interviews, as a way of knowing the teachers based on their experiences, thinking, and contexts. For example, in recent studies, Suazo-Flores (2017) used stories of an eighth-grade mathematics teacher's practice, to understand the teacher's personal practical knowledge in teaching the concept of area. Cox and Harper (2017) used narratives of how secondary mathematics PTs engaged with mathematics and with technology to articulate mathematical problem solving and problem posing in a technological environment. Oslund (2012) used elementary teachers' stories of new mathematics pedagogies to investigate aspects of their knowledge of mathematics for teaching. Lutovac and Kaasila (2014) used stories PTs told themselves, or others about themselves, as mathematics learners and teachers to investigate their future oriented mathematical identity. Nardi (2016) used stories told by mathematicians as they engaged with conversation on the teaching and learning of mathematics to capture their perspective. Finally, Chapman and Heater (2010) used a high school teacher's stories of teaching to understand how the teacher transformed her practice to an inquiry-oriented perspective.

In these studies, except for Suazo-Flores who also focused on herself, as discussed later, the mathematics teacher educators used stories to know something about the other (i.e., the participants). In such studies, the goal is not narrative as a way of knowing through relationships between author and self, author and reader, or reader and self. Instead, stories are treated as objects to be analysed to identify categories or themes across them that characterise the authors of the stories in specific ways. Polkinghorne (1995) refers to the thematic approach as *analysis of narratives*, which is grounded in pragmatic reasoning, as opposed to *narrative analysis*, which is grounded in narrative knowing. For Polkinghorne, both are narrative inquiry or ways of knowing, but the products are different in that the former is a traditional qualitative research report while the latter (discussed in the next section) is usually in the form of a story. In the work of mathematics teacher educators, there tends to be more focus on the analysis of narrative than narrative analysis, which could be a consequence of actual or perceived expectations of the field regarding appropriate research methods.

## Narrative in researching self

Narrative, in researching self as a category of examples, deals with mathematics teacher educators who have used narrative in studying themselves through *self-based research methodologies* involving narrative analysis as the way of knowing. These methodologies include *narrative inquiry*, *self-study*, and *autoethnography*. They “privilege self in the research design, recognising that addressing the self can contribute to our understanding of teaching and teacher education” (Hamilton, Smith & Worthington, 2008, p. 17). “Narrative inquiry, the study of experience as story, [...] is first and foremost a way of thinking about experience. [...] To use narrative inquiry methodology is to adopt a particular narrative view of experience as phenomena under study” (Connelly & Clandinin, 2006, p. 477). Self-studies “focus on the nature and development of personal, practical knowledge through examining, in situ, [one’s] own learning, beliefs, practices, processes, contexts, and relationships” (Berry & Hamilton, Para. 1). Autoethnography involves describing and systematically analysing personal experience in order to understand oneself within a culture or group in which one is a member (Ellis & Bochner, 2000).

In addition to the focus on self and experience, these three self-based research methodologies require the use of stories in ways that vary in the context of the studies. For example, narrative inquirers use a variety of stories of experience of self and others in the study, auto-ethnographers write extensive personal stories in the study, and self-study researchers write reflexive journals that include stories of critical moments relevant to the study. The use of narrative is also related to the position of the ‘I’ in each methodology as discussed by Hamilton *et al.* (2008):

For narrative inquiry, the self in relation to others holds privilege in a storied, usually written, form. In autoethnography, it is the cultural I shaped by cultural contexts and complexities that takes the foreground. Where the other methodologies focus on relation or culture, self study researchers focus on practice and improvement of practice, closely attending to self and others in and through their practice. (p. 25)

In the work of mathematics teacher educators, the self-study methodology has received more attention than the other two, which tend to be more dependant on narrative analysis. The following are recent examples of how the teacher educators have used these three methodologies to investigate themselves.

*Narrative Inquiry.* Chauvot (2009) used narrative inquiry to investigate the knowledge she drew on, and her growth as a novice mathematics teacher educator-researcher, from her doctoral program into her third year of a tenure-track faculty position at a United States university. The outcome focused on the knowledge she drew from to fulfill her role as a mathematics teacher educator-researcher. Bailey (2008) investigated her practice while working with preservice primary teachers in mathematics education. She outlined her experience of how narrative inquiry supported a deep exploration of her thinking about mathematics curriculum (how and why) and proposed that it is a valuable means to deepen

one’s understanding. Suazo-Flores (2017) framed her study of an eighth-grade mathematics teacher’s personal practical knowledge as narrative inquiry in which she also explored her own personal practical knowledge in relation to the teacher. The outcome for her was new understandings of the mathematics concept and its teaching, interactions with students, and ways of working with teachers.

*Self-study methodology.* In Kastberg, Lischka, and Hillman (2018), Kastberg reported on her use of self-study to build questioning in her teaching as a relational practice. She focused on developing a perspective on relationship to construct relationships with her PTs based on receptivity in order to grow in relationship with the PTs. Marin’s (2014) self-study focused on her transition from teacher to teacher educator based on her experiences and struggles in teaching a mathematics methods course for the first time. The self-study helped her to uncover her assumptions, challenge her beliefs, frame her practice, and understand who she was as a teacher educator, resulting in changes in her self and practice. Hjalmarson (2017) conducted a self-study of her first-time teaching of an online course for mathematics specialists to support their development as mathematics coaches working with K-8 teachers to enhance mathematics teaching and learning. Her focus was on what guided her design decisions in facilitating learning in the online project-based course for mathematics specialist students.

*Autoethnography methodology.* One study using this methodology is Ward (2017). Ward used autoethnography to explore her experiences, both successes and challenges, as she worked to teach mathematics using a social justice framework in a summer enrichment camp with four and five-year-old children. The outcome informed her way of working with PTs of early childhood mathematics education. For example, she spends more time listening to them and is slow to make judgments about their beliefs based on their actions in the field or things they say or write for class.

In these studies, how the use of narrative unfolded was dependent on the researchers. However, narrative as a way of knowing is inherent to these self-based methodologies in which researchers use stories to construct and tell a larger narrative of new learnings, meanings, or understandings about themselves and possibly others. In such studies, the researcher engagement with stories as a way of learning includes relationships between author and self or reader and self where author and reader is the researcher as participant of a study. For example, the researchers learn through the process of writing stories of themselves regarding what to write about and how to write about it and resonating in their stories regarding what stands out for them and revealing particular aspects of themselves. They could also resonate in stories of others to focus on themselves. In the work of mathematics teacher educators, there tends to be little focus on these self-based methodologies despite their potential benefits. An underlying influence is the concern about the acceptance of non-traditional approaches to research within the broader mathematics education community. But such approaches could bring an important, different perspective to the field to understand the mathematics teacher educator.



## Narrative in supporting teacher learning

In Chapman (2008) I discuss ways in which I have used narrative as a way of knowing to support teacher learning. Here, I focus on two examples of the ways I have used and investigated the use of narrative to support PTs' learning; one involving pedagogical stories and the other problem-solving stories.

Pedagogical stories could be based on real or imagined teaching situations. For example, PTs could write stories about: (i) the teaching of their teachers when they were students of mathematics, (ii) their own teaching during their practicum, (iii) the teaching of others observed during their practicum, and (iv) teaching as imagined. Following is a specific example.

In secondary mathematics education courses I have taught, early in the courses, the PTs were required to write a story of a memorable experience teaching a mathematics concept during their practicum. Figure 1 is an example of their stories. Later in the courses, they were required to rewrite their stories by considering, for example, how students were engaged in learning the mathematics, teacher-student interactions or discourse about the mathematics, student thinking and the mathematics concepts being taught where they identified an issue.

In this example of the use of narrative, the focus was on the PTs' learning through writing and rewriting their stories. This required them to engage in narrative knowing through an author-self relationship. For example, the PT in Figure 1 explained that, for his initial story, he chose the percent lesson to show how the students were thinking and the difficulty they encountered in trying to understand the concept. However, writing the story, and thinking about his role in the lesson, shifted his attention to the difficulty he had in trying to help the students to understand the concept. He became aware of some of his taken-for-granted beliefs about the students, for example, percent is an easy concept for students to learn and if he explains a concept clearly the students will "get it". He raised concerns about his knowledge of the concept, in particular, alternative ways to interpret/represent it. He wondered about possible issues with his instructional approach, which he had considered to be engaging because students actively participated in the discourse. His re-storying of the percent lesson later in the course allowed him to focus on the self he wanted to become as a mathematics teacher based on pedagogical ideas, such as multiple representations, inquiry-based learning, and discourse in teaching mathematics, covered in the course.

The second example of using narrative to support PTs' learning involves PTs exploring and learning about problem solving through personal experience. I focus here on an approach I investigated to enhance the guideline given to the PTs to draw their attention to what to notice and include in the narrative. I reported on this study in Chapman (2013) and include aspects of it here but with more of a focus on narrative knowing. In this approach, secondary mathematics PTs in a mathematics education course (a different year and different PTs from the first example above) solved problems and recorded their experiences in two different ways (W1 and W2), each with different problems. W1 involved writing a narrative account of their experience while W2 involved a form of *self-dialogue* consisting of 'stuck' and 'aha!' (based

on Mason, Burton & Stacey, 1982) moments in the experience. Following is one of the problems used in W2 [1]:

For the end of season squash tournament there were 27 entries. The tournament was arranged on a knockout basis with the loser of each match being eliminated. A number of players received a bye in the first round so that from the second round onwards the number of players going forward at each stage was halved. Norman and Theresa, the squash captains, met to arrange the draw. Their first problem was to decide how many matches would be needed in the first round and hence how many players should have byes. Norman was worried, he didn't really know how to begin, but Theresa with experience of organising tennis tournaments on similar lines was very quickly able to say how many rounds would be needed, how many byes to give and how many matches there would be in the whole tournament. What are the numbers involved? How many matches would need to be played in a tournament with N players? (Bolt, 1989, p. 17)

None of the PTs had an issue interpreting 'bye', used in sports for a situation in which players advance to the next round in a tournament without playing an opponent. Figures 2 and 3 provides excerpts from two of the PTs' self-dialogue for this task.

While my goal for the first example was the PTs' learning, in this example it was both the PTs' and my learning. The PTs engaged in narrative knowing through an author-self relationship based on the aspects of the problem solving experience they considered meaningful to include in their stories and self-dialogues. I engaged in narrative knowing through an author-reader and reader-self relationship based on the aspect of their stories I considered meaningful to understand the PTs, and my thinking, and to inform my practice. My reading of and resonance with the stories clarified my understanding about what was important to them, and taken for granted, based on what was told or not told. It also highlighted what was important to me, and taken for granted, regarding how I tried to support their learning of problem solving as a lived experience though the narrative approach. For example, their W2 stories did not include the affective factors in their W1 stories and their W1 stories did not include the focus on the cognitive in terms of the stuck-aha cycle. Thus combining W1 and W2 seemed likely to be more representative of the lived problem solving experience and offer the PTs better opportunity to develop deeper understanding of themselves as problem solvers and of problem solving from the perspectives of a problem-solver/learner and future teacher of problem solving through narrative knowing. This resulted in me considering ways of working with the two approaches individually and in combination.

## Conclusions

The examples of the use of narrative in the work of mathematics teacher educators discussed in this article highlight the important role narrative has and can play in supporting mathematics teacher educators' research, learning and teaching. Narrative analysis, in particular, offers opportunities for meaningful ways of knowing to develop our self-awareness

Me: Can we have a percent greater than 100?  
*There was a brief moment of silence and then one student said, "no" and the others agreed. Only one student said, "yes".*  
 Me: Okay Daniel, if we can have a percent greater than 100, what does that mean?  
 Daniel: Well, you told us that 100% means a whole, and we can have more than a whole something, so we can have a percent greater than 100.  
 Me: Daniel you are absolutely correct, we can have a percent greater than 100.  
 Carmen: I don't agree Mr. S. How can anything be bigger than 100%?  
 Me: Good question, Carmen.  
*I then drew a picture of a pizza on the board, cut into 4 pieces.*  
 Me: Carmen, do you see that this pizza is cut into 4 pieces?  
 Carmen: Yes.  
*Next, I shaded the 4 pieces in.*  
 Me: Okay Carmen, how many pieces of this pizza are shaded in?  
 Carmen: I know how to do this stuff, 4 pieces or 100%.  
 Me: That is absolutely correct Carmen. Now if I draw another pizza with 4 slices and shade in 2, what is the percent increase?  
 Carmen: Two  
 Me: The percent increase, you are right that 2 is the increase, but not the percent increase. What is the percent increase formula? Yes Steve.  
 Steve: The percent increase is equal to the increase, which is 2 over the original amount, which is 4 multiplied by 100%. This is 50%  
 Me: Right Steve. When we have 2 over 4 we change it to a decimal first. The decimal is 0.5 and then we multiply by 100% to get 50%. Therefore, we have increased our amount of pizza by 50%. Do you agree with this Carmen?  
 Carmen: Yes, but 50% is less than 100%.  
 Me: You are right Carmen. But we can also say that we have 150% of the original pizza when we add the 2 other pieces.  
*About 10 hands went up.*  
 Me: Just wait a minute please. Okay, when we want to know what 150% of 4 is what do we do first?  
 Philip: First we change the percent to a decimal by dividing by 100%, so we get 1.50, then because 'of' means 'times' 1.50 and 4 to get 6.  
 Me: Does anyone notice anything about the numbers 4 and 6?  
 Kim: They add up to 10.  
 Me: Yes Kim they do. But what can you see that relates to the pizzas on the board?  
 Kim: Oh 6 minus 4 is 2 and that is the number of pieces added to the 4 pieces.  
 Me: Great job! That is totally correct. So to get from 4 to 6 we increase by 50% but 6 is 150% of 4, right?  
*The students looked very confused. [...]*  
 Me: How many of you are even slightly confused?  
*About 76% of the class put their hands up.*  
 Me: Okay, Lane where are you having the problem understanding?  
 Lane: Well, I see how you got the 50% increase, but I see the 6 pieces of pizza as 100% not 150%.  
*Actually, most of the class nodded their heads and a few doubted that a percent greater than 100% could really exist. [...]*

Figure 1. An excerpt of a PT's story of a memorable lesson taught during practicum ("Me" is the PT who authored the story).

of our identities, experiences, and biases in order to nurture and energize our practices and to support teachers' learning and their self-awareness of their identities, experiences, and biases. While discussion of challenges in working with narrative analysis is beyond the scope of this article, the examples offered illustrate what is possible. In general, the field of mathematics education could benefit from more attention to narrative as a way of knowing and narrative

analysis. As previously stated, the nature of mathematics teacher educators' professional knowledge and their learning to develop or enhance this knowledge are under-researched areas that need more attention given the growing emphases in many countries on teacher quality (Beswick & Goos, 2018). The self-based methodologies could make a meaningful contribution to address this situation.

## PT1

- [Stuck] I don't see how to ensure all evens, because a number like 14 will give 7 remaining, then a bye is required.
- [Aha] The numbers should be powers of 2 ( $2^n$ ).
- [Stuck] I'm not sure that this doesn't eliminate some legitimate possibilities.
- [Stuck] I don't see how to check with-out just picking numbers at random to test that are even but not  $2^n$
- [Aha] Trying with random even numbers smaller than 27 [...]
- [Stuck] Starting with 16 players means 11 byes in first round, then where do they go?
- [Stuck] I don't know [...] if we are not to have any byes after the first round, it's ambiguous. If we can, changes the problem completely.
- [Aha] Will allow byes in any round [...] Each round that finished with an odd number also had a bye [...]
- [Stuck] I can't be sure that will always be the case.
- [Aha] Try testing numbers near 27 to see what pattern develops. [...] figuring out [...]
- [Aha] Number of matches is  $N-1$  always. [...] Number of rounds is given by the power of the next highest power of 2 [...]
- [Stuck] I can't nail down the pattern for the byes, although I can see one developing [...]

Figure 2. Sample of PT1's self-dialogue.

## Note

[1] Chapman (2013) includes examples of problem and PTs' stories for W1.

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## PT2

- [Stuck] I'm not sure what to do with the odd number of players.
- [Aha] Try to divide 26 by 2 since half of the players have to get byes.
- [Stuck] I cannot see how that is going to work since the 2nd round will have 10 matches which isn't half of 7 matches in the 1<sup>st</sup>.
- [Aha] Try having all the players playing in round 1 except one. [...]
- [Stuck] I'm not sure what to do with this extra person in the 3rd round. It makes sense to give the person a bye to the 4th round but the problem only mentioned byes in the 1st round.
- [Aha] Maybe the first round isn't the only place that players received a bye. [...] Therefore there are 5 rounds 2 byes and 26 matches. So in a tournament with  $N$  players, there will be  $N-1$  matches.
- [Stuck] Looking back on the problem, I have a few problems with the odd numbers which don't divide evenly. [...]
- [Aha] Maybe that is how we can figure out how many byes we'll have all together. Every time we get an odd number of winners in a round we'll get a bye. [...]
- [Stuck] I'm not sure if the question with  $N$  players is using the same method as the problem uses.
- [Aha] Try some other tournaments with different numbers of players and see how many matches they get [...]

Figure 3. Sample of PT2's self-dialogue.



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