LABELLING ANGLES: CARE, INDIFFERENCE AND MATHEMATICAL SYMBOLS

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> Sir TOBY BELCH: *What a plague means my niece, to take the death of her brother thus? I am sure care's an enemy to life*
>
> William Shakespeare, Twelfth Night

Care is fraught with dangers: confusion, frustration and grief. These dangers may well be "enemies to life," but they are also places for growth, learning and joy. In this journal, Hackenberg (2005a) has initiated a conversation about care, mathematics and teaching/learning. Hackenberg describes what it means for her to care for a student by attending to the student's learning of mathematics. She names this way of being with students "mathematical caring relations." In a subsequent exchange of ideas, Falkenberg (2005) recognizes how the idea of mathematical caring relations adds density to general notions of care. He also shifts the conversation toward thinking about the needs of students. In response, Hackenberg (2005b) writes about her work with a particular student and describes the pedagogical content knowledge that she drew on to care for the student. In a later article, Hackenberg further clarifies the complex interactions between affective and cognitive domains that make caring mathematical for her (Hackenberg & Sinclair, 2007)

In this article, I add a further voice to this conversation. I too am interested in exploring care in the context of school mathematics. I wonder how teachers deal with the tension of caring for individual children in their particular contexts and caring for ideas in mathematics that emphasize generality. Specifically, I think there are analogous tensions in caring for people and caring for mathematical ideas. I believe that tensions exist between general rules and principles about what it means to care and the particulars of caring for individuals in their living contexts (Noddings, 2003). For me, there are similar tensions in caring for ideas in mathematics that have to do with general rules and principles, while also attending to particular examples. Sir Toby Belch might well claim that care is an enemy to life (and, perhaps, to mathematics), but I am interested in the tensions around care in the mathematics classroom.

An ethic of care and mathematics

According to Noddings (2003), reciprocity is one feature of a caring relationship. Reciprocity traces the movement of care—literally the give and take of the relationship. Care is not unidirectional; when the person receiving the care—for example, a student—acknowledges and recognizes the care, he or she completes the relationship. The giver of care receives something in return. The reciprocity at work in day-to-day classroom interactions nurtures a sense of connection between teacher and student. It is through reciprocity that care is completed as the carer and cared-for receive from and respond to one another.

Caring for ideas is rooted in experiences of caring for people, though the response that sustains care for mathematics may be considered an aesthetic experience. Some aspects of care are of particular interest in mathematics. One example in the literature is that of equality. Tahta (1993) has described the work of Lusiane Weyl-Kailey, a French therapist and former mathematics teacher. At times, she worked with children on mathematics. Other times, they worked on family issues. In some cases, the mathematics and the family issues were linked. For instance, Weyl-Kailey worked with Theo on mathematical equality, which helped him "to face his family as an equal" (Tahta, 1993, p. 48). Mathematics can provide a space that may not be available elsewhere for thinking about and working through relational issues, because there is the opportunity for developing understanding of and facility with a concept, which can then be applied to other areas.

The field texts from my doctoral work (Long, 2008; Long, 2009) present several aspects of care specific to mathematics. I focus, here, on one aspect that reverberated across the field texts and across time: indifference. I use one classroom episode to probe further into what it means for a teacher, Karen Marks, to care both for her Grade 6 students (ages 11-12 years) and for mathematical ideas. I chose the classroom episode on the basis of my interest in the situation, its potential to engage the reader, and its promise in illustrating dimensions within school mathematics. At the end of the episode, I revisit the experience by connecting it to research in mathematics education and to care.

Indifference, though also an important consideration in other subject areas, seems particularly problematic in school mathematics [1]. The indifference I examine in this study is the indifference that is part of mathematics, the indifference that I see as linked to generalizations. There is also the indifference of people toward mathematics, used as a way of coping with subject matter that is not of interest to them. This sort of indifference may come when a person feels disconnected from mathematics because he or she does not understand concepts or it may come from a person's view of mathematics as a whole (as removed from real life, as boring, and so on). I have personal experience with tensions around indifference as both a teacher and a learner. For this reason, I introduce the notion of indifference through my own memories of teaching and learning mathematics.

A rough map of indifference in school mathematics

An ethic of care indicates concern and response; the two people in a caring relationship matter to one another. Some aspects of mathematics are consistent with indifference (perhaps this could be called *not mattering* or *not caring*). For example, if I write $3x + 4y = 48$ or $3s + 4t = 48$ and specify the variables appropriately, it does not matter which expression I use; the two expressions communicate the same relationship between the variables. If I want to offer pre-service teachers an example of 3-digit addition with double regrouping, it does not matter to me whether I offer 349 + 274 or 486 + 137. Both give the same answer and both involve regrouping in the ones and the tens when using a conventional algorithm. The differences in the addends might not matter mathematically or might not matter to the teacher, but they may well matter to the student who has difficulty adding 6 and 7 but knows the sum of 9 and 4. Instances of indifference, also described as *not mattering* or *not caring,* are sites of tension for exploring care in the mathematics classroom.

When I work with pre-service teachers, we often look at patterns. Inevitably we find a multitude of ways to solve a problem, but when I ask, for example, how we could find the 511th term of a pattern, the real work begins. The teachers have become intimate with the early terms of the sequence. Though they have seen a pattern and have specialized by using several examples, it is usually difficult to come to the generalizations. Though I may intend to care for the pre-service teachers by asking them to engage in mathematical thinking, they may not experience this as care. The pre-service teachers may (or may not) be receptive to a caring relationship with me as their instructor, but being receptive to mathematical ideas is yet another layer of care.

I think of the general as being in profound connection with many particulars. Generalizations are powerful commonalities found through visualizing, observing, communicating, asking repeated questions, and seeking mathematical relationships. (This process could be described as receptivity to mathematical ideas.) Indifference is important to the mathematical power of generalizations. It does not matter which term of a sequence I am looking at; a generalization must describe it and include it.

The power of a generalization lies in its concise description of a mathematical relationship. A generalization imbues the particular elements with sameness (or indifference). Non-mathematical generalizations, such as generalizations about students and learning, can also be useful as teachers plan tasks, recognize potential difficulties, and reflect on their practice. Generalizations (mathematical and non-mathematical) live in tension with the uniqueness of each student's knowledge, experience and humanity. Teachers dwell in this tension between generalizations and particulars.

Two accounts of learning to label angles

Taking as examples the work of Hackenberg (2005a, b) and Hackenberg and Sinclair (2007), I offer an episode that holds this tension of generalizations and particulars. This episode is a reconstruction of classroom events surrounding a student's question about labelling angles. I include two accounts of the same episode, one written by me, the other by the teacher, Karen. As a part of the research process, we reflected on the previous week of teaching and discussed moments that we had noticed. We decided on a moment to explore further and we each wrote an account of the moment. The two accounts of the "labelling angles" moment highlight aspects of teaching and learning mathematics that are related to care.

I first present my own recollections of the episode in order to help the reader get a sense of the context Karen considers in the account that follows.

> Once all the students have presented about their health projects, Karen invites everyone to the carpet. Alex arrives from the washroom just in time. She asks everyone to think about angles, which they have studied before. Nicholas and Alex define the different types. Then students show acute, obtuse, right, and straight angles with their hands. Karen introduces the notion of a reflex angle and uses her hands to demonstrate. She asks everyone to stand up and leads us in trying to show each angle with our bodies, with the waist as the vertex.
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> As we all take our places again, Karen writes each of the five terms on the board and chooses random sticks for people to draw an example of each. The right angle is drawn towards the left. Khalil says, "That's a left angle," and giggles. Karen continues to convey that in order to identify the angle, we put a dot at the vertex and an arc between the two rays forming the angle. Kqaren describes how to name an angle. She places a dot on each ray near the arrowhead. She says that each dot is assigned a letter and the angle is labelled with an angle symbol, $\angle$, along with the letters, with the middle dot being the middle letter.
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> She proceeds by labelling each of the angles. When she labels the straight angle $\angle$ABC, Emily asks why it is called an angle when it is straight. Karen says that it is part of the angle family and that it is still an angle. Emily, sitting at the front of the room, quietly asks a question. Karen broadcasts the question for the rest of the class; it is about why the symbol for an angle is an acute angle. Karen goes on to say that the same symbol, $\angle$, is used no matter what kind of angle is being labelled; it is just a fast way to write "angle."
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> Karen notices that the Gerbera daisies on her desk are drooping and indicates to the students the angles made by the flower head and the stem. She goes through each type of angle. She then mentions that they will use a protractor to measure the acute and obtuse angles on a worksheet. She hands out protractors on transparencies. She reminds students to find the zero because there are two sets of measurements on the protractors in opposite directions. She says, "Remember how that drove you crazy, Emily?" As students begin to

work on their sheets, Karen circulates amongst the students, who are working in groups of two.

Karen's account of teaching the labelling of angles lesson resonates with my experience of the day and also with memories of my own teaching. Though I present her words second, I do not consider them to be of secondary importance. On the contrary, her account helps to reveal care for people and ideas and gives an insider's perspective on the episode

> On Tuesday, we were discussing angles - acute, obtuse, straight, reflex. The students were drawing, measuring, and classifying angles. As Emily was working, she began to wonder why a straight angle is classified as an angle at all. Isn't it just a straight line? She asked me that question, and I wasn't really prepared (I should have been - it's natural). I tried to tie it to the point in the middle that marked that middle part of the angle (hmm, what's that called, anyway?) and that obtuse angles and reflex angles that are nearly straight are still angles. Actually, I'm not making sense to myself even now. Really, what I was expecting Emily to do was accept it because "I said so." In the same lesson, she wondered why this symbol (∠) is used to indicate "angle" even when the angle itself looks nothing like that. Another good question that I wasn't sure how to answer! The written language of mathematics has other examples of arbitrary symbols, and luckily for me Emily was willing to accept that that's just the way it is. I loved the fact that she was wondering about those things and asking those questions. She is clearly trying to make sense of the mathematical ideas and isn't satisfied to just take things on my say-so

Inquiring into the accounts through the arbitrary and the necessary

Here is an example of indifference in mathematics: the symbol for all angles is indifferent to the size of the specific angle labelled; an acute angle, [2] in fact, a particular angle of about 45°, stands in for all angles. Emily's question about why the symbol used to label a straight angle is actually an acute angle, ∠, and not a straight line, points to the importance of considering conventions in school mathematics

Hewitt (1999) distinguishes between the *arbitrary* and the *necessary* in mathematics curriculum. Mathematics can be viewed as arbitrary social conventions or as necessary properties that can be worked out. He asserts that conventions such as labels and symbols for mathematical ideas are considered arbitrary if "someone could only come to know it is true by being informed of it by some external means - whether by a teacher, a book, the internet, *etc.*" (p. 3) while the necessary aspects of mathematics involve properties and relationships that a person can figure out based on what he or she already knows. Hewitt further distinguishes between arbitrary conventions and necessary properties by stating that the former lie "in the realm of memory" (p. 2) while the latter are "dependent upon the awareness students already have" (p. 4). In this article, when I use his term *necessary*, I do not mean to imply that there is but one *necessary* way to interpret or solve mathematical problems. I use *necessary* to point to how humans have created mathematics with connections that follow some internal logic. Humans draw on our abilities to make sense of those connections and expand on them. Using the ideas of *arbitrary* and *necessary*, I re-examine Emily's questions and Karen's written account. I later return to explore care in these same contexts.

Emily's questions about why a straight angle was labelled with an acute-angled symbol indicate that she was prepared to write |ABC so that the symbol matched the type of angle and reflected the difference in the type of angle. During the same class period and the next day's class, I observed students using ⌣ XYZ to label obtuse angles that were similar in orientation and measure to the angle of the symbol. Having spent much of their class time attending to the different types of angles, students counted on the notation being consistent with and reflecting the differences they had studied. During the class period, there was a subtle shift in attention from classifying the angles to the notation that is used to label them [3]. The notation does not vary; it is indifferent to the type of angle being labelled.

One way to think about students' use of unconventional (or invented) symbols is to frame the situation with ideas of what is arbitrary and what is necessary. Karen's students already knew a lot about angles, especially what they looked like and their associated names. They had worked hard to learn the various types of angles, so, I believe, they wanted the symbols to reflect and carry the understanding that they held. They drew on their knowledge of angles and of symbols in other areas of mathematics to come up with symbols for the different angles. They tried to work out for themselves the relationship between the object (the angle) and the symbol for that object. They looked for the necessary in the arbitrary. But the necessary and the arbitrary aspects of mathematics do not overlap, though they may be related. There can be no necessary in the arbitrary. Emily and her classmates bump up against the conventional symbols as they try to work out their own notation system. And though the symbols the students use seem to be incorrect, "the recorded form cannot fully convey the meaning attached to it by the recorder" (Tahta, 1981, p. 3).

Hewitt suggests that the essence of mathematics can be found in the necessary, in the realm of what can be "worked out or found out" (1999, p. 5). Though the angle symbol was created arbitrarily and accepted by the wider mathematical community, Karen's students worked at connecting their understanding of angles to different visual symbols for different types of angles. But the accepted symbol for all angles is still an acute angle: ∠. It is a symbol to be learned and memorized, not one to be worked out; the symbol is fixed.

Consider the angle symbol in the spirit of Magritte's painting "La trahison des images (Ceci n'est pas une pipe)" and Foucault's discussion of the painting (1983): just as representations (Magritte's painting of a pipe and the written words beneath) are not what they represent (a physical pipe), the symbols of mathematics are treacherous: they are not what they symbolize. The symbol used to label a right angle (or any angle) is itself an acute angle (see Figure 1).

Symbols are an important consideration in school mathematics. They merit further discussion. Skemp (1987) wrote

Figure 1. La trahison des symboles (Ceci n'est pas un angle)

about various functions of symbols. In his terms, writing |ABC or ⌣ XYZ can be described as "recording knowledge (and) helping to show structure" (1987, p. 46). Emily and her classmates worked on angles and used their symbols to make a permanent record of how they are named and labelled. In addition, the symbols showed the structure of the differences amongst the angles by using a different symbol to represent each type of angle. [4]

There is something special about how close the symbols are to the objects they represent (the signified). Pictograms, as described by Pimm (1987), are "stylized icons in which the symbol is closely related to the meaning" (p. 141). The symbol looks like the object: □ for a square, ○ for a circle, and △ for a triangle. These three symbols are still general; they can stand for any object in that geometric class. The symbol □ can be used for any square as long as the letters that follow it identify the particular square in question. Emily and her classmates recognize the general nature of symbols and use an appropriate symbol to label any straight angle and any obtuse angle. But when Karen uses an acute-angled symbol to label a non-acute angle, Emily is perplexed. There is a "serious mismatch between the system of symbols which children are required to learn, and their own spontaneous representations" (Hughes, 1986, p. 78).

This mismatch appears not only in geometric symbols, but also in other areas of school mathematics, such as the symbols used to represent operations on numbers. [5] In this classroom episode, Emily's pictogram is perhaps too much like the angle it is representing with respect to standard mathematical symbols. The students' invented symbols for straight and obtuse angles are very specific and are consistent with the differences they have been attending to amongst the types of angles. However, pictograms, though they are visually like the objects they represent, must still be general enough to be useful as symbols. Imagine having to match the shape or type of angle (or other geometric shape) exactly when using a symbol. The usefulness of symbols for communicating with others and for working quickly and flexibly with the ideas that the symbols represent would be negated (Pimm, 1987). It is this generality, or indifference to the particular angle in question, that makes a symbol mathematically useful. Emily and her classmates use their visual and conceptual understanding to develop a coherent system for labelling angles. Though angle labels belong to the arbitrary world of mathematics, students in Karen's class draw on their experience of the necessary relationships amongst mathematical ideas to invent symbols that are consistent with their knowledge.

In this complex moment of teaching and learning, it is important to consider the subject matter. Looking at it in terms of the arbitrary and the necessary sheds some light on how mathematics is intertwined with the classroom context and the roles of teachers and students. Karen's account of labelling angles acknowledges the importance of the arbitrary as well as her understanding of her role as teacher in informing her students of mathematical conventions (though this role is not always easy). She values Emily's thinking as Emily tries to create connections to what she already knows. In other classroom interactions, Karen encouraged her students to work out necessary relationships by providing engaging tasks, asking questions, encouraging discussion, and supporting their thinking. Emily has learned that to do mathematics is to work things out and think hard. Karen tells her students how to label angles because it is a mathematical convention. But Emily sees the symbol and label for an acute angle as setting up a pattern for labelling the different types of angles, a pattern that reflects the differences she is attending to amongst the angles. She works out what she sees as the *necessary symbol* for the straight angle. Though, in Hewitt's terms, a necessary symbol is an oxymoron, in Emily's terms, it is exactly what she is working on. What Emily sees as a necessary symbol is actually an arbitrary convention accepted by the wider mathematical community. These arbitrary aspects of mathematics are *not* mathematics, but they might be described as "paramathematical" (Phillips, 2002, p. 35) or as being in the penumbra of mathematics. But even more is at work in this classroom: it is enlightening to consider an ethic of care in the relationships among teachers, students and mathematics. Through the arbitrary/necessary lens, the nature of mathematics is the focus. In the next section, I revisit Karen's account with care as the lens.

Inquiring into the accounts through care and indifference

The moment of practice described in the accounts was a literal stopping point for the students and their teacher as they discussed and puzzled over the angle symbol. It is a figurative stopping point for me as I re-examine the moment through an ethic of care. Care involves entering into a relationship with another person or with ideas. Karen cares for her students. Aspects of her care are evident in the episode and account as she responds dynamically to students. She looks at them, paying attention to their verbal and body language. She plans tasks and questions for them. She improvises angle types on a Gerbera daisy. And the students respond by asking questions, sharing their understanding, trying out angles with their hands, drawing on the board, and listening attentively. Care takes many forms in this classroom.

For her part, Emily is working hard at understanding mathematical ideas. On previous days, she has shown perseverance in completing mathematical tasks and has willingly shared her insights. Response is valued in a caring relationship; Emily is receptive to the ideas she is working on in mathematics and she has received something in return (pleasure, understanding, an *it makes sense* conclusion). In this case of labelling angles, the mathematics is not giving back; it is not responding as expected because Emily is working with the arbitrary [6] She spends much time categorizing and learning the properties of angles She draws on that knowledge to imagine a symbol for a straight angle (|ABC) that reflects what she has come to know about it and about angles in general. But this conflicts with the symbol Karen uses and Emily asks why this is so

It is through the caring relationship that already exists that Karen responds to Emily's question. There is reciprocity, a back-and-forthness of care, as Karen takes the inquiry seriously and answers in different ways, watching Emily's reactions, monitoring Emily's response, and responding accordingly.

In this situation, Karen cares both for the student asking the question and for the mathematical ideas When she hears Emily's question, Karen knows that her student is thinking mathematically. Karen takes for granted the symbol for angles; as it is a mathematical convention, it is not something to be understood. Emily's query causes Karen to re-evaluate the symbol and the connection to what it is symbolizing. Karen writes, "[Emily] wondered why this symbol (∠) is used to indicate 'angle' even when the angle itself looks nothing like that " As she cares for Emily's mathematical learning, she can also be said to be caring for Emily through mathematics. Karen also works at caring for mathematics directly as she stops to think about Emily's question and allows herself to be open to reconsidering the mathematical ideas, though that receptivity is veiled and is only evident in the slight pause in the rhythm of conversation between Emily and Karen In the moments that followed the unanticipated question, there were many things to consider: understanding Emily's question clearly, responding to Emily in a way that honoured her mathematical efforts and her question, navigating the arbitrary and necessary worlds of mathematics, engaging other students (or not) in the explanation, using the examples on the board, *etc* As Karen later put it, "Why do we do it that way? It sort of causes you to continually question what you know and why you know it and how you're going to try and help them know it "

Drawing the threads together

Teaching is replete with these moments, but what makes this particular instance fascinating is that care for people and care for mathematical ideas are simultaneous and related in complex ways In terms of the arbitrary and the necessary, I think that there is more possibility for a perceived response (such as a feeling of "I get it!") from working on the necessary aspects of mathematics than on arbitrary ones In this account of Emily's questions about labelling angles, students expect to receive something back from their work on mathematics. Through her students, Karen also receives a response, a sense of pride in their learning and an opportunity to engage in her own mathematical work The arbitrary aspects of mathematics, such as symbols and labels, have interesting historical facets, but it takes a different kind of work to receive pleasure (or pride or accomplishment) in this domain. The reciprocity and receptivity that I see as being essential to caring for people is at work in caring for the necessary aspects of mathematical ideas. It comes with profound attention and work.

The indifference that is an integral part of mathematics is strongly tied, though not limited, to the arbitrary. I return to the account about Emily. When we label any angle, we use an arbitrary symbol, ∠, and some letters. Emily did not know this and, as she was trying to work out what she thought of as the necessary symbol (based on the type of angle), the care that she put into her work was blocked by the arbitrariness of mathematical conventions and the indifference to context that those conventions reflect Because she had developed a caring relationship with Karen, Emily was comfortable enough to ask about this blockage. As a way of sustaining care both for Emily and for mathematics, Karen pursued Emily's line of questioning. Though to label a straight angle as |ABC would be considered a mistake in conventional school mathematics, Karen pays close attention to what is behind the symbol and what it means for Emily's understanding of mathematics. Her care is immediate, and it continues as she reflects in her account about what she could do differently.

The aspects of mathematics that are arbitrary, or that are taught or learned as arbitrary, offer experiences that differ from the mathematics that is necessary and based on relations within the subject matter. Arbitrary aspects offer no reciprocity for the student's effort to understand. As well, there is no opportunity to practice the receptivity (or openness to the other or to ideas) that is essential to care It is still important for teachers and students to attend to the arbitrary aspects of mathematics, such as labelling angles, as these aspects can help people work more powerfully on the necessary aspects of mathematics.

By caring for one another and for (necessary) mathematics, we can extend both our personal relationships and our relationships with ideas, just as Karen and Emily did in this account. Through both ways of caring, we can give and receive, feeling deep connections to one another and, within ourselves, for mathematical ideas. Caring for students and caring for mathematical ideas are not independent forms of care, but interplay in complex, and sometimes synergistic, ways Teachers can draw on care for their students to care for mathematical ideas and care for mathematical ideas can become a site for expanding care for students.

The idea that what is necessary in mathematics gives something (generally something positive) back to students and teachers opens possibilities for planned curriculum, educational materials, professional development, and pre-service teacher education. It also brings a knotty new thread into the thoughtful discussion begun by Hackenberg. To echo the beginning of this article, care is no enemy to life or to mathematics Care is, perhaps, a way to think about developing sustaining relationships with both people and ideas in the teaching and learning of mathematics.

Notes

[1] The word *indifference* comes from the Latin *indifferentia*, which means "want of difference, similarity" (www etymonline com). I take *generalization* to mean: a statement created by looking for similarities across specific cases. Both indifference and generalization deal with similarities

[2] The symbol for angle first appeared in a 1634 series of elementary mathematics texts by the French mathematician Pierre Hérigone (Shreves, 1989, p. 362) To denote an angle Hérigone used both the symbol that is now widely accepted, ∠, as well as the symbol <, which had already been used

for over a decade to mean *less than* The first symbol endured despite a number of variations that appeared over the years For a remarkably clear compendium of the invention and use of mathematical symbols, see "The History of Mathematical Symbols" by Douglas Weaver and Anthony D Smith, available at http://www.roma unisa edu au/07305/symbols htm

[3] Though labelling angles is not one of the specific outcomes in the Alberta Program of Studies for Grade 6, the notation is part of what I would call the *understood* curriculum Karen understands that notation is part of what it means to work with angles. In addition, this notation is commonly used in materials prepared by the local school district, including a worksheet students used later that afternoon

[4] Wilder (1968), a topologist and philosopher, differentiates between *symbolic initiative* which is the ability to "assign symbols to stand for objects or ideas, set up relationships between them and operate with them on a conceptual level" (p 5) and *symbolic reflex*, which might be described as learned responses to symbols While recognizing the importance of symbolic reflex in mathematical work, Wilder cautions teachers and students that symbolic reflex must involve understanding of the purposes and processes involved In this episode, Emily and her classmates are demonstrating symbolic initiative as they create symbols for their understanding of different types of angles

[5] For more on the spontaneous symbols that young children use to represent number, addition, and subtraction, see Hughes (1986)

[6] Of course, ideas cannot care for people. I have used personification here and in other parts of the article to draw attention to the indifference that is part of what makes mathematics powerful but also might be experienced as being in tension with attention to the particular

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[T]he abilities of young children are most likely to be elicited by problems that arise naturally in a context which the children find interesting, and where the rationale for working out an answer, using a symbol or writing something down is clearly spelt out. As Margaret Donaldson has pointed out, their difficulties frequently start when they are required to move 'beyond the bounds of human sense'

Any new approach to number must therefore recognise the fact that most children have an impressive range of mathematical abilities when they first start school. Yet this makes it even more puzzling that so many of them find school mathematics difficult and confusing. Even for more competent children, mathematics often becomes a set of tricks and procedures which are applied fairly indiscriminately There is a striking contrast between the lack of thought and care displayed by older children in their mathematics work at school [.] and the persistence and logic shown by much younger children as they grapple with mathematical problems before they start school. One has to ask: what is going wrong?

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