Psychodynamics of Mathematics Texts

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I often think it odd, that it should be so dull, for a good deal of it must be invention.[1]

Nothing is more common than for students to "freeze" at being asked to do a mathematics problem, a response for which there may be no conscious reason. If there are no conscious reasons then such freezing must be an outcome of unconscious process. This might not seem a very helpful conclusion but the work of two unusually scientific psychoanalysts, Ignacio Matte Blanco and Robert Langs helps to explain and tackle the problem. Moreover their work is entirely agnostic in respect of claims of unconscious contents; in their different ways, their work deals only with the functioning of unconscious processes.

In this short paper I would like to argue that Matte Blanco's work can help us think about, and Langs' work maybe resolve, this "freezing" of some people at the sight of a mathematics problem.

It seems reasonable to assume that such counterproductive freezing can be encouraged or retarded by the quality of exposition by teachers and by the textbook that they use. Although teachers vary in how they use textbooks, a small number of texts are continually reprinted year after year, so these texts must be enjoying some level of support from teachers. The quality of exposition in these texts must therefore be one source of messages about mathematics present in many a classroom. It thus seems a fair question to ask whether these expositions may be doing something to encourage freezing. Since I conceptualise this freezing in terms of an idea from Matte Blanco, I begin with a brief introduction to this. I will then examine a couple of frequently reprinted mathematics texts to see whether there is evidence that they encourage freezing.

Symmetrisation

Matte Blanco [2] introduces his ideas with a startling example of a woman suffering from schizophrenia who said: "My arm is my body". Since people suffering from schizophrenia tend to believe things very literally, she probably at that moment genuinely believed this. By contrast most other people have no difficulty in distinguishing a body from one its arms, whether by size, shape, function, and so on. That is, we treat a body and one of its arms *asymmetrically*. This the schizophrenic woman did not – rather she equated them. In so doing she treated body and arm *symmetrically*.

Take a further example, also quoted by Matte Blanco, of another schizophrenic who consulted his dentist after being bitten by a dog. One explanation of this would be that whereas most people would have no difficulty distinguishing a man's teeth from those of a dog, this man may well have symmetrised these, perhaps under some heading like "bad teeth". Thinking that the problem was one of bad teeth, the man referred to a dentist. As in the previous case, two things that needed to be treated asymmetrically were being treated symmetrically; such a replacement I shall call a *symmetrisation*. Of course, normal consciousness can see similarities too, and it is this interplay of symmetry and asymmetry that Matte Blanco calls a bilogical structure [3]. A symmetrisation destroys some of the asymmetries that would otherwise be present in someone's thinking. Matte Blanco thinks that symmetrisation is the work of strong feelings; symmetrisation is exactly what strong feelings produce. It is, he thinks, the marked tendency to symmetrisation that distinguishes psychosis, such as schizophrenia, from normal consciousness.

Like other psychoanalysts, Matte Blanco takes it that schizophrenia and other psychoses result from an invasion of consciousness by unconscious functioning. Psychosis then is taken to be a window through which the strangeness of unconscious functioning can be seen. Matte Blanco believes that this strangeness of unconscious functioning exists precisely in symmetrisation; he holds that examples such as those just quoted above show that where unconscious functioning differs from consciousness, it does so by treating symmetrically what consciousness would treat asymmetrically.

These two examples were very extreme ones, inasmuch as each symmetrisation was very radical. By contrast, many outcomes in non-psychotic situations, are much milder. For example, many of my students have difficulty in imagining time as going back much more than say a hundred years. It is as if all time before this has been symmetrised out in their imaginations.

Matte Blanco stresses that all thinking involves what he calls bilogical functioning: it must simultaneously work with similarities and differences. It is my suggestion that the act of freezing up at a mathematics problem, is a symmetrisation. To see that asymmetries are being lost, note first that problem solving requires that one initially picks out one or more salient points; the saliency of these details has asymmetric importance at that moment as compared to the other details. Secondly, there is the asymmetry of the path of thinking down which one must (asymmetrically) travel towards a solution. Such a path would also make use of some mathematical concepts from elsewhere, and for this to happen some similarities between the present problem and past ones must be seen. Problem solving thus emerges as a very delicate outcome of sophisticated bilogical functioning, and as such would be very easily destroyed by a symmetrisation.

By comparing the first two symmetrisations by the two schizophrenics with the latter one by students, we see that symmetrisations form a continuum that stretches from the radical or severe, to others that are much milder. Freezing

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up would seem to be a symmetrisation of mid-range severity. A more radical symmetrisation than just freezing would be someone who had a so-called "mental block" against mathematics in general; such a block certainly destroys the asymmetries of mathematical problem solving. Once one believes that such a block results from symmetrisation, it is a short step to thinking that it might well be an outcome of unconscious functioning. This would also tie in with the frequent failure for people with such a symmetrisation, of all demonstrations that mathematics is not nearly as difficult as it is taken to be; such appeals would tend to fail since they would not usually be addressing the *unconscious* reasons for the block. Once such a block has been formed it is usually very hard to dissolve.

Secure frame

So much for a conceptualisation of the problem. So far as resolving it goes I find useful an idea I have adapted from Robert Langs [4]. Langs has the idea of the *secure frame*, which he takes to be those conditions in which a patient (in the type of psychotherapy that he has pioneered) can feel as safe as possible. In this paper, I wish to argue that some concepts in mathematics, such as equivalent fractions, can function in the learning of mathematics, as a mathematical version of this secure frame. I will argue that expositions of mathematics in terms of such secure frames minimise the likelihood of symmetrising freezes

One kind of secure frame in mathematics is a concept like equivalent fractions, that can get to the essence of a number of apparently disparate areas of mathematics, such as work with fractions themselves, work with "percentages", "decimals", what it means to "convert" one to another, and so on. By actually studying these fields as instances of equivalent fractions, and speaking explicitly of decimal fractions and percentage fractions, the student in my experience is most likely to come to approach spontaneously these fields in terms of the operation(s) involved in the secure frame - in this case, the multiplying of a fraction above and below by the same value. So the operations involved in a secure frame enjoy a privileged position. Such privileging confers on them an asymmetrical status, which once explicitly recognised by the student, is experienced as both cognitively deserved *and* affectively supportive at the same time It would seem only appropriate that it is in such stout asymmetry that there resides an effective antidote to the baneful operation of Matte Blanco symmetrisation.

This sort of secure frame in mathematics has I believe a further bonus. This is that a properly chosen frame will help a student *attune* to a particular branch of mathematics. Every branch of mathematics has its characteristic music, as every mathematician knows. I believe that it is in this attunement to some branch of mathematics, that we can locate that hard to express, but very real, phenomeneon of mathematical intuition for that branch. Although not so often remarked on directly, it seems plain that since mathematical intuition is not something that occurs in consciousness, it must be an outcome of unconscious functioning. If this involves attunement, then mathematical intuition in some field must include an unconscious attunement to that field.[5]

With these ideas in mind I wish to study a couple of the most widely used mathematical texts in Britain.

The qualifying examination for secondary school education in England and Wales since 1987 has been the General Certificate of Secondary Education, the GCSE. There are three levels of difficulty at the GCSE: Foundation, Intermediate and Higher. I will restrict myself here to looking at two frequently reprinted and widely used GCSE mathematics texts namely: *ST(P) mathematics* by L. Bostock *et al* [6] and *Mathematics for GCSE* by A. Greer [7]. I shall consider the treatment in those books of the following topics since they are generally found to be more than usually difficult and thus may be supposed to be stumbling blocks for many people: fractions in general, especially the addition of fractions; the "solution" of all but the most trivial equations; how to correctly change the dependent variable in a formula which has more than three letters.

Fractions

Addition of two fractions is a notoriously difficult topic and I shall firstly follow its treatment in the text of Bostock *et al*. They say in the Introduction to their book 3A that it "has been designed for those of you who are hoping to attempt the highest level GCSE papers in Mathematics". They approach fractions with a declaration on the bottom of page 1: "The value of a fraction is unaltered if both numerator and denominator are multiplied by the same number". This is described as a fact and is promptly followed by a discussion of how to find a common denominator using the lowest common multiple without any mention that this only works because of the so-called fact. The discussion is not in terms of equivalent fractions, a feature which makes their whole discussion an exercise in cameos. Thus when they introduce so called "percentages" they do so by observing that "65% means 65 out of 100", but a few lines down they say:

$$7/20 = 7/20 \times 100\% = 35\%.$$

This makes the symbol % in the above appear to behave as if it were some constant entity *in the numerator*. Of course % really means 1/100 here, but it actually appears as if % is an operator, like the derivative or the integral, and I doubt if this is cognitively clear or, thus, affectively encouraging to many 15-year-old students!

Greer in his three volumes (for the three levels of difficulty) does similar things in his "Percentages" chapters. These discuss - as most books do- "changing" fractions (and "decimals") into "percentages". This language helps students freeze at the idea of percentages since they have not been properly linked to what is already known, namely fractions. Indeed percentages appear to be things that sometimes, but not always, result from "changing" a fraction or a decimal! Such language implies that percentages are not fractions, which will puzzle students. Greer proceeds like Bostock *et al*, for in his intermediate volume he converts 17/20 into a "percentage" as

$$17/20 = 17/20 \times 100/1\ \%$$

and simply declares: "to convert a fraction or a decimal into a percentage multiply it by 100".

If all this seems like a random witches'-brew of notions then I think that I have fairly represented how our two texts discuss these topics.

We have already noted that the appropriate secure frame in this area is equivalence. Some students fail to form the concept of a *family* of equivalent fractions. One failure is to see just two numbers, and not to form the concept of a numerator and denominator of a single fraction. The opposite conceptual failure results from seeing a fraction as simply one entity, so that all separate behaviour of numerator and denominator is symmetrised out. A fraction is either one thing or it is not; and if it is one thing, then symmetrisation leads to a refusal to believe that you can go around multiplying above and below by some value and still have the same fraction. The symmetrisation here is that of the operation of division; as a result there is either no relation between numerator and denominator, or else these are fused together with superglue! Either way, division is the asymmetry that one has to continually re-insert.

Equations

It seems to me that students need to have explicit exposition of two secure frames here. The first is the algebraic expression, which seems to me to be the mathematical version of the phrase in language - lose it and you can't say anything. One needs then to know of those operations, such as simplifying and factorising, that leave the value of an expression unchanged. The second secure frame is that of equivalent equations, where one (linear) equation is equivalent to another if one has gone from the first to the second by making the *same change* to the left and right hand expression.

Let us now see how authors approach (linear) equations Bostock *et al* proceed as follows in their Book 3A: "Consider the statement $3x + 6 = 2 - x$. This is an equation......Solving an equation means finding the number that the letter stands for so that the two sides are equal." They then give a shopping list of things to do:

1) remove any brackets;
2) collect any like terms on each side;
3) collect letter terms on one side (choose the side with the greater number to start with remembering that, say, $-2x > -3x$);
4) collect the number terms on the other side.

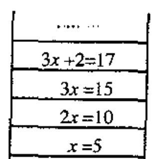
This is just the sort of list which is enough to make plenty of students freeze right off mathematics altogether! One odd thing about it is that having just introduced expressions, our authors say nothing about an equation being the equivalence of two expressions. Indeed immediately prior to this section is an exercise on ways of "simplifying" such expressions, and the somewhat arbitrary set of instructions is simply trying to restate part of this. Having avoided equations as the equality of two expressions, they not surprisingly fall back on that old vague notion of doing very arbitrary looking things to one or other "side."

In his higher level book, Greer introduces algebra with a chapter entitled Basic Algebra. The chapter is actually concerned with what one can and cannot do with algebraic expressions, but nowhere could I find him saying so. Indeed the term "algebraic expressions" first seems to appear during the chapter in a casual remark: "Note that multiplication signs are often missed out when writing algebraic expressions so that, for instance, $2y$ means $2 \times y$....". Greer begins his first chapter on equations, entitled Linear Equations, by declaring: "Linear equations contain only the first power of the unknown quantity. Thus $5x - 3 = 7$ and $x/4 = 9$." Some eighty pages later under the heading "The Equation of a Straight Line" we learn that "Every linear equation may be written in the standard form: $y = mx + c$." I could not discover any explicit statement that the reason for calling linear, an equation where the variables were all of first power only, was because this gives a straight-line graph!

In his intermediate level book, Greer introduces equations by using the familiar scale-pan balance, here to look at $x + 2 = 7$. This of course is a well known model of equations but I suggest that it is actually a very poor substitute for teaching through the requisite secure frames, and that symmetrisation is a good way to discuss this. To begin with there is the obvious fact that the two scale-pans of a balance have some abstract similarity to the two expressions in an equation. The value of the unknown will emerge of course when the unknown is by itself in one pan with the scale in balance.

In saying these things, however, we have said all that the model has in common with an equation but we have omitted many differences. For one thing, given an equation like $x^2 = 5$, how does one obtain the value of x using some *physically* real operations with a scale pan? Much more seriously, a scale-pan presents the student with an exceptionally poor notion of the algebraic expression, which as I have already argued, needs to function in mathematical thinking as a secure frame. In other words, to think of an equation as a balanced scale is a Matte Blanco symmetrisation: yes, there are a few attributes common to the equation and the scale in balance, but to think of an equation as a balanced scale is to lose crucial differences, such as those just mentioned. As with the schizophrenic who consulted his dentist after being bitten by a dog, it is crucial differences that have been symmetrised out.

A better picture for equivalent equations is, I suggest, a ladder;



One plainly climbs up this ladder from the simplest rung, $x = 5$, to some given equation, by steps that consist of making, *at each step*, the same change to each expression. Accordingly, it becomes obvious that a way of climbing down to the simplest rung again, is for each step down to be a particular change to each expression which is the inverse of one used to climb up in the first place. (There is more than one ladder up or down - shades of *Snakes and Ladders*.)

Formulae

Another apparently arcane and magical sounding idea is the "transformation of formulae". This language, and even more the language of "transposing of formulae", leads all too many students to engage in the "transformation" or "transposition" of $a = bcd$ to $b = acd$! For this really is "transposing" the position of a and b!

Though errors of this sort are encouraged by such misleading usage of "rearranging" and "transposing", this does not at all prevent Greer from using both in the same exposition: "The formula $y = ax + b$ has y as its subject. By rearranging this formula we could make x the subject. We are then said to have transposed the formula to make x the subject...." These sentences are then followed by a shopping list like that in Bostock, *et al* and for the same reason: he too has not built his discussion from work on expressions, and so follows an arbitrary looking, discouraging list of six instructions.

By contrast it is obvious, that, provided one has firstly introduced the idea of the independent and dependent variables from graphs, the new dependent variable must simply be one expression of an equation equivalent to the given one.

Final comments

Though I have been critical of the texts discussed they are not obviously worse than many other school texts. The question thus arises as to just why the authors of mathematics texts generally make so little attempt to make the mathematical content attractive? For one thing the examples are often needlessly dull; this is readily observable when contrasted with a text by Mark Bindley [8] which manages to include some very memorable contemporary mathematical examples and some illustrations from the history of mathematics. It is also the case that much more thought is required to show how through equivalence apparently disparate areas are just new forms of the same thing.

All this is I think strengthened by yet another Matte Blanco symmetrisation. This is that those who write mathematics texts are in the first place drawn from that utterly unrepresentative minority who attune to mathematics for its own sake; furthermore, they tacitly symmetrise themselves with those who read their texts. That is to say, many students who do not spontaneously attune to mathematics and for whom the subject is thus experienced as hard and boring, are using texts written by authors who tacitly symmetrise out such feelings, since they do spontaneously attune to mathematics. As a result of this symmetrisation, authors do not seem to ask themselves in any intense and ongoing way what can be done to overcome a lack of attunement which they themselves do not share.

Invoking Matte Blanco also tacitly explains why such symmetrisation would not be noticed. This is precisely because such symmetrisation is occurring non-consciously, and probably unconsciously. The difference between nonconscious and unconscious here lies in the level of resistance that there would be to examining whether such symmetrisation is occurring, and if so, to resolving to write differently in the light of the new insight. Such a new consciousness would lead authors to replace (symmetrising) fusion with empathy, an altogether more subtle blend of closeness and distance.

I hope that I have established that knowledge of Langs' notion of a secure frame and of Matte Blanco's symmetrisation have something to contribute to the writing of mathematics texts. One useful contribution of the latter is to banish the notion of any single student as the ideal audience for such texts, since that would inevitably mean symmetrising all readers with that one single reader. There are several well worn instances of such symmetrisation. For instance, the feminist criticism that mathematics texts have treated the entire body of students as if they knew about physics or engineering, from whence examples in mathematics texts and examination papers have often been drawn. This is plainly a symmetrisation, of the whole student "body" with an engineering-oriented, (mainly male) part. Pursuing this line of thought further, one might speculate whether any single textbook could possibly attune to such a multiplicity of demands. I suspect that no textbook could and that any that attempted it would be literally and metaphorically so heavy as to be even more intimidating, and thus freeze-encouraging, than bulky textbooks already tend to be. The only way I can see to tackle this conundrum is through a flexible network of workshop materials. Perhaps one contribution of Matte Blanco symmetrisation to the creation of mathematics is the potential death of the standard school mathematics textbook, in spirit and in fact.

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