How Ordinary Meaning Underpins the Meaning of Mathematics

CHRISTOPHER ORMELL

In this paper I shall attempt to give a straightforward, non-technical account of the principal ways in which ordinary meaning underpins the meaning of mathematics. The arguments I shall use were first announced in my third monograph on understanding [Ormell, 1985]. The account I shall offer results essentially from a sustained application of the Wittgensteinian principle that the meaning of language can be "read" from the uses to which it is put in society. To ask about the ordinary-language sources of the "meaning" of mathematics is to ask about its uses.

1. The problem

The problem arises from two strange facts: (a) that the total body of recognised uses of mathematics in society appears to be very small, possibly about one percent of the body of known mathematics, (b) that there is no coherent account of the role, purpose and *modus operandi* of applications in the established canon of mathematical knowledge. (a) is strange because, at the level of elementary mathematics, there appear to be a great many uses around. It is commonly believed that mathematics has a lot of uses in technology and industry. Hogben [1951] wrote a book before World War II called *Mathematics for the million*. In it he portrayed "uses" as the main *raison d'être* of mathematics, but mathematicians, such as G.H. Hardy [1967] , were quick to point out that Hogben knew very little mathematics. If he had known more, he would have realised that his theory of the *raison d'être* of mathematics collapsed completely at the higher levels, where the knowledge obtained was predominantly *not* applicable to the real world. [1]

Hogben also failed to say clearly and succinctly what the *role* of applications of mathematics was. In this he simply followed the prevailing tradition that mathematics did not attempt to understand (b) why *other people*, typically engineers and scientists, wanted mathematicians to solve "their" engineering/scientific problems by applying mathematics. I have called this attitude elsewhere the "defensive stance" adopted by mathematicians vis-à-vis other disciplines [Ormell, 1991]. To put it in ordinary language, mathematicians simply "did not want to know" what people in other disciplines did with *their* "applications". The Official Story was that:

i. Mathematics is a powerful instrument for solving problems in science and technology,
ii. Mathematicians have no interest in, or anything to say about, what the mathematical "solutions" to these "problems" achieve. [2]

It is common knowledge that mathematicians as professionals are inward-looking [3] and that they seem to lack any vision of what their knowledge is used for in society. Most textbooks of higher mathematics contain sections called "Applications" at the end of each chapter. These sections say it all. The so-called "applications" of mathematics to the real world listed in these sections consist normally of questions taken from the mathematical discipline and covered with a thin, perfunctory, and totally unconvincing camouflage of ordinary words. Thus in a handbook of "Mathematics for biologists" we find "applications" like: "Find the mass, m grams, of an organism which is growing in accordance with the law:

$$dm/dt = km, \text{ where } k \text{ is a known constant."}$$

It is difficult to believe that any other academic discipline has such a pathetic official understanding of the public uses to which it is put. Of course there are exceptions to the general rule that mathematicians do not concern themselves with these matters. Some outstanding mathematicians such as Von Neumann[4], Weiner, Lighthill[5], Potts[6] *have* taken an interest in the real world and what one can do with mathematics in it. They have not been sufficiently interested, though, to work out a general account of what the application of mathematics achieves in social terms.

Officially, the social purpose of applicable mathematics is a mystery. Figures such as those named above will generally claim, with extreme modesty, that they know a *little* about how to apply mathematics to one *small* corner of the real world, but that they would not presume to understand what is happening elsewhere. It is only when one compares this response with the response of professionals in other fields (to the application of *their* expertise) that one realises just how odd this is.

(b), then, like (a), is very strange!

2. The solution

In the period 1957-1960 the computer achieved for the first time the level of reliability needed to do extensive, sustained, calculations and other logico-mathematical operations. Industrialists and applicable mathematicians began to enjoy using this greatly enhanced computing power to formulate mathematical descriptions of whole situations as opposed to simply looking at isolated parameters of those situations. This new way of "applying" mathematics was called *mathematical modelling*. What happened generally was that a set of equations was used to *mimic* the behavior of a set of variables in a real situation. One could adopt the sort of attitude towards it that one would have to a working model, e.g. a miniature steam engine.

While mathematical modelling in this sense was clearly an interesting tool in industry, it hardly had earth-shattering implications. It might tell you what would happen chemically in the principal reactor of a petro-chemical plant (after 12, 18, 24... hours) *if* you set the input, output and heating controls in a certain way. It was useful to be able to do this, but it only delivered the same kind of information as an experienced operator might know by direct personal observation.

Gradually, however, it became clear that "mathematical modelling" could be used in a more ambitious way than this. Mathematical modelling could be used to mimic situations which were only "possibilities", i.e. did not actually exist. There were two principal kinds of such "possibilities":

A. Possible new theories in science which might account for a body of well-attested information,

B. Possible new gadgets, systems, arrangements, designs in development/technology.[7]

Mathematical modelling enabled the modeller, as it were, to *discuss* the implications of the possibility" he/she was investigating. A classic example from science was that Newton discussed the possibility that the law of gravitational attraction might be an inverse *cube* law. He showed that such a law implied that planets would go into spiral trajectories, falling towards, and finally into, the Sun. In a classic example of technology/development, Renford Potts used a mathematical computer model to discuss the feasibility of the "Dial a Bus" system proposed for Adelaide. He showed that the proposed system could not work in any remotely economical way. He was fully vindicated by subsequent events, after his modelling results had been foolishly disregarded. See MAG (1979) [8]

Literature on failed theories and failed technological innovations is pretty scarce. Nevertheless it is generally believed that only about one proposed technological innovation in a hundred is successful, and that only about one scientific theory in twenty finally achieves a period of consensus support from the appropriate research community. These figures tell us that there are certainly hundreds, more probably thousands, of scientific and technological "possibilities" which are initially "discussed" in society for each theory or gadget that actually achieves public acceptance. By the same token, modelling mathematics has a useful target hundreds, possibly thousands, of times *larger* than was previously recognised. The scope for the useful application of mathematical modelling is very much larger than was always casually assumed. The main "use" of most of this modelling is negative. It tells us that something which sounds extremely plausible (like the Adelaide "Dial-a-Bus" plan) is *not,* actually, viable.

So there are two principal insights which hugely enlarge the "perceived applicability" of mathematics. The first is that, when we apply mathematics to a given topic, event, situation, proposal.... we can usefully use the mathematics to discuss a great range of relevant *possibilities*. We are not, as many hasty commentators have assumed, limited to the actual and the concrete. When NASA used mathematical modelling to preview their lunar missions in the late 1960s they looked at four major kinds of trajectory from Earth to Moon. Simply in terms of the "general form" of the trajectory there were four times as many possibilities to discuss as there might have appeared to be if one assumed that they simply modelled the trajectory actually adopted in the Apollo missions.

The second enlargement is that discussed above: when we realise that mathematical modelling has been applied to many failed projects, we see that almost all the evidence of the mathematical modelling of the past has fallen out of the historical record.

The view of the applicability of mathematics which emerges from the combined effect of the two enlargements justifies, in my opinion, the use of the term "hyper-applicability". This new hyper-applicability is underwritten by the abundant computing power now available in most educational, scientific, and industrial organisations. Mathematics' hyper-applicability, as thus revealed, is not directly "to" the real world as such, but "to" *proposals* to explain it scientifically and to change it technologically/developmentally.

The realisation that the applicability of mathematics is rather larger than it was previously thought to be *has,* I think quietly disseminated itself across quite a wide sector of opinion in both mathematics and mathematics education, though frequently only in the form of a vague awareness of the new horizon. Further reflection shows almost at once, however, that society exerts a distinctly suppressive effect on possible solutions to scientific and technological/developmental problems. Such suppressive pressure was justified in the past by the finite amount of thinking and computing power available properly to "discuss" putative solutions to scientific, technological, and commercial/social/developmental problems. It was necessary to sieve out what were immediately dismissed as the "nuttier" suggestions! Once we recognise that we possess a splendid method—mathematical modelling—to tease out the predictable implications of proposals of all kinds, the need for this draconian censor on the practical/scientific imagination disappears. In other words, realisation of the new potency of mathematics serves as a powerful spur to human imagination, and hence to the creation of yet more targets for application. The sky is the limit to the *potential* amount of imaginative energy which could be usefully invested in scanning future "possibilities". By the same token the sky is the limit for the useful applicability of mathematics. We are, of course, talking about mathematical models of proposals which will normally be handled on a digital computer, via an application program, rather than old-fashioned paper-and-pencil work.

Applications of mathematics to the modelling of plausible "possibilities" may be called "projective" modelling. They can also be described as "IF-if modelling", because the "little ifs" we pursue within each particular mathematical line of enquiry all lie embedded in a "big IF", the plausible "possibility" we are working on.

Now, at last, we have a substantial body of potential "uses" of mathematics. But there is a snag: they appear to be mainly "potential", destined—hopefully—to materialise in the projects of the future. It looks as if mathematics may presently become plainly and visibly "hyper-applicable", but that it is only weakly applicable (in this explicit sense) here-and-now.[9] We have already seen, however, that this conclusion is a mistake. Not only is the total body of actually executed applications of mathematics likely to be much larger in the future than was previously expected: in the past it was considerably larger also. Even if we assume that only *half* the failed scientific theories of the past were "discussed" mathematically, and only a *tenth* of the failed inventions were so "discussed", we end up with a figure for the body of applicable mathematics that was actually executed which is *ten* times the usual presumption. (The "usual presumption" being that only *successful* theories and gadgets were pre-checked mathematically.)

3. Levels of applicability

Why do ordinary citizens, politicians and engineers want to "discuss" the implications of a developmental possibility, say, a *a tunnel under the Bass Straight from Victoria to Tasmania?* The answer is clear: we are interested in certain dependent variables, like the total cost, the likely time of construction, how to minimise any environmental damage, the improvement in driving time from Geelong to Hobart, etc. Our purpose is to tease out implications such as the values of the three latter variables when the first is optimised. This purpose may be called "commercial/developmental design". We want to get the "best possible" version of the design. ("Best possible" must cover environmental costs as well as financial ones.)

Sometimes we do modelling to look for "best possible" designs in personal projects, technical, technological, and scientific experimental contexts. At the level of scientific theory, we do modelling to explore the observable consequences of promising new hypotheses.

It is possible to distinguish *eight* levels of purpose on which we use mathematics to model proposals to describe or change the real world:

8. Explorations of the consequences of new scientific theories, and variations of old theories: we use mathematics to look for their observable implications.
7. Discussion of the design of scientific experiments: we use mathematics to try to get the best possible "signal" from the results of the experiment about the real world.
6. Discussion of consumer/industrial designs hinging on the exploitation of new scientific effects: we use mathematics to try to optimise the design.
5. Discussion of technical designs using established, well-tried scientific/technological principles: we use mathematics to try to optimise the design.
4. Discussion of options and variations in corporate and public developments: we use mathematics to try to minimise the cost while maximising the user-benefits and minimising the environmental damage.
3. Thinking about and planning personal building/making projects: we use mathematics to preview the projects and hence as the basis of our decision what to do.
2. Thinking about and planning personal acts, such as visits, social events, activities: we use mathematics to preview these transient items and hence as the basis of our decision what to do, as in (3) above.
1. Everyday checking-up (on what change we receive, materials supplied, etc): we use elementary calculations to *review* the transaction and verify that it has been correctly carried out.

Above level 8 it is possible to distinguish *four* further levels of purpose when we apply mathematics to *itself*:

12. Meta-mathematics: the most rigorous, most comprehensive, overview of everything on levels 1-11: we use mathematics to try to optimise consistency, elegance, and integration of the total body of knowledge.
11. Modern mathematics: a working version of level 12: it is also concerned with the efficiency and economy of all the methods employed on levels 1-10.
10. Modern discrete mathematics: meta-inquiries arising from he use of discrete modelling on levels 5 - 8: we use mathematics to generalise, unify, fill gaps, and solve puzzles which arise from this discrete modelling.
9. Traditional pure mathematics: meta-inquiries from the use of elementary mathematics on levels 1-4, and continuous mathematics on levels 5-8: here again we use mathematics to generalise, unify, fill gaps, and solve puzzles on the levels 1-8. See Ormell (1982, 1983, 1985).

The point about this taxonomy of purposeful application is that it begins to show the structure as well as the extent of the "hidden" use of mathematics in society. The levels of hyper-applicability, 1-8, are levels on which mathematics is essentially simply a means to particular non-mathematical (scientific and technological) ends. Such mathematics may be described using Richard Feynman's term "Babylonian mathematics", or Chevallard's term "proto-mathematics" [Chevallard, 1989]. It is a "modelling kit"for use on the various levels, and these uses, once understood, can be seen to be extremely numerous. Indeed, it is no exaggeration to describe the potential uses of mathematics as "oceanic".

Now if Wittgenstein is right, if the ultimate substance of meaning is to be located in socially valued uses, this gives us an account of the basis of the "meaning of mathematics" as recognised in society. This, for the ordinary citizen, is what mathematics is about: this is the purpose: this is the function.[10]

By the same token, one would expect a method of teaching mathematics which keeps this possibility-previewing role in the forefront of the pupils' attention, to make more sense than one which does not. I have have been engaged in experiments involving teaching mathematics from the modelling point of view for more than twenty-five years, and I have always found that this is the case. Many children sit up and take notice for the first time when they finally realise (a) that mathematics is useful, (b) that it is used to clarify proposals, (c) that the proposals it will clarify for you can be very interesting ones, concerned with novel rearrangements of objects or ideas. This, more than anything else, banishes the awful incomprehension many children feel about *why* they are being asked to do such a grey and boring subject (as it appears to them).

4. Modus operandi

Mathematical modelling on the levels of hyper-applicability, 1-8, can be represented by the schema:

$$S + D = C \qquad \text{Ormell (1982)}$$

where S stands for a *given situation,* D stands for a proposed *development* in that situation, and C stands for the *consequences*. We do the modelling by working with formal/symbolic expressions which mimic S, D and C. The + sign is not ordinary addition, but a convenient shorthand for the process of "adding" the development to the situation. Sometimes we know S and D and would like to focus on C: this is the case when it has already been decided to "do" D in the real situation, e.g. the "Dial-a-Bus" system mentioned earlier. More often we know the consequences, C, we would *like* to end up with, and the problem is to find the D which, when "added" to S, will produce this desired C. Occasionally we know D (what was done) and C (what happened) and the puzzle is to work out what the initial situation must have been: this may be termed "retrospective" modelling, or more generally, "reviewing".

On each of the levels 1-8, then, we can use modelling in a predicting, previewing, possible previewing (discussing) or reviewing mode. In each case we are initially faced with a kind of *fog* produced by the inability of the unaided human mind to "see" the consequences, or the antecedents, of a situation. The modelling has the general result of clearing the fog. We "zoom in", as it were, mathematically to see the details of S, D or C in an accurate, sharply focussed way.

The main conclusion arising from this analysis, then, is that the "good" delivered by the mathematical modelling of real situations is a transient good. The "fog", having once been cleared, tends to stay cleared. So doing mathematical modelling is rather like cooking meals: once the meal has been prepared, it is soon eaten! Once a modelling analysis of a situation has been done, and has been understood and verified, by those interested in the results, its job is done: the fog goes: we take the results for granted, and there is no point whatever in repeating the exercise. Often the results are built-into a new concept or are printed as a standard graph, formula or table. (We are talking about the direct, adult, *use* of mathematics in society. The "use" of mathematical challenges/problems in educating children is another matter.)

By the same token, the main target for mathematical modelling is a moving, disappearing target: essentially it is to *new* ideas ("possibilities") which look initially plausible in science, technology, industry, and development.

5. The meaning of so-called "pure mathematics"

It is now possible to give an account of the "use" of so-called "pure" mathematics. Pure mathematics starts with the *fact* of a large body of modelling on levels 1-8. This exists, and is virtually "part of the real world", because it delivers "goods" which are valued by society. "Pure mathematics" may be described as the attempt to apply scientific curiosity to this "reality". We seek to understand such strange facts emerging on levels 1-8 as that any natural number can be expressed in lots of ways in the forms $a + b$, $a - b$, and $a : b$, where a and b are also natural numbers, but cannot always be expressed in more than *one* way in the form $a \times b$, that the sum of the roots of a quadratic equation always appears to be $-b/a$, even when the roots are complex, that the integral of any power of x is another power of x except in the case of $1/x$... and so on. Often the motive driving inquiry at this stage is simple interpolation — curiosity about the forms/structures which lie in between the ones which have been used for modelling.

Such "scientific inquiry" leads, of course, to new structures and concepts. Investigations of the solubility of quadratic equations led to the concept of imaginary numbers, which might be described as a form of *coding* representing the "distance" of the "impossible roots" (i.e. expressions containing $\sqrt{-1}$) from being real roots. Such forms then become starting points for new exercises in scientific curiosity. "Pure" mathematics may therefore be described as a "great root tree" of results, initially derived from modelling mathematics, but subsequently growing prolifically be a process of self-regeneration.[11] For centuries its results, though predominantly useless in the real world, contained a few pearls of conceptual power which enabled computations to be achieved in areas otherwise quite inaccessible to ordinary paper-and-pencil calculation. These results were considered *priceless*. They provided tiny patches of "computing power" in an otherwise computerless world. The computer may appear, at first, to have devalued them, and certainly they have ceased to be priceless in the way they were before the computer era. But we are becoming increasingly aware that, for best results, mathematics should be used hand-in-hand with computer programs. To achieve the "magic" standards of speed we have come to expect in application programs, it is often necessary to infuse mathematical simplifications into the program. These bits of "pure" mathematics then take on the role of *computing accelerators,* available for use both in modelling the real world and in exploring the "mathematicians' mathematics" of levels 9-12.

6. Consequences for education

The first and major consequence for education is probably that it is idle to try to teach children mathematics in the manner of levels 9-12, i.e. "mathematicians' mathematics" or "non-modelling mathematics" [12], until the children have come to appreciate quite a lot of the weight of the meaning, purpose, and reality of modelling mathematics on levels 1-8. The questions which "mathematicians' mathematics" tries to answer are an expression of the same kind of curiosity as natural science, but when tackled prematurely, i.e. in the absence of a preliminary awareness of the usefulness, magic [13]. and power of modelling, often look hopelessly arbitrary, artificial, boring, unintelligible, and pointless. I'm afraid we have made the mistake of trying to teach children mathematics from a "mathematician's" viewpoint for more than two thousand years. Traditionally mathematicians have been quite scornful of "useful" mathematics. Most of us used to regard it as pretty boring. We only began to take an interest on the predominantly "useless" theoretical levels 9-12. We did not really try to understand how much "useful" mathematics there was, or what it did for people. We completely underestimated its scope and use in the past, because the records had disappeared.

We were hardly aware, however, that we presupposed the "importance" of mathematics. This importance, it is now abundantly clear, stems from the uses to which mathematics is put in society on levels 1-8.

In my opinion we ought to offer *all* children a good initiation into mathematics as a modelling kit for previewing and discussing what they and others are going to do. If we do the job even half-decently they ought to learn far more modelling mathematics than children did in the past, though it would be a mistake to push them too fast. (We need tests for *thorough assimilation* so that we can tell when pupils are ready to proceed and when they are not.[14]) The potential interest of modelling to children is limitless: mathematics can easily, I believe, become the most fascinating subject in the curriculum, because we can use it over and over again to pose questions like the viability of the Dial-a-Bus, and the Tasmanian Tunnel, and thousands of equally mind-stimulating thing closer to home. In each culture there are "burning" issues which we can skirt round and illuminate, thereby broadening the awareness of the children to the kind of world *their* world will be in ten or twenty years' time.

In a word, the hyper-applicability of mathematics in society can give us a vast new canon of stimulating problem-challenges for children. They can be used at all stages in teaching maths. As initial examples, illustrations, worked analyses, discussion problems, routine problems, assignments, projects and examination assessments.

The imaginative quality of mathematics education is capable of being transformed for most children by the use of this methodology.

As the years pass we will find, I am sure, that some especially able children begin to take an interest in modelling mathematics as a fact, as a "given" thing, and therefore as a target of rational curiosity. For them it will be possible to begin an initiation into "mathematicians' mathematics", though I would caution against going too fast. (There might be a temptation for some people to say "Good! Now we can go back to the old methods!") At each stage we must ascertain the level of symbols/concepts the pupil is coming to take-for-granted. These symbols, these concepts, have become unquestionably "real" for the child, and can become a focus for natural, scientific, curiosity.[15] But don't let's abandon everything we know about good pedagogy! We should still use a problem-solving pedagogy with these students, respecting their curiosity and watching carefully to see that we do not abuse it (i.e. over-extend it) in the interests of getting academic glory for ourselves and our institutions![16]

Notes

[1] In my monograph [1982] I offered a notional graph which illustrated how the "perceived applicability" of mathematics changed (in the 1930s) with the amount of mathematics one knew. The graph rises steeply at first, but then bends over and levels off. Anyone who was only familiar with a little mathematics may have been tempted to extrapolate from the early, steeply-rising part of the graph, and hence obtain a totally unreal picture of the applicability of mathematics as a whole *at that time*. The effect of the arrival of the computer has, however, completely changed the picture. It was the main stimulus leading to the new view of mathematics as "hyper-applicable" to the real world, and hence to a greatly enlarged "perceived applicability".

[2] The first part of this pair of attitudes is prominently present in the U.K. Cockcroft Report [1982]. The second part is also there—by default, i.e. it is displayed by the fact that the Report is satisfied with slogans of this kind and little further illumination of what this "problem solving" is *about*, or what this"method of communication" is used *for*, is offered.

[3] This theme is explored at length in Kline [1980] and Davis and Hersh [1982].

[4] See Von Neumann [1954].

[5] See Lighthill [1972].

[6] Potts contributed a chapter to Lighthill [1972]

[7] These; "possibilities" are not figments of the imagination, but *real* possibilities in the sense that politics is sometimes defined as the "art of the possible" We are talking about things which are unquestionably relevant to a practical or theoretical proposal, which need to be "taken into account" when we discuss that proposal

[8] It is possible to demonstrate the gist of Potts' analysis using a computer model very easily. First generate, say, twenty pairs of random points in a rectangle (taken to represent the City of Adelaide) and get the computer to show them as directed line intervals on the screen. They fall across it like hatpins of variable size thrown onto the floor at random. The difficulty of "clubbing together" sets of these requested journeys (to make up viable minibus trips) becomes immediately obvious

[9] Various levels of "perception" of the applic*ability* are in play here. The "hyper-applicability" of mathematics which is the theme of the present paper is apparent to anyone who seriously thinks about the matter. It is, in plain terms, a potential applicability. The commonly perceived "applicability" is determined by what people have seen of what has actually been executed. In the case of the historical examples, however, people have "seen" rather little, because the mathematical discussion of failed projects in technology, development and science has usually gone straight into the waste paper basket

[10] There is also the reverse effect of this realisation: that Wittgenstein's account of meaning, which seemed for many years to give such a poor account of the meaning of mathematics, is vindicated after all. This strengthens the Wittgensteinian analysis in all sorts of ways. Here, too, a general awareness of this fact has permeated quite widely around educational circles, and may be seen in the increased use of Wittgensteinian ideas in education, philosophy of education and particularly in the philosophy of mathematics education. In the present author's opinion, however, Wittgenstein's *Reflections on the foundations of mathematics*[1956] is one of his least successful works, and one which reflects the fact that he was never initiated into the central methods of modern mathematics.

[11] In other words, very like a tree, much of the substance of which comes, not through the roots, but out of the air.

[12] Actually, as Sir Michael Atiyah has pointed out, "modelling" is a method extensively used in modern *pure* mathematics as well as in applicable mathematics: in the sense that we try to construct representations of key aspects (subsystems of) formal systems which enable us to disregard much of the currently irrelevant detail. See Atiyah [1979].

[13] This "magic" is much more important as a motivator of children than what they often see as "boring", i.e. ordinary pedestrian examples of,

utility. The "magic" arises essentially from scenarios which set out consciously to explore the practical implications of highly unusual or innovative ideas These are "protective modelling scenarios" The first examples were devised by the author for the Schools Council *Mathematics Applicable* Project (1969-78) Since 1978 many other examples have been devised under the auspices of the Mathematics Applicable Group, based at the School of Education, University of East Anglia

[14] The author worked for five years(1981-86) with Dr Ibrahim Abdel-Ghany of Minia University, Egypt, on this problem The results consist of a new concept "application readiness", tests (for "application readiness") and a new test methodology (topic and subject anonymity). See Ormell [1989a, 1989b] for further details.

[15] This is an example of Thom's general principle that we must let the concepts of a given stage of mathematics become unquestionably "real" for the child before we move to the next stage See Thom

[16] Revised version of a talk given to the International RECSAM-Deakin Research Symposium on Cultural Aspects of Mathematics/Science Education, October 1990.

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This educational truthful 'hardness' of the real is seen by Plato at first most evidently in mathematics which plays the crucial mediating role in the education system of the *Republic*. Mathematics leads us beyond the lower (softer) education [..] which consists of (carefully selected) stories music poetry, where an uncriticized imagery is the best vehicle of such truth as can be grasped Understanding of how mathematics is independent of sense experience leads us to understand how the Forms are; and necessary mathematical relations suggest necessary relations between Forms [..] Measuring and counting are "felicitous aids" by which reason leads the soul from appearance to reality The paradoxes of sense experience inspire us to philosophy and our respect for and satisfaction in necessary form, whether in mathematics or in nature tends to fortify our rational faculties. (Philosophy since Plato has largely favoured just these starting points)

Iris Murdoch
