DYNAMIC CONTEXTS AND IMAGINED WORLDS: AN INTERDISCIPLINARY APPROACH TO MATHEMATICS APPLICATIONS

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While students are able to recognize that questions in other disciplines involve mathematical knowledge and that personal knowledge is a resource for solving problems

> (t)he difficulty lies in their ability to integrate these knowledges with knowledge of content areas other than mathematics. We contend that this skill is one of the most essential elements of mathematical power. (Kastberg, D'Ambrosio, McDermott and Saada, 2005, p. 15)

In this article, I report on a teaching experiment in which undergraduate algebra students studied exponential growth as a means of understanding the effects of infectious diseases on personal income internationally. The social science context of economic epidemiology in this application differed from many treatments of context in mathematics because the context was dynamic – it was a site of active learning in which students encountered social science learning goals alongside mathematical tasks. I outline the teaching opportunities and dilemmas posed by a strongly interdisciplinary approach to context, one in which the issues and forms of analysis of each discipline, in this case mathematics and social science, are treated equally. This discussion is intended to help frame research questions on mathematics applications presented through significant non-mathematical contexts.

Deep contexts: beyond figure and ground

While several prominent approaches in mathematics education make extensive and critical use of context, they do this with rather different boundaries and different means of linking mathematics to context. In the Realistic Mathematics Education (RME) approach, curriculum design focuses on "rich contexts demanding mathematical organization" (Van den Heuvel-Panhuizen, 2005, p. 2). These contexts may be based in concrete experiences that are familiar to students, or they may be imagined worlds that have mathematical dimensions. Contexts should be "real in the students' minds and they can experience them as real for themselves" (*ibid.*, p. 2). Imagined realities are important for RME contextualizations because the students come to feel ownership over the problem as they select which aspect of the context they will describe mathematically.

In a well-known problem context, for example, a lift having a capacity of 14 people, students are asked how many times the lift must go up if 269 people want to use it during rush hour (SEAC, 1992). Cooper and Harries (2002) show that students are able to imagine aspects of the real-world context that justify different answers, and they are able to make this contextual selection in multiple ways. Attending to different aspects of context leads to different forms of mathematizing that many mathematics educators view as legitimate.

Even given the openness of interpretation and the recognition of imagination as a field of mathematical relevance, RME, Problem-Based Learning (PBL), and similar approaches to designing authentic problem-solving tasks still tend to use a limited approach to context when viewed from the perspective of interdisciplinary potential. Context is most often viewed dichotomously as a field of figure and ground, in which the students' goal is to separate the mathematical figure from the ground of extraneous real-world circumstance. Different students may appropriate the mathematical figure from the context differently, leading to different forms of mathematizing, but once this choice is made, the remainder of the context drops out of sight. A study (Fuchs, L., Fuchs, D., Finelli, Courey, Hamlett, Sones and Hope, 2006) of explicit instruction on real-life problem solving strategies illustrates this orientation to context quite clearly:

> Teacher: Yes, a lot of information in real life is irrelevant for answering our questions. We ignore irrelevant information. This is another tip about real-life situations [refer to poster]:
>
> IGNORE irrelevant information. (p. 308)

This direct instruction on problem-solving strategies yielded significant improvements in students' ability to transfer these strategies to new mathematics problems. It enhanced assessed mathematical skill. We do not know, however, if there are hidden costs associated with the radical suppression of non-mathematical content. Nor do we know how to teach students to think mathematically while retaining a sense of value and importance for some aspect of the authentic situation.

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Another study from PBL suggests that context can enhance motivation. PBL

> promotes links among subject matter disciplines and presents an expanded, rather than narrow, view of subject matter (Blumenfeld, Soloway, Marx, Krajcik, Guzdial and Palincsar, 1991, p. 372)

but the character and limitations of these linkages are not described. The authors observe,

> we have little systematic empirical information about what problems students actually find valuable, interesting, or useful enough to work on for long periods. (*ibid.*, p. 376)

Because students usually do not approach mathematics with the values and perspectives of a mathematician, it is possible that the valuable, interesting and useful aspects of problems are contained as much in the contextual ground as in the mathematical figure.

I take the perspective that a stimulating presentation of context will allow the development of multiple subjective viewpoints on the context. I contrast this subjective, issues-oriented approach to the conception of context as setting. The lift problem, for example, as it was configured by Cooper and Harries, is strongly dynamic in the sense that it supports different mathematical interpretations. As a human situation, however, once students choose the aspects of the context that they will mathematize, no further learning experiences support their development of a sense of value or interest in the situation itself.

For the purposes of this article, I define a dynamic context to be one that supports non-mathematical learning goals and that includes activities that encourage students to develop unique, subjective evaluations of the context – in relation to or apart from mathematics. This approach to presenting contexts for mathematics applications *i.e.*, using information that is not necessarily essential to producing an answer, can, just as in RME tasks, be powerful in making the problem meaningful and support the case that mathematics is an activity worth doing.

Teaching approaches with social contexts

Several approaches make explicit use of social data for context, including ethnomathematics (Ascher, 1991, 2002; Borba, 1997; D'Ambrosio, 1985; Eglash, 2002; Frankenstein, 1997; Lipka and Adams, 2004; Zavlasky, 1973), critical mathematics (Skovsmose, 1994), culturally-responsive mathematics (Ladson Billings, 1995) and mathematics for social justice (Gutstein, 2003). These approaches differ in how dynamic the contexts tend to be, what use the curriculum makes of context and whether there are learning goals associated with the context.

In the case of critical mathematics, Skovsmose (1994) describes numerous social discussion topics on the theme of "pocket money" and household economics: "What are the reasons for differences in salary?" (p. 64) or "Why is a new youth club built just around the corner?" (p. 66). Here, discussions seem to serve the purpose of creating engagement through a detailed imagined context rather than to achieve specific social learning goals.

While ethnomathematics accounts are generally very well researched (Ascher, 1991, 2002), they often do not make full use of context in developing dynamic social science learning goals. Take, for example, the famous example of the *quipu*, base-10 Andean counting devices. *Quipu* mathematics is presented from primary school classrooms through undergraduate liberal arts general education classes and teacher education classes. The secondary or undergraduate levels could include historical data that suggests *quipu*s were used for indigenous resistance against Spanish colonialism through the act of confession. In the mid 1570s, Catholic priests recommended that the penitent tie *quipu*s to assist the memory of sins during confession, but the policy was reversed in 1583, as *quipu*s were by this time understood as a means of recalling indigenous religious rites. Some indigenous leaders had people in their community tie *quipu*s to help them recall false sins to offer in confession (Harrison, 2002, pp. 268-9). As Andean mathematics became a threat to colonial control, Church authorities advocated the destruction of *quipu*s. A dynamic context for presenting *quipu*s could use ethnohistorical data to develop subjective positions around the theme of mathematics as an implement of state power and local resistance.

The major studies of culturally relevant teaching focus on classrooms that primarily serve a particular ethnic group (Gutstein, 2003, 2006; Ladson Billings, 1995a, 1995b, 1997; Matthews, 2003), but researchers have framed the approach with fully diverse classrooms in mind. They recognize, for example, that a teacher can teach in a culturally relevant way without sharing the students' heritage by creating activities based on students' cultural experiences or shared classroom experiences (Cahnmann and Remillard, 2002, p. 198). Researchers have found, though, that teachers are hard-pressed to develop learning experiences that treat both mathematics and social topics with significance. Matthews recommends the "mathematizing of social issues affecting students and their communities" (2003, p. 63). His observation that "building on students' cultural knowledge is essentially an active, dialogical endeavor" (2003, p. 80) will be useful in considering how to build contextualized curricula.

Teachers within the mathematics for social justice approach tend to have clearly identified, if broad learning goals for the role of context (Frankenstein, 1997; Gutstein, 2003, 2006; Weissglass 2000). Gutstein identifies three social justice learning goals: the development of "sociopolitical consciousness [...] a sense of *social agency*, [...] and positive cultural/social identities" (2006, p. 332). Frankenstein (1997) also provides models of discussions with specific social awareness goals, most importantly, recognizing racism and the mathematical methods of its implementation. In this view, mathematics is a tool to develop awareness of social issues.

Taken together, teaching approaches that value contextualized mathematics suggest that contexts

- create a rich, imagined world
- draw from shared social, cultural or classroom experiences
- encourage a sense of ownership
- encourage dialogue
- develop critical perspectives

In other words, contexts should create personal engagement, classroom interaction and the perception that a mathematical application is important enough that it can make a difference in the world. The case for dynamic contexts for mathematics applications is thus two-fold – students need to learn about the world and they need to develop a subjective position on it before they will truly have a sense of purpose for doing mathematics. Moreover, mathematical actions alone may not be sufficient to create this level of investment for all students, but a dynamic context may capture a broader range of imaginations. Different students make mathematical sense of contexts in different ways (Boaler, 1993) and, similarly, we can expect students to have varied reactions to contextual content alongside a mathematics application. Contextual information is not just relevant or irrelevant but could add motivational, inspirational or new mathematical modelling ideas.

An epidemiological context: disease and household income

In the spring of 2004, two of my classes of students studying developmental intermediate algebra at the University of Minnesota worked on exponential growth through the dynamic context of economic epidemiology focusing on the effects of infectious disease on personal income internationally. Students also completed standard textbook problems involving population growth, radioactive decay and continuously compounded interest. A third class studied exponential growth in a traditional manner through textbook problems. In addition, this group studied the limit definition of *e*, graphing increasing and decreasing exponential functions, and approximating solutions of exponential equations. Few of the students in the three classes were familiar with the base of the natural logarithm, and many had questions about articulating the order of operations on their calculators. The unit was part of an ongoing research and teaching project on issues-based algebra applications in the social sciences.

All the developmental algebra classes at the University of Minnesota during these years were strongly multilingual and multi-ethnic (approximately 50% of enrolled students were non-white, mostly African-American, Euro-American, Somali, Latino, Hmong and Native American). The class diversity was a significant resource for discussions on the impact of infectious diseases. The primary data source for students' explorations was the United Nations Development Programme (UNDP) Human Development Indicators document (2002) along with economic growth penalties for HIV and malaria reported in Sachs and Malaney (2002) and Piot, Bartos, Ghys, Walker and Schwartländer (2001)

The UNDP data set allows students to gain an immediate sense of worldwide economic disparities. Students used exponential growth to predict changes in personal income (using Gross Domestic Production (GDP) *per capita*) in various countries over a given number of years. Then they used data on the growth penalties exacted by malaria or HIV to estimate household income if these diseases could be eliminated (Piot *et al*, 2001; Sachs and Malaney, 2002). A sample calculation (see Figure 1) shows how a student has used Tanzania's GDP *per capita* of $501 and the annual

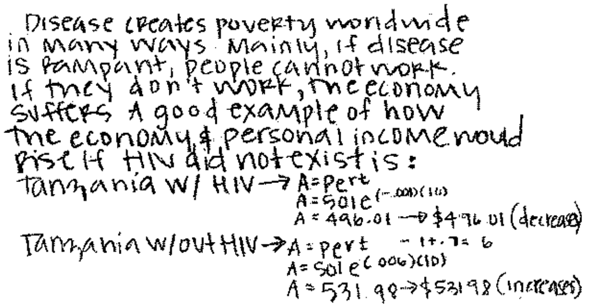

Figure 1: Predicting personal income in Tanzania.

GDP *per capita* growth rate of −0.1% from Human Development Indicators, to estimate the change in personal income over 10 years. In the second calculation, she has used Tanzania's HIV prevalence and the graph in Piot *et al.* (2001) to estimate the growth penalty for HIV in Tanzania and adjust the *per capita* growth rate

The contextual theme of this unit was to understand associations between infectious disease and underdevelopment. Contextual learning goals were centered on United Nations' perspectives on infectious diseases and assessment of standard of living, as presented in the UNDP Human Development Indicators data set. Some of these learning goals were factual. Students learned, for example, that the United Nations considers malaria, HIV/AIDS, and tuberculosis to be the world's most damaging infectious diseases and they studied the geographical distribution and prevalence of the diseases. They learned that UN assessments of standard of living are based on GDP *per capita*, literacy rates, and life expectancy. Open-ended contextual questions were also included in class discussions, for example, "What aspects of life should be included in a measurement of standard of living?" and "How are disease and poverty interrelated?" From here students compared the UN assessment of standard of living in the Human Development Indicators document with mainstream American values. Many students expressed surprise that the United States was not ranked first in UN standard of living estimates. The class also developed a critique of the mathematical model by suggesting that unequal distribution of wealth within a country can confound economic predictions using aggregate data

When asked to consider how disease and poverty are related, students quickly suggested that poverty exacerbates the effects of disease, giving responses along the lines of "No money to visit the doctor." The converse, that the presence of an infectious disease is a cause of economic underdevelopment, was a more elusive concept for many of the students, but they developed an understanding of this effect through their estimates of personal income. As one student observed, increased programs for malaria prevention in Africa "would be a good investment. When the disease is eradicated in our problems, the income goes up".

Understanding global differences in resources and assessments of standard of living is the topic that generated the most dialogue and disparate views. Many students expressed surprise and sympathy as they encountered global disparities. Other students continued to see unequal access to resources as an unquestioned aspect of life, as one young woman wrote, "it takes more to take care of me." I believe that humanistic perspectives often take longer than one semester to develop and may be difficult to assess at all. The opinions and experiences expressed by students' peers often seem to affect students more than those that I express

On the last day of the unit, students completed a test that included traditional exponential growth problems, objective epidemiology problems (similar to the problem in Figure 1) and open-ended questions that allowed students to articulate mathematics and contextual understanding. Some sample responses to a question on relationships between poverty and disease can be seen in Figures 2 and 3.

In Figure 3, the student refers to the HIV growth penalty graph in Piot *et al* (2001). He estimates that the growth penalty for a population with 5% HIV prevalence is -0.5% and for 30% HIV prevalence, it is -1.4%. Many students made reference to these average growth penalties for malaria and to graphs used to estimate the growth penalty for HIV given GDP *per capita* (Piot *et al.*, 2001; Sachs and Malaney, 2002). Other students made reference to homework problems, as in Figure 1. A few students used the exponential growth equation to construct scenarios that were not covered in class, see Figure 4, in which a student modeled unemployment due to infectious diseases.

With this in mind, as a student's teacher for a single semester, I define the limits of my role to be the creation of classroom situations in which students encounter data on disparities, learn techniques of mathematical analysis and listen to the varied viewpoints of their classmates.

Dynamic contexts create imagined worlds

The context of an application, even the thin context of a standard textbook word problem, invites students to participate in an imaginative act or fantasy world. Student reflections at the end of the semester indicated the character of these imagined worlds. As part of a broader assessment of

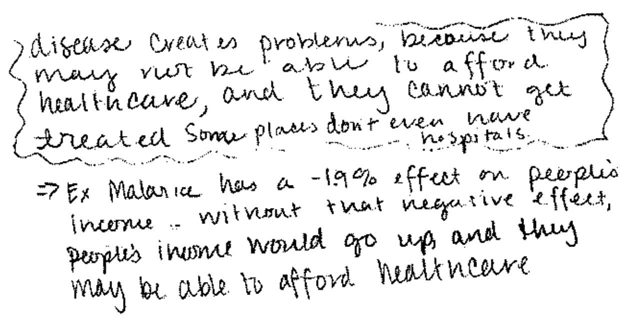

Figure 2: Malaria growth penalty.

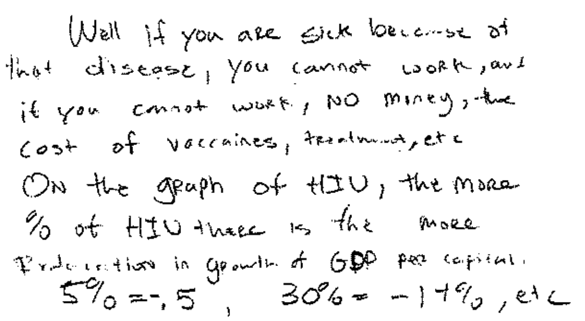

Figure 3: HIV growth penalty.

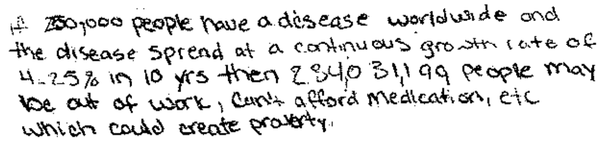

Figure 4: A model of unemployment due to infectious diseases.

the class, student focus groups were asked, "What was the most memorable topic of the class for you?." Some students indicated the topic that they found most difficult, while others selected a topic that they enjoyed or that they had initially found confusing, but had later mastered. A few of the students in the class that had studied exponential growth in a traditional manner selected this unit:

Ally: The HIV/AIDS . just because you got to relate it to actual things that are actually with ... people, like it was not just math and numbers, it was like comparing to people and life in different countries.

Andrea: Just because you can actually see it ... yeah, because you learn new stuff about other countries, like I never knew about so many countries, the US is fourth (*sic*) and we always think we are the highest standard of living and really we're not. It just gives you more information.

Brendon: The HIV and malaria and in other countries. I think that had an impact on me. Just like I didn't know how diseases could bring down your income so much .. or how many people could die or how they were living because of the disease.

All of these students described in vivid terms an imagined world rooted in the context of an exponential growth application. They spoke of a reorientation of thought, a newfound sympathy or a revised sense of nationalism. Ally expressed her connection in the very direct terms of relationship: "you got to relate it to . . people." Phrasing this as "got" rather than "had" seems to imply that Ally considered this as an opportunity to develop thinking that she valued. Andrea expressed her connection in the intellectual terms "learn stuff about other countries" and she applied this knowledge to develop a more reflective stance on the standard of living in the United States than she previously had taken. Students who found the epidemiological application as highly memorable expressed an awareness of human realities that many had not experienced directly, but that had opened up to them through mathematics.

All of these students appreciated the concreteness of exponential applications, whether experimental or traditional. Verbal anchors tied their discourse to the experiential world: "actual," "see it," "how they were living." Arguably, all of the students also valued computational processes, as in "I didn't know how diseases could bring down your income so much." Even Ally, who appreciated the disease application because it "was not just math and numbers," seems to have gained a deeper understanding of mathematics as a representation of reality, or "comparison," as she put it, "comparing to people and their life in other countries." It is important to note that Ally was one of the strongest algebra students in her class. Students are not affected by contextualized applications because they do less mathematics - indeed, the epidemiological application was more challenging than standard textbook applications - but rather because interdisciplinary contexts that extend slightly beyond the purview of mathematics give mathematics meaning

Discussion and dilemmas

If contextualized mathematics is to fulfill the vision of fostering dialogue and critical perspectives, we must rethink how mathematics teachers can prepare, present and guide students through interdisciplinary aspects. Dialogue and perspective are likely to be based on more than simply mathematics, and the multiplicity of student responses implied by this vision suggests that a static, factual view of context-as-setting will prove inadequate. The dynamic context approach described in this article situates context as a site for active learning in both mathematics and other disciplines.

In 1993, Boaler wrote that the popular impression of mathematics as impersonal, abstract and irrelevant "may be broken down by the use of contexts which are more subjective and personal" (p. 13). Embedding mathematics into authentic investigations of schooling or other familiar and significant community issues has proven to be an engaging and effective approach for children (Moses, 2001). The approach outlined here works somewhat in the opposite direction, starting with a context and dedicating class time to help students build a relationship to it through the interdisciplinary context. This is based on the rationale that for young adults who are beginning to plan their futures, issues and values may become just as motivational as everyday experiences. Interdisciplinary content linked to classroom experiences that develop subjective, values-based viewpoints on aspects of the context can make mathematics and the social problems that mathematics addresses, real in the minds of young adults.

It is not my intention to suggest that socially contextualized lessons replace other mathematics approaches. Instead, offering some lessons with non-mathematical learning goals may help students develop a sense of the importance of mathematics. There are several strong resources for teachers who wish to build lessons with dynamic contexts including case studies in environmentalism (Fusaro and Kenschaft, 2003; Schaufele, Zumoff, Sims and Sims, 2002), social justice perspectives (Gutstein and Peterson, 2005), and service learning projects (Hadlock, 2005). In developing a dynamic context for a classroom presentation of an application, some non-mathematical learning goals can be factual, like the basis of UN assessments of standard of living. Others can be open-ended. All learning goals should be included in student assessment. Classroom discussions of a dynamic context need not be restricted to mathematical actions, since the objective is to build subjective involvement with an application, but still, these discussions will often provide opportunities to critique, modify or improve a mathematical model. As Boaler points out, approaches to context that develop both personal meaning and a deeper understanding of mathematics are likely to improve transfer of learning to new situations (Boaler, 1993, p. 15).

To advocate that mathematics course content and assessment can include non-mathematical learning goals is admittedly difficult given traditional disciplinary boundaries in mathematics. A survey of secondary school teachers found that mathematics teachers consider their subject more bound by curricular sequence, content and common examinations than teachers in science, English and social studies (Grossman and Stodolsky, 1995). Many mathematics teachers are accustomed to knowing the answer to all the questions a student may ask, but once a discussion is opened to non-mathematical content and perspectives, this position of intellectual authority can be subverted. A student could contribute knowledge, data or perspectives that are outside of the teacher's experience.

While teaching these contextualized applications, I find myself considering at least two moment-by-moment concerns. The first is how to guide the contextual discussion and ensure that it is sufficiently significant (*cf.* Matthews, 2003). Prior planning of non-mathematical learning objectives helps teachers feel as if their responsibility for non-mathematical content is bounded and it may help them guide and moderate contextual discussions more effectively. Furthermore, these short, focused, guided discussions on contextual, non-mathematical issues can be class time well spent. In a time allocation study of an epidemiology unit in an introductory algebra class, only 29 minutes out of 48 hours of instructional time were dedicated to non-mathematical discussions; 96% of the students in the class considered the topic to be relevant to their learning of mathematics, and 60% cited it as the most memorable topic in the class (Staats, 2005, p. 194). Second is the desire to ensure that the students are learning mathematics. New questioning strategies can embed skill development and generate mathematically critical perspectives (Staats, 2005, 2006), questions that can be used even when the teacher no longer feels like the absolute authority on the knowledge that is articulated in the classroom:

> "If your assumptions are true, how would you use them to make a prediction or a comparison?"
>
> "What data does she need to evaluate her position?"
>
> "Could you express that figure in a different form?"

Can dynamic contexts tied to a student's major academic interest strengthen their commitment to that field and to using mathematics in that field? Do defined non-mathematical learning goals (either factual or open-ended ones) make mathematics teachers more comfortable introducing socially or culturally relevant material into their classrooms? Perhaps most importantly, what pedagogical skills are required to teach mathematics while retaining a sense of value for the context?

Conclusion

Dynamic contexts can involve learners as whole people, rather than as "schizo-mathematics-learners" (Valero, 2002, p. 5) who think only of mathematics when they are learning mathematics. Valero proposes a

> "re-humanised" view of students as whole learners, who have multiple motives for learning, and who live in a broad context which influences their intentions to participate in school mathematics practices [...] Stu-

> dents are important in mathematics education because they are full human beings to whom the experience of engaging in mathematical learning has to make a broad meaning. (Valero, 2002, p. 10).

Integrating significant, non-mathematical learning goals into mathematics applications is one means of inviting students to engage with mathematics as "full human beings." Instead of leaving the imaginative power of context implicit, as in traditional applications, guided discussion of issues allows students to construct parts of their identity within a context that also includes mathematics. The connection to mathematics that discussion fosters might not be a utility-based relationship, as in "I want to learn these computations because their outcomes are important to me," but rather an incipient, associative relationship, as in, "I care about this situation in general, and mathematics has something to say about it, too." Dynamic contexts can develop this lateral relationship between mathematics and student experience and values and help secondary and early undergraduate students decide whether their academic passions can involve mathematics.

The non-mathematical context of an application is an imagined world that may be a far more powerful dimension than we realize, because we rarely allow the context to be a site of active learning in mathematics classroom conversations. The values and viewpoints that students express may never be fully realized mathematically in the purview of the particular lesson but may nonetheless affect them. We will make sufficient use of the power of context once we are willing to engage the non-mathematical issues that motivate, engage and excite non-mathematicians.

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