Reflections on Forty Years of Work on Mathematics Teaching

CALEB GATTEGNO

Caleb Gattegno died in Paris in July 1988. He was in his seventy-seventh year.

The paper reprinted here first appeared in the Proceedings of the 1980 meeting of the Canadian Mathematics Education Study Group. It records what Gattegno himself wished to say about his work in mathematics education to that point.

He wrote a great deal throughout his working life, always reaching out to the public domain where he hoped his work would "be examined and its merits, if any, acknowledged". Yet, what impact has it had on the mathematics education community as a whole? At times Gattegno confessed his inability to make clear to others what he had found. He may have confused people by mixing together the roles of teacher, inventor, guru, and businessman. But now that he has left us it seems particularly sad that, for whatever reasons, he was denied those processes of informed scrutiny, testing, and evaluation, that he desired so much and that scientists in almost every other field are able to count on. -Ed.

About 1927-28, some of my cousins, and their schoolmates, about my age, asked me to help them in their mathematics studies. Although I was not yet 17 I took my job seriously and so I became a mathematics teacher at the secondary school level. In 1932 I founded the Mathematics Seminar in Alexandria where a small number of people trying to educate themselves lectured to each other on selected chapters of pure and applied mathematics. That became my teaching at the graduate level, from which at least four people came out as professional mathematicians.

With my children, around 1940, when they were very young, I learned that teaching mathematics at the elementary levels was a very different activity. My failures discouraged me and for years I did not attempt to touch that population.

From 1946 to 1953 I concentrated my study of teaching and learning on the secondary and postsecondary levels. Not only did I make some progress—as can be seen from my numerous publications of those years—but I also shook off Piaget's grip on my mind. I learned to gather evidence directly from people I worked with rather than go via the work of others. My studies of consciousness started just before World War II and I was able to apply to the study of the structures of the mind the mathematical structures that had been found by mathematicians, mainly following Dedekind. The Bourbaki books were not accessible outside France at that time, but soon after 1945 I could get acquainted with them. I found that their work affected my understanding of the work of the mind but also that their ideas needed alteration if they were to serve me properly. It was then that I reached my understanding of algebra as operations upon operations. Among the Bourbaki, Dieudonné agreed wholeheartedly, in 1950, that it also described his ways of working.

By 1947 I had already propagated the notion that mathematics was the work of *awareness on the dynamics of relationships per se*. Soon after, algebra appeared to me as being the dynamics itself and present in *all* mathematical activity. Thus although mathematicians classify themselves as analysts, topologists, algebraists, probabilists, number theorists, logicians, and a few other subdivisions, I reached a different conclusion since I came to the meaning of mathematics by studying mathematisation. Some of my writings from the years around 1950 clearly hint at that.

I worked in isolation for all was new in the field and I did not seem to know how to attract the attention of my friends, those good minds I was in contact with in Great Britain and the continent of Europe. They listened with interest but left me to do the needed spade-work.

In 1950 I formed the International Commission for the Study and Improvement of the Teaching of Mathematics. It counted among its original members Choquet and Dieudonné from France, Wijnsink from Holland, and myself from England. Soon after we co-opted Fletcher from England, Servais and Papy from Belgium, Puig Adam from Spain, Castelnuovo and Campedelli from Italy, and a number of others from a number of countries. The work of the Commission became known partly because of the luminaries among those named above and partly because of the two monographs I edited in French in 1954 and 1956 and which have been translated into Italian, Spanish and German (but not into English). The Commission still exists and meets once a year somewhere on earth.

Two offshoots of the Commission were ATAM (now ATM) in Britain, founded in 1952, and the Belgian Association of Math Teachers which also started work that year. Both involved me for a few years. In 1959 I was elected the first President of ATAM and soon after its first Honorary Member. In 1960 I resigned from the International Commission after ten years as its secretary.

I had encountered teaching with films in 1949 when I

met Nicolet. The following year in Debden (northeast of London), Nicolet and Jacquemard came together to work for one week on teaching mathematics to adolescents. At once I liked Cantegrel's films (with Jacquemard's scenario and Motard's animation) but I soon understood that Nicolet had the better formula. I offered to make Nicolet's work known. Unfortunately I did not manage it and when he died in 1966 although he was appreciated by a number of people it was by too few for him to feel he had not laboured in vain

On a lecture tour in Belgium in April 1953 I was told I had to become acquainted with the work of another obscure pioneer in the teaching of mathematis: George Cuisenaire. It proved to be a turning point in my career as a student of learning and teaching and some of you know that in a few years Cuisenaire became a household name among teachers of arithmetic all over the world. I founded eleven Cuisenaire companies, wrote twenty texts for students and teachers, met literally hundreds of thousands of teachers from 1953 to 1962, in 44 countries, on all continents.

But all this social activity was overshadowed by discoveries I made thanks to my finding that at last I could work with young children and let them teach me what to do to reach them, to understand their ways of working and from there to develop the *subordination of teaching to learning* in a number of fields. These words came to me in 1960 when I was interviewed for the *Christchurch Daily* (New Zealand), and was asked to characterize my work in a few simple words. I said, "I teach what I know, and that is, to subordinate teaching to learning". At that time I had done some work in the fields of science through my "open books" on the study of energy (1957-58), in the field of reading for natives (Amharic, Hindi, Spanish in 1958, and English and French in 1959), in the field of foreign languages (1954), and, of course, in mathematics.

Being busy in so many areas it was to be expected that my contributions to the teaching of mathematics would become more the expansion of what I had already done than of new findings. But life decided otherwise. In 1967 I was forced to recast the foundations of my teaching of elementary mathematics when I noticed that not all children using the Cuisenaire rods managed to master the basic numerical facts of addition. It then occurred to me that I had to replace the rods and some manipulations of them—which I had advocated for 14 years—by some awarenesses of what children can do with their fingers.

For a while I worked out the details of that approach which I published as the first chapters of "The Common Sense of Teaching Mathematics" and in articles you may have read. Today I know for sure that that appraoach has (1) made an explicit use of what children do so early and so well, i.e., learn to speak (which I had not fully integrated when I had asked them to work with rods), and (2) made an explicit use of two underlying mental powers which we can call *the perceptions of equivalence and of complementarity*

Coming closer to the workings of the mind in this way has been very fruitful, and still is. A large number of experiments with very young children and with so-called "learning disabled", have confirmed that we may have arrived at a point where we shall be able to propose ways of working that can meet the true dynamics of the mind and gather unsuspected crops far beyond what was achieved more than a quarter of a century ago when the rods were first used for teaching

We do not need to spend time, as we did in the past, to make children retain arithmetical facts. Hand calculators have made all the efforts in the direction of teaching computation a waste of time. Pressing buttons and reading numerals and signs for operations is all we need as preparation to do quite complicated arithmetical computations. They take no time and, if we are careful, they are even generally correct.

To belong to our age requires us, as educators, to know how to link the potential of the mind and the potential of our electronic technologies: computers and TV color monitors. Since I am aware that our present stage of human evolution can be defined in terms of energy, particularly the utilizations of extremely minute amounts of energy, I have called the present "the era of the nothings." If we consider the problem of utilising the public fad for electronic devices together with a serious subordination of these powers in order to achieve maximized learning by as many people as possible, we see our task defined.

Three lines of approach on this have presented themselves to me

> First, I have used the technology of computer graphics to make mathematical films.
>
> Second, I have submitted proposals to show how computers can be used in work with beginners so that in a very short time considerable and valuable mathematics experiences can be acquired and become functional
>
> Third, I am engaging in writing articles for various publications (including our own *Newsletter*) so that this way of relating to the powers of the computer will be examined and its merits, if any, acknowledged

Working on computers and on TV makes me more aware every day that we must all become experts (among other things) in working on "nothings". Our collective future depends on our capacity to produce a generation of minds who can synthesize what is relevant today and offer our children effortless learnings in all fields.

Nowadays a generation gap can happen every few days, not only between people of different ages but between adults, and an accumulation of lack of understanding is not in the interest of the inhabitants of the earth. When I concentrate on the future and on the era of the "nothings", I am working at being truly of my time