Dominant Epistemologies in Mathematics Education

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Since the devotional writings of the Pythagoreans, mathematics has been associated with the mystical. In a romantic search for 'origins', humans attributed divine foundations and transcendental powers to mathematics. Its exegetical force spawned a myth, a 'theory of everything', or 'anything', which appeared at the same time as the 'unifying glue' of an enlightened, rational human mind. This myth led to a pseudo-unification of the human mind and its different expressions, leading the way for the epistemic hegemony of mathematics.

In this article, I address the relationship between scientism, as expressed in Descartes' writings, and the dominant epistemologies in the field of mathematics education today. More specifically, I ask: 'What is the legacy of Cartesianism in the field of mathematics education and in the mathematics classroom?' Is the subsumption of differing voices within the purview of an 'epistemological elite' perceived as integration? Do mathematical sciences colonise other school subjects under the sceptre of interdisciplinarity? Is mathematics a global construct without any cultural or historical contingencies, or is there a place for ethnomathematics, or better the anthropology of mathematics? Is a development on the way from 'within' mathematics and not 'with' mathematics and 'with' the world and the life we live in it?

The dominance of Cartesianism in the field of mathematics education can be balanced, I will suggest, by means of an incorporation of humanist philosophies. The demands of humanism should not be those of the 'old humanist lobby' that Paul Ernest (1991) describes, which portrays a timeless clash between mind and matter, culture and civilisation, or even high and low culture. On the contrary, the demands of humanism will be seen as reflected in the philosophy of Vico, who presented an alternative view of mathematics as a human institution discursively formed in the course of history. This view considers knowledge in general as created and acquired in the process of the genesis and development of human societies.

The origins of mathematics and its exegetical powers

Mathematics operates through processes of decontextualisation and recontextualisation. The detachment and study of insights from activities, phenomena or science is followed by their subsequent re-attachment to these or other environments. Theoretical mathematics, which emerges most notably in Ancient Greece after the sixth century B.C., developed out of this belief in a readily-mathematisable world, as well as from an interest in non-utilitarian applications of mathematical knowledge. Although the sovereignty of the Ancient Greeks' mathematical thought has rightly been challenged by non-European accounts of the history of mathematics and by recent studies of the anthropology of numbers, they were nonetheless the first to add the question of 'why?' to that of 'what?'. The teleological dimension of doing mathematics, in the Aristotelian sense, was to be found in the simple activity of knowing and in the acquisition of scientific truth. A knowledge of mathematics in Plato's Academy (387BC–529AD) became the gateway to philosophy and its attempt to discover eternal and universal principles that would bring some order to the 'chaos' of existence. Equally important at that time was the creation of 'axiomatic worlds': in other words, mathematical theories elaborated around a set of axioms.

As a result, mathematics has been described for millennia as a theoretical subject divorced from its human origins. Freudenthal (1973) has remarked that:

> the number sequence is the foundation-stone of mathematics, historically, genetically, and systematically. [...] Without the number sequence there is no mathematics. (pp. 171-2)

Mathematical formalism is built on the sequence 1, 2, 3, ..., what mathematicians call the 'natural' numbers. This 'natural', Platonic dimension attributed infinity, priority and endlessness to mathematics, giving it almost divine foundations. Just as the existence of God does not require the existence of the world, so the existence of mathematics does not depend on its having earthly origins. Thus began a mathematisation of the discipline of mathematics itself, which would lead to the divorce of mathematics from the spheres of the senses, feelings, intuition, and non-exact practices. Mathematicians were discovering or creating mathematics, following syntactico-grammatical canons and protocols that dictated the acceptable and expected use of symbols. As a result, mathematical practice is full of imperatives: 'multiply x by y', 'integrate the function f', but lacking indexical elements of natural language, such as: 'I', 'you', 'here', 'elsewhere', that tie messages to the socio-cultural environments in which they were uttered (Rotman, 1993; Walkerdine, 1988).

While arithmetic and geometry were supposedly handed over to mathematicians by means of divine mediation (Derrida, 1989), the transcendental becoming of mathematics was strongly supported by its powerful force of exegesis. Mathematics promoted the illusion that we could illuminate the structure and function of the universe and domesticate the disorder and insecurity of human existence by translating the real into the arithmetical. This process resulted in the development of a mathematical and, by extension, scientific hegemony, established in the Enlightenment but maybe 'felt' even more today. Our lives revolve around and depend on the products and development of the mathematical sciences, a phenomenon that is accompanied by the gradual weakening of the voice of the humanities.

'Scientism/humanism' and the dominant epistemologies in mathematics education

Descriptions of the world serve as an understanding, as a 'theory' (literally, a 'way of seeing'), that drives action towards certain goals. Nonetheless, it is easy to overlook the fact that 'practical' policy decisions in mathematics education are dependent on theory. These decisions often are underwritten by tacit assumptions about the nature and the limits of mathematical knowledge, as well as by a philosophy for mathematical pedagogy and education more generally.

Over time, certain ideas we hold about mathematics have become naturalised. Conflicts among alternative epistemologies in the field of mathematics education seldom move outside the idioms of Platonism or Formalism. Thus, certain phenomena more easily gain 'referential consensus' within the academic community, which represents the first step towards acknowledgment and acceptance of those phenomena among a broader audience (Schneider, 1993). A number of mathematicians, for instance, support the view that:

> mathematics must be what has been labelled "decontextualised" [...] Purity in mathematics is just the limitation on the scope of the detachment and reattachment to mathematics itself. (Thomas, 1996, pp. 11, 16)

Ethnomathematics, or the anthropology of geometry and numbers, are considered then, as something *other*, a 'watering down' of 'real', decontextualised mathematics or in the best of epithets as 'useful' preliminaries, as proto-mathematics (see Chevallard, 1990; Thomas, 1996). As far as epistemological paradigms are concerned, research in education in general is still governed by an 'epistemological elite' who call for distance, objectivity and abstraction (Turkle and Papert, 1992). Ethnographic and autobiographical evidence that brings the cultural context of the research and the researcher's personal voice to the forefront may be criticised as 'soft', non-scientific and therefore inappropriate for publishing, financing or discussion.

The vicious circle of the mathematisation of the world and the mathematisation of the discipline of mathematics has resulted in the seductive equation of the philosophy of life with a mathematical philosophy. The continuance of the epistemological leadership of Platonism and Formalism in the field of mathematics education, I suggest, is partly due to the long-standing disagreement between scientism and humanism, which underlies assumptions about the origin and nature of mathematics. This argument is nothing new in the history of Western philosophy. Since the time of Plato, mind and theory were separated from matter and practice respectively (these dichotomies were not evident in the thought of pre-Socratic philosophers and the Sophists).

The ideas of René Descartes (1596-1650) played the foremost part in forming the scientism that marked the thinking of Enlightenment. Descartes, having developed an antipathy for the Humanities, became convinced that the true use of Mathematics should not be limited to the service of the mechanical arts but could be extended even to the very structure of the Universe. He added characteristically, that the mathematical sciences:

> should contain the primary rudiments of human reason, and [their] province ought to extend to the eliciting of true results in every subject. (Descartes, in Eaton, 1927, p. 51)

He rejected all merely probable knowledge and trusted what was completely known and incapable of being doubted. He welcomed numbers and figures as illustrations of self-evidence and certainty, suggesting that the first germs of useful modes of thought, scattered in the Divine element of the human mind, were aspects of the simplest sciences, those of arithmetic and geometry. Not unjustly then, his mathematically-deduced Method constituted the celebration of a rationalised mind, marking the era of a Universal Science resting on a Mathematics that touched the sphere of the Divine.

In contrast to Descartes' quest stand the ideas of the Italian philosopher, jurist and classicist Giambattista Vico (1668-1714). He is widely known as the founder of a philosophy of history and the formulator of the historical method. His 'reputation' as a thinker has been largely limited to his work on history, as his theories were open-ended and never constituted a system. Vico regarded historical process as one whereby humans construct systems of language, custom, law, government, etc. These systems do not unfold over time as a step-by-step realisation of a pre-existing, transcendental plan. It is 'men' themselves, in the process of genesis and development of societies, that create both form and matter together in the name of human institutions (Vico, 1744/1968).

Vico boldly attacked the basis of Descartes' Method: the assumption of a clear and distinct idea. To make the assumption that one's ideas are clear and distinct proves one's belief in them but does not entail their truth. His intentions were partly orientated towards the formulation of a principle of the necessary limits of human knowledge, a way of distinguishing what can be known from what cannot. He argued that things that can be known are only those that 'man', the knower himself or herself, has created (Vico, 1709/1990). Therefore, he held the view that true knowledge is constructed not essential. Nevertheless, to know things is not just to create them yourself, since it is of great importance *how* this knowledge has been created in the first place. It is this maxim that has received a warm welcome by constructivists in the field of mathematics education. Moreover, Vico emphasised the importance of the ability to express things known through the medium of language (*eloquencia*), a topic that directly relates to Skemp's (1979) discussion of logical understanding.

According to Vico, then, humans can have true knowledge of arithmetic and geometry because these sciences are based on hypotheses that men and women themselves have created. As Collingwood (1994) explains:

> Any piece of mathematical thinking begins with a *fiat*: let ABC be a triangle and let AB = AC. It is because by this act of will the mathematician makes the triangle, because it is his *factum*, that he can have true knowledge of it. (p. 64)

We are thus faced with a paradoxical situation in which humans can appropriate true knowledge of mathematics, as they themselves have created it, while at the same time true knowledge of nature is promoted through reference to mathematical ways of understanding. According to Vico's doctrine, though, humans cannot appropriate true knowledge of nature, even if the attributes of physical phenomena seem unambiguous and readily accessible through mathematics. Therefore, either the nature of mathematics is divine and the assumption of true knowledge of it is refuted, or the Cartesian demand for a mathematisation of human existence and the world is a myth. The oxymoronic characteristics of the situation are resolved by acknowledging that in the midst of their over-sophistication humans tend to seek for islands of certainty in the ocean of uncertainty; that is the nature of human life. Judging distant and unknown things by the standards of what is familiar and at hand is a customary practice (Vico, 1744/1968).

Vico also objected to Descartes' methodological monism. He did not oppose mathematical deduction but its uncontrolled and frivolous application to non-scientific subjects (Vico, 1709/1990). On the other hand, he did not hesitate to criticise the methodological anarchism in the humanities. Vico appears as a peace-maker between humanism and scientism, fighting on the side of the humanities without abolishing the use of mathematics and the sciences. He stressed the need to perceive man as a unity of intellect, fantasy, passion and emotion, influenced by the historical and social dimension of human life. In protesting a rationalistic model of education and the narrow intellectual mastery of a single subject, he declared that "the whole is really the flower of wisdom" (*ibid.*, p. 77) and that students should be taught the entire spectrum of the curriculum (Vico, 1728/1993). He supported the view that such activities of thought as poetry, rhetoric and history constitute a source of meaning and not of foolishness, as Descartes' rationalistic model of education suggested. Therefore, we may say that Vico replaced Descartes' dictum 'I think therefore I am' with the Socratic teaching of 'know thyself' through the study of the whole curriculum of thought.

Descartes' quest for the unification of all sciences and of the human mind itself under the *aegis* of mathematical methods represents the ultimate expression of the renunciation of interdisciplinarity in the name of an epistemological monism. I suggest that Cartesianism and the leading epistemologies in the field of mathematics education are based on convergent assumptions, metaphorically if not explicitly. The truthfulness of mathematical discourse and the mathematisation of the world are based on the assumption that mathematics as a subject exists somehow 'in nature', ingrained in the very substance of physical phenomena. Mathematics is treated as a body of true and pre-existing knowledge that can be reached through pious devotion to the 'Only Way for Rightly Conducting the Reason and Seeking for Truth in the Sciences' (Descartes, in Eaton, 1927). If the role of the mathematician as a discoverer is replaced with that of a creator - where the requirements of formalism come into play - the mathematician is placed in the position of the veracious Proto-Mathematician who, again, builds a rigorous, but not necessarily meaningful, divine-like construct (Cobb, 1989). Platonism, on the one hand, disassociated mathematics from its profane origins. Formalism, on the other, alienated mathematicians from their cultural attachment to the socio-historical process, since they claimed exclusive custody of mathematical culture and its becoming.

The legacy of scientism in the field of mathematics education

Reflecting on the extent to which our belief about 'what' and 'how' to teach in the mathematics classroom resides in our philosophy of mathematics and our conceptions of the limits of mathematical and human knowledge is the first step in embracing a critical stance in the field. It is a fact, as Dossey (1992) suggests, that the lack of consensus about a philosophy of mathematics segments and complicates the practice and teaching of mathematics. A large part of the construction of mathematical meaning is achieved through a signification process, the social creation of meanings through the use of formal signs. The removal of reference to first-order experiences is characteristic of mathematics. Aspects of value, emotionality, desire and the subjectivity of 'I' or 'you' are also suppressed along the way.

The power of such processes is constituted by the fact:

> that they can form descriptions of, can be read back onto, *anything*. It is in this sense that they are apparently without context. (Walkerdine, 1988, p. 187)

Remaining in this sphere of decontextualised abstractions, with no reference to the concrete, the learner of mathematics becomes a 'non-person', a simulacrum of him- or herself. A similar process is described by Turkle and Papert (1992) in the field of computer programming, where pre-packaged programs (black boxes) are used by programmers when planning a large job. When this black-boxing technique is promoted as the only appropriate approach for programming, it prevents students from knowing how programs work or applying alternative logics to assignments.

The 'non-person', the learner of mathematics, is offered the indolent forgetfulness of the legendary *lotos* - the fantasy of playing the role of God or the divine mathematician, the illusion inscribed in Descartes' *cogito*. Thus, all experiences that surround mathematical learning are considered exterior to the process of signification, overlooking the fact that emotions and mathematical meaning are both galvanised in the perceptual process leading to evaluation and judgement. The emphatic belief in the extended referentiality of decontextualised signification processes may be the 'stigma' of mathematics. Birthmark qualities attributed to mathematics, such as the power of 'tidying' things up, are combined with the consistent renunciation of the individuality of the subject in learning.

When transferred into the classroom, this Cartesian insistence on a fixed outside world masterable through the right Method leads to a spectator theory of knowledge (Benjamin and Echeverria, 1992). Students accept the teacher's narrations passively, since they represent the beliefs of a fixed external world to which students want to ascribe. False beliefs are to be replaced by true ones through a series of educational experiences, which are not determined, nor chosen, and as a consequence are not 'lived' by the students. Increasingly often, technology and other 'alternative' educational approaches are used in the mathematics classroom in order to enhance motivation for learning. Bruner (1965) cautions, however, that these steps may lead in the long run to passivity and spectatorship, with students "waiting for some curtain to go up" (p. 72) to arouse them. Facing the danger of being accused of being neo-Luddites, we can suggest that the computerisation of education and the world is a stage in the Cartesian process of the mathematisation of the world (Davis and Hersh, 1986).

Belief in the clear and distinct idea ultimately supports the hegemony of mathematics over other school subjects and sets the standards for their relationships. School curricula are compartmentalised as the relationship of mathematics to other school subjects is seen through its application in the teaching of those subjects and not *vice versa*. Attempts to reconcile mathematics with other fields of knowledge within the school curricula are destined to fail if they colonise other subjects.

An example of such a failure comes from the Greek educational system. In an attempt to 'enrich' the content of the mathematics syllabus, 'historical notes' are attached to the end of each chapter of the high school mathematics textbooks. These historical references mostly describe the 'discoveries' of great mathematicians and quaint episodes from their lives. Extracted from historical context, they serve as examples of the subordination of history to the teaching needs of mathematics while their educational potential is left unexploited. In the end, this attitude is nourished by and further nourishes the view that mathematics has an extra-discursive, unrestricted nature, independent of ephemeral and parochial knowledge of the values and feelings of different cultures and epochs.

The myth of an all-knowing human intellectual faculty is complemented by a regiment of 'math-atheists', who fail to adopt or refuse to subscribe to mathematics in the way it is served to them. There is a tendency to identify the math-atheists with the mathematically illiterate, those students who score low in mathematics tests. Their 'failure' enhances the position of the 'math-theists' who, by means of their good achievement, share the powers of the archetypal mathematician, enjoying the social recognition of the mathematically skilled. This, though, is only part of the story.

Students' behaviour towards mathematics may result from the development of a resentment mechanism. The powers of mathematics, social or intellectual, are rejected, or, in a desperate attempt 'to belong', students may adopt a superficial approach to learning, what Adorno (1990) described as 'semi-education'. This is the burning desire for communion with the commonly accepted and respected, a desire that is distant, though, from true knowledge and grasp of the subject. Semi-education, then, is not half-education or the absence of education but enmity towards education. It results in a mind conquered by the fetishistic nature of the commodity, demanding the 'purchase' and 'possession' of fragmented 'chunks' of knowledge. Unquestionably, those who are 'by necessity' math-atheists or math-theists are fundamentally different from those who are what they are 'by choice'.

In our society of spectacle, the domination of economy over social life has caused 'possessing' to become 'seeming', just as in past times 'having' took the place of 'being'. Fragments of scientific or intellectualised knowledge are subjected to commercialisation. They become exchangeable commodities and emblems of social prestige. Mathematics, for instance, has served as the social sieve for selecting the 'clever' from the less 'clever', as knowing Latin characterised the cultivated person in other times. Indicative is the role that mathematics played, and still plays, in screening and categorising people through IQ tests.

For parents, good grades in mathematics are synonymous with a passport to success and security (though, in fact, people who have excellent verbal skills actually make it to the top of the pyramid in Greece and certainly many other countries). Students enjoy admiration and recognition when they reveal a 'natural talent' in mathematics, even if his/her development in literary and artistic subjects does not follow that of mathematics. Within a professionally-oriented educational system, the student is kindly 'forced' to specialise his/her efforts towards specific ends. In Greece, this choice is silently made from the first years in high school - in other words, almost six years before the examinations that determine entrance to different state universities. As a result, school curricula are fragmented, even if they do not appear so, secluding students from vital fields of the human intellect. The reward of the students' efforts is expressed in entry to university departments: mathematics and the hard sciences, the medical sciences, literary subjects and humanities, and the social sciences. These departments form rigid and exclusive intellectual clusters between which students are forbidden to move. Since certification is a one-way street, students who fail the entrance exams attend courses in private universities, which are characterised by an almost complete absence of offerings in the humanities.

Specialisation serves the needs of a 'banking' model of education and more broadly of a civilised society with a set course towards future development. 'Deposits' from various intellectual faculties enjoy different 'weightings', with mathematics and the hard sciences being far in front. Having tasted the powers of control over life and natural phenomena, the learner of specialised scientific knowledge is confident of having achieved something important, something that will last. Dictated by the demands of specialisation itself, however, scientific knowledge is always being superseded and in a world enchanted by scientific technology, we must always face the possibility of playing the role of a renunciatory hero (Weber, 1919/1958). As a result, as Dewey (1900/1990) explains:

> The unity of education is dissipated, and the studies become centrifugal; so much of this study to secure this end, so much to secure another, until the whole becomes a sheer compromise and patchwork between contending aims and disparate studies. (p. 72)

Subjugated knowledge in the field of mathematics education

As Koyré (1991) comments, the 'man' in the past could not calculate accurately since s/he did not have the means (calculating or measuring devices) and the language for it. This may lead us to assume that mathematical culture is the construct of the mathematicians and scientists who control the language and the means for doing mathematics. Who is to decide, though, if a Brazilian carpenter is a 'math-charlatan' or whether 'hands-on' mathematics is not 'real' mathematics, since it relates to concrete things? And what is 'real' mathematics after all? An example from Greek reality may elucidate the hidden ideologies and the power games characterising apparently neutral definitions of mathematical knowledge. In the late 1980s, when mathematics education started to gain recognition within the Hellenic academy as an autonomous field, an undeclared 'war' began between professors of mathematics departments and those from the humanities. The former insisted that they were entitled to be the masters of this new *vogue*, since they possessed the knowledge of the science of mathematics. The latter claimed on the other hand that they could best enrich the teaching of mathematics with their knowledge of philosophy, pedagogy and psychology. Are we to assume then, that mathematics is of central importance in teaching the subject to children and pedagogy is something on the side, a helpful but not essential supplement? Is it just another game of knowledge and power, a struggle over defining what acting or thinking mathematically should mean?

The mobilisation of arithmetic and geometry initially served the economic and technological needs of the time. Today if one were to ask a random sample of people what mathematics is all about and why it is so important, the most common responses would point to the uses of arithmetic in everyday transactions. Many would also probably respond more generically that mathematics 'sharpens your mind' (a response reflecting the social and intellectual over-valuation of mathematics). Even if we teachers detect in this line of thinking the unfortunate reduction of mathematics to arithmetic, these impressions testify to the fact that for many people mathematics in everyday life is understood in terms of its 'explicit' (Thomas, 1996) applications to calculating, estimating or measuring practices of which 'real' mathematicians may not be very proud.

Great 'discoveries' in the history of mathematics might better be understood in historical terms as the crystallisation of broader currents of thinking and problematisation in human societies. For instance, what determined the development of probability theory in the seventeenth century? For thousands of years, humans have been fascinated by the 'goddess of chance' and have generated various means to prognosticate outcomes, to foresee the future. An 'informal' discourse of 'predicting' or 'determining the odds' can still be found in a plethora of ideas concerning chance, randomness, luck, the possible, the likely, hazard, accident, mere opinion, signs, evidence and even authority (see Hacking, 1975).

Academic probability as we know it today is concurrently differentiated and intrinsically related to this informal discourse and everyday uses of the concept of 'the probable'. A gambler does not necessarily calculate the *a priori* probability for winning before betting his money in blackjack. He also expects to have 'more chances' to win after constantly losing for hours as he is not aware of the law of large numbers. Similarly, a farmer looking at the sky and sensing the direction and moisture of the wind might assume 'it is probably going to rain tomorrow'. The everyday uses of the concept remind us of the subjective character ingrained in the concept of 'the probable'. Its mathematical-scientific use, though, strives to decrease the aleatory nature of probability and promote a rationalised theorising of reality, a stochasticised world.

Ethnomathematics, a reaction against the Western hegemony of mathematical thinking, is a relatively recent area of research and epistemological paradigm in the field of mathematics education. Influenced by the methods and findings of anthropology, mathematicians have moved towards the study of mathematical traditions of non-Western civilisations and of social practices that involve or reflect mathematical meaning. These attempts, as Nunes (1992) suggests, have developed along two trajectories. The first, based on a notion of cultural relativism, stresses the differences among mathematics cultures (all equal, all different), while the second assumes the basic unity of the human mind and studies other cultures in order to determine invariant principles underlying mathematical activities. On the level of practice, the introduction of ethnomathematics into the classroom has been used by teachers to render mathematics more appealing to children. This strategy, however, might ultimately reconfirm that 'real' mathematics is something *other* than mathematics that possesses a cultural context. That 'culture' is to be found exclusively in 'colourful' non-Western societies and lower-class *millieux* would seem to enforce the broader hegemony that is being questioned in the first place.

The development of ethnomathematics clearly reflects the need on the part of some scholars to move away from an oppressive Cartesian theorisation of mathematics. The attribution of labels such as 'ethno-' and 'proto-' to this type of mathematics, however, would seem simultaneously to 'elevate' certain humble behaviour to the status of mathematics while 'reducing' it to a preliminary or lower status. In other words, theorists of ethnomathematics have fallen into the same pitfalls of anthropological liberalism. In identifying spheres of difference within one's own culture or abroad, orientalism, exoticism and a certain bourgeois voyeurism tends to be reproduced. By studying the differences in order to deduce commonalities across cultures, one still reads other cultures through the texts of the dominant western discourse of mathematics.

The rules and structures that regulate mathematical discourse have set the standards required for referential competence and development of the discipline. In the course of history, 'real' mathematics became:

> [an] ideal signification [...] a truthful discourse (Foucault, 1984, pp. 77, 79)

one taking the place of truth, directing attention away from certain practices. Metaphors, and the exploration of metaphorical thinking, for instance, was left to the poets, the linguists and the literary critics. A twelve-year-old boy calling a wooden cylinder 'a circular prism' because he cannot remember its 'proper name' would possibly give teachers an opportunity for a good laugh during recess. The study of metaphors in the mathematics classroom (Pimm, 1987), though, has opened new visions of reflecting on the construction of mathematical knowledge and possibilities for genuinely bridging the humanism/scientism divide.

In invoking the union of mathematics and poetry, of course, there is always the danger of recreating hierarchies. Doing mathematics is sometimes paralleled with writing poetry, because of the imaginative, aesthetic rewards that both actions carry for the subject. The 'wonder' of mathematics, as an all-explaining recipe, has also occasionally inspired the poets. In these uses, poetry is subordinated to mathematics and its educational potential in the mathematics classroom is left unexploited (the need to rely on poetry in order to express the sublime nature of mathematics' exegetical powers, however, undermines in a way the alleged exegetical powers of mathematics itself). Poems have also been used as word problems to render mathematical problem solving more interesting and enjoyable to children. Although indirectly, this approach still abides in the previous philosophy.

In current research, the present author has asked children to write their own poetry. The only restriction imposed for each poem is the particular theme that the poem should address, e.g. a certain number or a geometrical object, the four operations in arithmetic, feelings about mathematics as a school subject. Poems are read aloud after having been written and then 'analysed' so as to unravel and discuss mathematical understandings or feelings. Treating poetry as a medium of communicating mathematical meaning and exploring its educational potential in the mathematics classroom represents a willingness to subject mathematics to the demands of metaphorical thinking and the faculty of language in general.

A number of recent commentators have pointed out a traditional resistance to thinking of mathematics as bound up with other linguistic practices (Rotman, 1993; Walkerdine, 1988). In a collaborative study (Potari and Triadafillidis, 1997), the author analysed children's verbal and written descriptions which were solicited during the haptic exploration of three-dimensional objects and the construction of physical models based on these written descriptions. The explored objects, which were taken out of their everyday context, were familiar to the children (e.g. a glass bottle of soda). Since they were not allowed to use the name of the object in their descriptions, children had to reflect on the characteristics of the object. Our attention was concentrated on the potential value that different modes of communicating mathematical understanding might carry for children. Language was not considered to be distinct from reality, nor a mere tool for transmitting mathematical ideas through preordained Cartesian channels of communication, but as a constitutive element of the mathematics classroom. The interplay of various forms of textual representation, including written description, verbal exchange, physical models and even haptic exploration strategies, does not so much point to the integration of different representational media in the mathematics classroom, as to the crossing of boundaries between mathematics and other disciplines.

Vico (1744/1968) once remarked that the human mind is naturally impelled to take delight in uniformity. Mathematics seems to have fulfilled this cognitive yearning, claiming Oedipus' riddle-solving ingenuity. Since it became customary to assume a secret origin behind any beginning, in mathematics this origin was found in the sphere of the divine. By means of inaugural mediations, mathematics then was handed over to mathematicians by a divine proto-mathematician. A transcendental proto-history marks not only the desired origins of mathematics but also its development and the totality of its becoming (Derrida, 1989). Thereafter, all manifest discourse runs parallel to an 'already-said' (by the divine mathematician) yet concurrently 'never-said' (as no one has ever heard it), incorporeal discourse (Foucault, 1993, p. 25). Mathematicians and scientists are challenged to discover and follow this incorporeal discourse.

The production and reproduction of a truthful discourse follows the creation of binaries: Humanities-Sciences, Mind-Action, Ethnomathematics-Real mathematics, Emotion-Rationality, Culture-Civilisation, Man-Woman. The adherence to an all-explaining theory translates binary objects according to the understanding of the theory s/he professes. Thus, binary objects, appearing as antithetical concepts, acquire a pseudo-descriptive nature and serve the needs of pre-existing forms of continuity. As a result, adherents to antithetical theories not only evaluate the world differently, but experience it differently as well.

Vico, with his teaching, brought the process of constructing all kinds of knowledge under one roof, that of the historical process. Placing an emphasis on *how* true knowledge is constructed calls for reflecting on our teaching approaches, but more than that on reconsidering the set boundaries among various faculties and expressions of the human mind. According to this line of thought, the only difference between proto-mathematics, ethnomathematics, humanistic mathematics and 'real' mathematics as an academic discipline lies in their degree of power in determining what can be said and regarded as factual or true of mathematics and what can be rejected as subjective and idiosyncratic.

The breaking of any binary pairing is not the real issue. The myth of a mathematised world may very well be a mere presupposition and not a true statement. The project of 'Taking the God out of mathematics and putting the body back in' (Rotman, 1993) is not an attempt to de-mystify mathematics driven by feelings of resentment or revenge. It is simply, following Vico, the decision to become prospectors of this world instead of claiming to be pioneers of a new universe.

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