THE ONTO-SEMIOTIC APPROACH: IMPLICATIONS FOR THE PRESCRIPTIVE CHARACTER OF DIDACTICS

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In this article, we reply to a question posed by Gascón & Nicolás (2017) about the prescriptive nature of didactics of mathematics research:

> To what extent, how, under which conditions can (or must), didactics set value judgments and normative prescriptions in order to provide criteria about how to organize and manage study processes? (p. 26)

In their work, Gascón and Nicolás analyse the answers given to this question by several authors, applying the specific perspective of the Anthropological Theory of the Didactic (ATD, Chevallard, 1992). To contribute to this discussion, we react here to the challenge posed by Gascón and Nicolás, by using the principles and theoretical tools developed in the Onto-semiotic Approach to Mathematical Knowledge and Instruction (OSA) (Godino, Batanero & Font, 2007).

We begin our reflection by explaining the conception of didactics assumed in OSA, and next, we synthesise its epistemological, ontological, semiotic and educational-instructional assumptions. We hope this synthesis will facilitate the comparison and articulation with other theories in future works, as Gascón and Nicolás propose. Finally, we describe the notion of didactical suitability which allows us to give an affirmative answer to the question of the prescriptive character of didactics research.

Didactics as science and as technology

Epistemological reflection on the nature of didactics is essential to guide didactic research since this reflection conditions the formulation of its central questions. Two authors concerned with this reflection are Steiner (1985) and Brousseau (1989). Given the extreme complexity of mathematics education problems, Steiner (1985) highlights two extreme positions:

> - Some authors affirm that mathematics education cannot become a scientific field and, therefore, the teaching of mathematics is essentially an art.
> - Other authors, believing that the existence of didactics as a science is possible, reduce its complexity by selecting a partial aspect of the same. For example, the analysis of the content to be taught, the construction of the curriculum, the improvement of teaching methods, the development of student's skills, or the classroom interaction, to which they attribute a special role, and, consequently they support different definitions and visions of didactics. (p. 11)

In a similar way, Brousseau, in *La tour de Babel* (1989), describes an initial meaning of didactics of mathematics as the art of teaching, that is, the set of tools and procedures intended to make known, in our case, mathematics. Brousseau, in addition, distinguishes two scientific conceptions: the *applied multidisciplinary conception* and the *autonomous conception*. As a link between these two scientific visions, he also distinguishes a technical conception in which didactics is the set of teaching techniques intended to improve mathematics teaching; that is the invention, description, study, production and control of new teaching tools, such as curriculum, objectives, evaluation instruments, materials, textbooks, software, *etc*. Brousseau's multidisciplinary conception coincides with the second tendency pointed out by Steiner. In it didactics appears as a label to designate the teaching resources needed to carry out the technical and professional training of teachers. Didactics as a scientific discipline would be the research field focussed on teaching and is supported by other classical scientific disciplines, such as psychology, semiotics, sociology, linguistics, epistemology, logic, neurophysiology, pedagogy, paediatrics, or psychoanalysis.

Lesh and Sriraman (2010, p.124) also reflect on the nature of mathematics education as a field of inquiry, by posing the following questions: Should mathematics educators see themselves as applied educational psychologists, applied cognitive psychologists, or social applied scientists? Are they similar to scientists in the field of physics, or other pure sciences? Or rather to engineers or other design-oriented technicians whose research is based on multiple practical and disciplinary perspectives—and whose work is guided by the need to solve real problems, as well as by the need to elaborate relevant theories? Lesh and Sriraman consider mathematics education in this last sense, that is, as a science oriented to the design of processes and resources to improve mathematics teaching and learning.

The OSA framework attributes both a scientific and technological character to the knowledge produced by didactic research. On the one hand, it addresses theoretical problems related to the ontological, epistemological and semiotic nature of mathematical knowledge, as far as such problems are related to the teaching and learning processes (the scientific, descriptive, explanatory or predictive component). On the other hand, didactics should intervene in these processes to improve them as much as possible (the technological-prescriptive component). While description, explanation and prediction are the main goals of scientific activity, prescription

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and assessment are the main goals of technological enterprise; however, technological action also includes elements of applied research when solving specific problems.

Didactic problems, principles and research methods

The articulation, hybridisation and modular construction of theories from an anthropological and onto-semiotic approach is at the base of OSA (Font, Godino & Gallardo, 2013) [1]. This approach assumes the pertinence and potential utility of advancing towards a theoretical system that provides an articulated solution to the epistemological, ontological, semiotic-cognitive and educational problems involved in the teaching and learning of mathematics. Didactics is viewed as the techno-scientific discipline responsible for giving a coherent answer to didactic problems, because the principles, methods and results of other disciplines can be contradictory when focusing on the aforementioned problems in isolation. OSA assumes, therefore, an expanded conception of didactics, as related to mathematical teaching and learning processes, knowledge and practices (genesis, development, diffusion, transposition and use), as well as to the optimisation of these processes in educational contexts.

In the following sections, we enunciate the specific questions (Q), basic principles (P) and methods (M) proposed by OSA to address the solution for each of the epistemological, ontological, semiotic-cognitive, educational-instructional, and ecological problems. In this way, we intend to facilitate the comparison and articulation of OSA with other theoretical frameworks, as well as to clarify its position on the prescriptive nature of research results in didactics. We use the interpretation proposed by Radford (2008), of a theory as an instrument to produce understandings and as a type of action based on:

- A set, Q, of paradigmatic research questions.
- A system, P, of basic principles, which contains implicit visions and explicit statements outlining the boundaries of the discourse universe and the research perspective adopted.
- A methodology, M, that includes data collection techniques and their interpretation supported by P. (p. 320)

Epistemological problem

We enunciate the epistemological problem of mathematics in the following terms:

> *QE_1: How does mathematics emerge and develop?*

To answer this question, we assume an anthropological [2] and pragmatist (à la Peirce) view of mathematics; therefore, people's problem solving activity is the key element in the construction of mathematical knowledge. This epistemological vision becomes operative in OSA with the notion of *mathematical practice*, conceived with an institutional and personal relativity which leads to the following epistemological principle:

> PE_1: Mathematics is a human activity focused on the resolution of a certain type of problem-situations. This activity involves the implementation of systems of practices through which the responses to the problem-situations are given.

A mathematical practice is "any action or expression (verbal, graphical, *etc.*) performed by someone to solve mathematical problems, communicate the solution obtained to others, validate or generalise these solutions to other contexts and problems" (Godino & Batanero, 1998, p. 182). A mathematical problem is any situation requiring mathematical activity (characterized by the objects and processes involved (symbolising, stating, arguing, generalizing *etc.*; numbers, shapes, functions *etc.*)

A second principle asserts the institutional and personal nature of practices:

> PE_2: These practices can be idiosyncratic of a person or shared within an institution. There are no institutions without members, or individuals detached from the various institutions to which they are inevitably linked (family, school *etc.*).

The people involved in the same class of problem-situations constitute an institution; their mutual commitment to the same problem entails the realisation of *social practices* that tend to have particular characteristics and are generally conditioned by the instruments available in the institution, its rules and modes of operation. The distinction between personal and institutional practices highlights the dialectical relations between them; on the one hand, people share modes of action within the institutions in which they take part; on the other, institutions are open to their members' initiative and creativity.

The third principle refers to the possible breakdown of practices which allows their detailed analysis:

> PE_3: Problem solving is carried out through the articulation of sequences of operative, discursive and normative practices. Such sequences of practices take place over time and are described as processes. The mega-process of problem solving can be broken down into more basic processes (representation, algorithmising, argumentation *etc.*).

These principles are linked to the epistemological question on the genesis of knowledge and give rise to the following method of inquiry:

> ME_1: The institutional genesis of mathematical knowledge is investigated in OSA by 1) the identification and categorization of problem-situations (phenomenology); 2) the description of the operative, discursive and normative practices used in their resolution.

Given that the systems of problem solving practices are relative to the contexts of use and the institutional frameworks in which they are addressed, in OSA the knowledge is viewed as relative to the institutions and contexts.

Ontological problem

The ontological problem of mathematics is described in the following terms:

> *QO_1: What is a mathematical object? What types of objects intervene in mathematical activity?*

In addition to being a human activity, mathematics is also a logically organised system of objects. In OSA, a mathematical object is any material or immaterial entity that intervenes in mathematical practices, by supporting and regulating its realisation; that is, there is no mathematical activity without objects or objects without activity. As the practices can be seen from the social (institutional, shared) or personal (individual, idiosyncratic) perspectives, the objects can also be conceived from the institutional-personal duality which originates the following principle:

> PO_1: In institutional or personal mathematical practices, various kinds of objects intervene that fulfil different roles: instrumental / representational, regulatory (setting rules on practices), explanatory, justifying.

To analyze the mathematical practices carried out in the resolution of problems, OSA proposes the notion of *onto-semiotic configuration* and identifies different types of objects and processes involved in these practices; this tool helps in carrying out a microscopic level of analysis. By explicitly recognising such objects and processes, we anticipate the potential and effective learning conflicts, can evaluate students' mathematical competences and identify those objects (problems, languages, definitions, propositions, procedures, arguments) that should be remembered and institutionalised at appropriate moments in the instructional process:

> PO_2: The onto-semiotic configuration articulates the notions of practice, object and process, as well as the dualities from which these notions can be considered in the institutional and personal analysis of mathematical activity.

Semiotic-cognitive problem

We describe the semiotic-cognitive problem of mathematical knowledge in the following way:

> *QSC_1: What is knowing a mathematical object? What does the object O mean for a subject given a time and circumstance?*

Knowledge is interpreted in OSA as the set of relationships that the subject (person or institution) establishes between objects and practices; such relationships are modelled by the notion of *semiotic function.* A semiotic function is understood as the correspondence between an antecedent object (expression, significant) and another consequent (content, meaning) established by a subject (person or institution), according to a criterion or rule of correspondence.

Any entity engaged in a semiotic interpretation process, or *language game,* is an object and can play the role of signifier (meaning) in semiotic functions; since the systems of operative and discursive practices are objects themselves, they can be components of semiotic functions. From these assumptions the following principle results:

> PSC_1: The knowledge of an object O by a subject X (either individual or institutional) is the set of semiotic functions that X can establish in which O intervenes as expression or content. Each semiotic function implies an act of semiosis by an interpreting agent and constitutes knowledge; it depends on the circumstances fixed in the act of interpretation.

Speaking of knowledge is speaking of the content of one (or many) semiotic functions, and therefore there are a variety of types of knowledge in correspondence with the diversity of semiotic functions that can be established between the various types of practices and objects. From this the next principle develops:

> PSC_2: The 'meaning of the object' (institutional or personal) [3] is interpreted as the correspondence between an object and the system of practices in which the object occurs.

The systems of practices put at stake in solving problem-situations depend on the individuals and communities of practices (institutions). Hence, meanings, and therefore knowledge, are relative. However, it is possible to reconstruct a global or holistic meaning of an object through the systematic exploration of the object, contexts of its use and the systems of practices required in solving problems. This holistic meaning is an epistemological and cognitive reference model for the object's partial meaning or meanings and constitutes an onto-semiotic-cognitive methodological tool:

> MSC_1: A method to delimit the different meanings of mathematical objects, and, therefore, to reconstruct the epistemological and cognitive reference models is analysis of the practice systems (personal and institutional) and of the onto-semiotic configurations involved in them.

The notion of institutional meaning of mathematical objects entails the recognition of a plurality of meanings, so that in each specific situation, it will be necessary to reconstruct its specific meanings. Nevertheless, these particular meanings should evolve towards a progressively richer model. This pragmatist positioning of OSA leads to a conception of understanding as competence, and not as a mental process; consequently, a subject understands a certain mathematical object when s/he uses it competently in different practices.

Educational-instructional problem

The educational-instructional component of didactics studies the teaching and learning processes going on in any 'didactic institution', in order to optimise these processes. The primary questions come next:

> *QEI_1: What is teaching? What is learning? How they relate?*

The instructional model assumed in OSA is based on the principles of cultural/discursive psychology (Lerman, 2001) which attributes a key role to the zone of proximal development. Contrary to constructivist models, the student's autonomy in the learning process is the product of that process and not a prerequisite for it. However, given the central role that the anthropological perspective of knowledge gives to problem-situations, the activity involved in its resolution, the search, selection and adaptation of good problem-situations and the students' involvement in their resolution is another principle of meaningful mathematical instruction. From this assumption, a mixed-type instructional model is derived in which the construction and transmission of knowledge are articulated in a dialectical way (Godino, Batanero, Cañadas & Contreras, 2016). The main features of this model are summarised in these principles:

> PEI_1: The aim of learning is the students' appropriation of institutional meanings and objects, to address the solution of certain problem-situations and help them develop as persons.

> PEI_2: The identification of students' personal meanings is an essential component of the educational process, since it conditions the intended institutional meanings.

The institutional meanings finally implemented in an instructional process may be different from the intended and reference meaning, due to the restrictions imposed by the students' cognitive capabilities, the available resources and the social and educational contexts. However, the intended and implemented meanings of institutional objects in a given educational context should constitute a representative sample of the global reference meaning.

The notion of *didactic configuration* constitutes the main methodological tool for the micro level analysis of instructional processes (Godino, Batanero & Font, 2007). It is defined as any segment of didactical activity (teaching and learning) included between the beginning and end of solving a problem-situation and includes, therefore, the students' and teacher's actions, as well as the tools planned or used to tackle the task. Consequently, the analysis of the didactic configurations that constitute a *didactic trajectory* is another methodological tool:

> MEI_1: To investigate instructional processes, analysis of the didactic configuration (network of teaching and learning actions and tools used to study a problem-situation) and didactic trajectory (sequence of didactic configurations) is carried out.

Three components constitute any didactic configuration:

a) An epistemic configuration (system of institutional practices, mathematical objects and processes required in solving the task);

b) An instructional configuration (system of teaching functions, students and instructional tools, as well as their interactions); and

c) A cognitive-affective configuration (system of practices, mathematical objects and personal processes describing learning and the affective components involved).

The need to identify each of these components gives rise to the following problem:

> *QEI_2: What types of interactions between people, knowledge and resources are required in the instructional processes to optimize learning?*

The relationships between teaching and learning are not linear, but cyclic and complex. In moments of inquiry, the student interacts with the epistemic configuration without teacher intervention (or with minor influence). This interaction conditions teaching interventions and needs to be planned in the instructional configuration, perhaps not entirely in its content, but in its nature, necessity and usefulness. The cognitive trajectory accounts for the students' learning progression produced as a result of the teacher's actions, when interacting with other students, using determined instructional resources and with regard to a specific teaching content. This principle follows:

> PEI_3: The optimization of instructional processes requires taking into account macro and micro level factors and will be in many cases local; hence with fixed certain conditions it is necessary to investigate the circumstances and resources necessary for this optimization.

Ecological problem

The ecological problem analyses the diversity of factors and norms that condition the teaching and learning processes, as synthesised here:

> *QEC_1: What factors condition and support the development of instructional processes and what norms regulate them?*

Didactics of mathematics has been interested in factors and norms that regulate teaching and learning processes, particularly authors who ground their work on symbolic interactionism (*e.g.,* Blumer, 1982). This research takes into account the generally implicit norms, habits and conventions that regulate the functioning of mathematics classrooms and condition to a greater or lesser extent the knowledge built by the students. In addition, other different factors affect the didactic system, such as, for example, the students' age or their capabilities, the preparation of teachers or the resources used in teaching. Both factors and norms relate to the six facets (epistemic, cognitive, interactional, mediational, affective and ecological) that are considered in the analysis of instructional processes:

> PEC_1: Identification of the factors and norms conditioning the instruction processes is essential to:

- Assess the relevance of teachers' and students' interventions, when taking into account the set of factors and norms that condition teaching and learning.
- Suggest changes in the types of norms that help improving the work and control of the instructional processes, with a view to an evolution of personal meanings towards the intended institutional meanings.
- Identify ways to act on some factors that influence the system: for example, manners of improving the student's attitudes or ways of helping students with greater or lesser capacity.

Problem of instruction optimisation: criteria of didactical suitability

The ultimate goal of didactic research is improving students' learning, which requires a series of criteria to ensure such optimisation, as reflected in the following question:

> *QIO_1. What kind of actions and resources should be implemented in the instructional processes to optimize students' mathematical learning?*

Finding well founded answers to this question is the central aim of didactic research and can be addressed directly (research based-design) or indirectly (descriptive-interpretative research), by applying different theoretical approaches. Didactic knowledge obtained as the result of such investigations is of a different nature: clarifying the nature of mathematical objects, both from the institutional and personal point of view, didactic principles on how to improve teaching and learning, and instructional resources. On the basis of these results of didactic research, OSA adopts the following postulates:

> PIO_1: Instructional principles and resources are not considered as general rules or laws, inferred in a positivist manner, but as suitability criteria or preferential action upon which a certain consensus has been generated in the mathematics education community.

> PIO_2: These criteria have to be applied locally, so they must be adapted and interpreted by the teacher and refer to each of the facets involved in the mathematical instruction processes: epistemic, ecological, cognitive, affective, interactional and mediational.

The notion of *didactical suitability* has been included in the theoretical system that configures OSA, as a systemic criterion for optimising mathematical instruction. It is defined as the degree to which the process (or a part of the same) has certain characteristics considered as optimal or adequate for succeeding in the adaptation between the students' personal meanings (*learning*) and the institutional meanings (*teaching*), taking into account the circumstances and available resources (*environment*). This general criterion of suitability has been specified for each of the facets considered in taking into account some OSA assumptions and tools, and a system of empirical indicators of suitability for the various components has been proposed (Godino, Batanero & Font, 2007; Breda, Pino-Fan & Font, 2017). For example, for the epistemic facet, the following partial criteria can be formulated:

> PIO_3: The meanings of the institutional objects intended in each educational context should be a representative sample of the global reference meaning of the object and take into account the restrictions of the context and subjects involved.

The achievement of high didactic suitability requires a balance between the different partial criteria related to the different facets, by taking into account the context in which it takes place. Suppose, for example, that there is a consensus that a criterion is that the students have learnt (cognitive criterion), another is that they have been taught relevant mathematics (problem solving, modelling, *etc.*) (epistemic criterion) and another is that students have been interested and engaged (affective criteria). It is relatively easy to achieve one of these three criteria in isolation; however, it is more difficult and valuable to achieve a certain balance between the three. Metaphorically, a ship sinks if the load is not balanced.

Suitability is related to changing temporal and contextual circumstances, so a reflective attitude on the part of the teacher and other agents who share responsibility for the educational project is required. This implies the assumption of an axiological rationality in mathematics education that allows analysis, criticism, justification of the choice of tools and goals, justification of change and ultimately answering the generic question of what aspects can be modified to achieve progressive improvement of mathematical instruction processes.

The notion of suitability is inspired by Peirce's consensual theory of truth and its subsequent developments and adaptations made by authors such as Apel (1991) and Habermas (1997). In this theory, 'true' is, in principle, a statement for a user when s/he believes that any other rational subject would be willing to assign the same predicate to the statement. The truth is neither thought of in relation to a world separated from ideas, nor as 'conformity' with transcendent ideas, but as what could be defended against a set of interlocutors and accepted by them.

Final reflections

OSA assumes a broad conception of didactics as a discipline, in considering that it must address descriptive, explanatory, and predictive goals that are characteristics of scientific knowledge; and prescriptive and evaluative aims, specific of the technological knowledge. Consequently, didactics should provide results that allow effective action on a part of reality: the teaching and learning of mathematics in the different contexts in which it takes place. In addition, it must take into account the four types of problem areas, epistemological, ontological, semiotic-cognitive, educational-instructional, described in this article and their interactions.

Regarding the prescriptive dimension, didactics should offer provisional principles (standards or suitability criteria in OSA) agreed by the community interested in mathematics education. These principles and norms are useful at two times: 1) a priori, the suitability criteria guide the way in which an instruction process should be developed, 2) a posteriori, the criteria serve to assess the teaching and learning process effectively implemented and identify possible aspects to be improved in the redesign. To generate these principles, researchers in mathematics education should discuss and collaborate with all other sectors interested in improving mathematics teaching (teachers, parents, administration, *etc.*). This will lead to a consensus that generates principles to guide and value the instruction processes, in order to achieve a suitable teaching of mathematics. It is recognised, however, that the identification of suitability criteria, both general and specific, requires a research agenda that is open to discussion and development in the mathematics education community.

On the other hand, didactics concerns the study of human beings interacting in very diverse contexts, that is complex, dynamic, open, heterogeneous systems engaged in multiple and diverse interactions. These systems have chaotic aspects, where small changes can lead to large deviations. Since small changes take place at the micro level, they should be studied as possible explanatory factors of the changes observed at the macro level. Consequently, didactics should contemplate the use of analysis units at the micro level (a task, or a teacher-student interaction of a specific nature), and at the macro level (a field of problems, a long-term didactic trajectory, the sociocultural context).

The principles explicitly stated as characteristics of a theory should be interpreted as tools, while the methods are ways of applying these tools to solve specific problems and questions in the field. Moreover, when interpreting theories as tools, we can propose the following questions to reduce the proliferation of theories:

a) Assimilation of theories: Can we discard theory T when the tools in theory Q suffice and are more efficient in didactical analysis and design?

b) Accommodation of theories: Which changes should be introduced in the theory Q tools system to accommodate theory T tools?

c) Compatibility of theories: Are some principles of theories T and Q incompatible? Which principles?

Given the systemic, modular and inclusive character of OSA, the principles and method stated in this article are not unique and definitive. This is a first step open to new development and precisions as a consequence of dialogue with other theories which is already going on.

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Notes

[1] Additional publications in which OSA is described are available from the web site, http://enfoqueontosemiotico.ugr.es

[2] Bloor (1983) describes Wittgenstein's perspective on mathematics as an anthropological phenomenon (chapter V) within his social vision of knowledge. This philosophical view on mathematics is consistent with that assumed by the ATD (Anthropological Theory of Didactic, Chevallard, 1992). The relationships between OSA and ATD were discussed in D'Amore & Godino (2007).

[3] This is a creative interpretation of Peirce's pragmatic maxim.

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Editor's Note

The two articles preceding this note, and the article following it, respond to an article in issue **37**(3). In that article Josep Gascón and Pedro Nicolás summarise and analyse comments from several mathematics educators addressing the question of how and if 'didactics' can make normative prescriptions. They close with a set of questions related to teaching and different approaches to educational theory. They also invite "all members of the community of didactics of mathematics to give their own answers to these or related questions, in the form of communications sent to *For the Learning of Mathematics*."

There have been several submissions that take up this invitation, some of which appeared in issue **38**(3). It is pleasing to me that we are receiving such responses. As David Wheeler remarked in the first issue of FLM, "a journal is a social enterprise, a product of people talking to people." Readers are encouraged to react to whatever they read, as a contribution to the ongoing conversation that is FLM. Responses need not be long. As David also noted, "There is space here for swift spontaneous personal writing. Insights worth telling don't always have to trail bibliographies after them."