

provide some rich applications of congruence of triangles and other concepts taught in high school geometry, and opportunities for valuable practice in constructing counterexamples” (p. 403).

Perucca and Torti (2023) prove congruence theorems for convex n -gons for arbitrary n . For example, that for $n > 6$ it suffices to know the lengths of enough diagonals to show congruence. The assumption of convexity is important. Knowing the exact position of $n - 1$ vertices and all sides and all but two angles is not sufficient to determine an n -gon up to congruence because the two sides at the missing vertex could point outwards or inwards. There remain open research questions in this area.

It is an instructive mathematical exercise to prove congruence theorems for circles (it suffices to know the radius or the diameter or the circumference or the area). One could also consider, for example, rhombi, trapezoids, T-shaped figures and so on. The class can, in principle, take any agreed-upon figure and analyze which conditions guarantee that the shape and size cannot be changed.

Congruence theorems can also be investigated for more complicated shapes in higher dimensions. One could investigate similarity theorems in the same spirit. Let us rejoice in the fact that there are plenty of open questions that are accessible research exercises.

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Teachers’ subjectification as a dialogue with standardized assessment

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Di Martino and Baccaglioni-Frank (2017) point out the need for a critical approach to standardized assessment that fosters “*Informational Potential*, seen as the information that can actually be obtained by interpreting and analysing students’ performance results on standardized tests (this includes understanding the limitations of such potential) [and] *Developmental Potential*, seen as the educational

opportunities offered by a critical approach to standardized tests and by a re-elaboration of the informational potential.” (p. 39).

In the present communication, we focus on an *operative* use of standardized assessment for pre-service teachers’ professional development based on data collected by the Italian National Evaluation Institute for the School System (INVALSI) and framed in mathematics education. In particular, we create a link between the Italian National Evaluation System and pre-service teacher professional development programs in order to *improve* mathematics teaching school practices.

Our model is made up of the following elements:

- an interactive database containing structured information regarding Italian standardized assessment that contains 1793 test items, spanning 10 years of the Italian National Evaluation System activity;
- a theoretical framework for pre-service teacher training that conceives teachers’ learning as a process of *subjectification* (Radford, 2020).

The first element encompasses Informational Potential and the second one Developmental Potential.

Theoretical background

Our understanding of teachers’ professional development ascribes a key role to the socio-cultural environment in defining individual agency and the production of mathematics teachers as new subjectivities, *i.e.*, a process of *subjectification*.

The Theory of Objectification (TO) outlines a dialectical co-production between individuals and their cultural and historical reality. Radford conceives the individual as:

an entity in flux, in perpetual becoming—an entity who, through practical activity (like play) is continuously inscribing herself in the social world and, in doing so, she is continuously produced and co-producing herself within the limits and possibilities of her culture. (p. 43)

The cultural-historical context defines who we are, how we conceive of ourselves and how others conceive of us, the “fabric of our subjectivity” (p. 46), the space, the constraints, and the forms of our agentic maneuvering. It is also true that we hold a reflexive relationship with our cultural context, which means that “we *react agentially* to such a context” (p. 4).

The cultural-historical context unfolds as a *symbolic superstructure*—termed by TO as ‘Semiotic Systems of Cultural Signification’: a network of distinctive traits that makes up the fabric of a culture and its society.

We have selected and adapted from Radford the systems that are specific to the production of prospective teachers’ subjectivities and have informed our professional development activities:

- Epistemology and ontology of mathematics
- Systems of truths
- Forms of rationality
- Accepted mathematical teaching practices, problems, and situations

- Conception of teachers and students
- Socio-mathematical norms
- Conceptions about education
- Language and languages
- Assessment

Differently from TO which frames teacher-student social interactions, our model looks at researcher-pre-service teacher modes of activity. Our objective is to realize pre-service teacher professional development based on social interaction, activity and collaboration, accomplished as *joint labour* (Radford, 2016).

Within the domain of standardized assessment, the notion of pre-service teachers' professional development, outlined in this section, unfolds the *developmental potential* proposed by Di Martino and Baccaglioni-Frank. In fact, standardized assessment mediates the dialectical interplay between the individuals' agency and their cultural-historical environment.

To complete the description of the sociocultural scenario where the pre-service teacher professional development takes place, we need to introduce the artifactual dimension that intertwines pre-service teachers' and the researcher's agentic reflection against the backdrop of Semiotic Systems of Cultural Signification. In the next section, we describe a new tool, based on standardized data, that along with other ideal and material artifacts, mediates the activities in which the pre-service teachers and the researchers are engaged.

Gestinv database

The Gestinv database [1] collects in a structured way a broad range of information regarding the mathematics tests that the Italian National Evaluation System issues annually and that involves all Italian students of grades 2, 5, 8, 10, 13. Gestinv rests on a theoretical framework [2] that informs both the construction and the selection of the items and its complex structure. Its structure triggers the use of mathematical content knowledge, mathematics education theoretical tools, and ideas about teaching and learning that together amount to a conceptual and cultural tool (see, *e.g.*, Ferretti, Santi & Bolondi, 2022). This tool allows users to carry out research concerning the national tests, available from 2008, according to mathematical content and their relationship with the National Guidelines, the results of the tests (percentage of correct, incorrect, invalid and missing answers and, for multiple choice tasks, the percentage of each option), school level, content keywords and statistical features (characteristic curves, distractor plots, *etc.*).

The impact of the Gestinv database has been assessed both quantitatively and qualitatively, through standard indicators such as the number of registered users (more than 16,643), the number of accesses (on average, 200 every day), the time spent on the website and other parameters. These data, along with its structured information, recommend Gestinv as a tool to be implemented in the design of teacher professional development models (Ferretti, Gambini, & Santi, 2020). Within the domain of standardized assessment, Gestinv unfolds the Informational Potential proposed by Di Martino and Baccaglioni-Frank. Furthermore, Gestinv allows our model to accomplish an educational

implementation of standardized assessment, via a fruitful interplay between pre-service teacher training and classroom teaching practices. Indeed, the widespread use of this tool in everyday school practice allows for a bridging of pre-service teacher training and classroom teaching practice that acknowledges the needs of the Italian educational system highlighted by standardized assessment.

Our model

In regard to the individuals involved in the positioning process, our model has been devised specifically for pre-service teachers, who are still in tertiary education and within one or two years will start their profession as teachers. An aware and critical positioning is crucial for their professional development. It brings new possibilities of thinking and action in the social dimension of education and mathematics teaching practices. From a theoretical point of view, the shift of subjectification from students to teachers is special. Pre-service teachers are trying to inscribe themselves in a new dimension as teachers to realize their project of life.

The researcher designs and fosters *joint labour* in order to position the pre-service teachers against the backdrop of the educational environment with its epistemological, political and ethical issues. Such a role can be assumed only by an expert in mathematics education, who is able to navigate through the complexities of mathematics and its teaching-learning processes.

With respect to the design of the activity carried out as a joint labour, the lesson plan we propose during the teaching training is inspired by Activity Theory (Radford, 2015). The activity plan follows the following schema:

Introduction of the activity. The researcher discusses with the pre-service teachers the mathematics education constructs, also looking at learning difficulties, that will be useful for the activity. Moreover, the researcher presents some of the functions of Gestinv that the teachers will use. Last but not least the researcher addresses the mathematical content selected for the activity from a conceptual and epistemological point of view. The mathematical content is discussed with the teachers according to some of the principles of Bartolini Bussi's (1996) *mathematical discussion*.

Analysis of an example. The researcher discusses a didactical macro-phenomenon emerging from standardized evaluation with the whole group of teachers. Such macro-phenomena are recognized by the pre-service teachers via cross-research related to more than one parameter, when they use Gestinv.

Group activity. The group divides into sub-groups of four or five members. The researcher in charge of the course assigns a task covering specific mathematical content, a learning difficulty, or a cognitive process, that are developed according to the Learning Objectives and the Goals for the competences development of the National Guidelines. The members of the group strongly interact with Gestinv. The group activity aims at the construction of a multimedia product, an artifact,

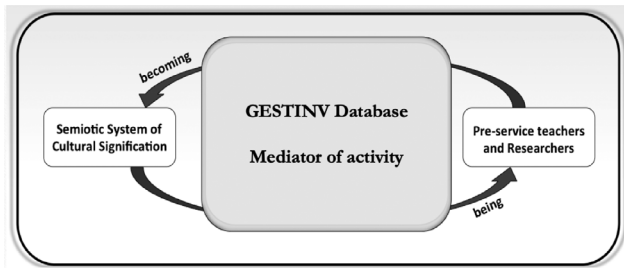


Figure 1. The model for pre-service teachers' professional development.

the design of an activity for students *etc.*, which should highlight teachers' reflections, convictions and beliefs.

General discussion. The sub-groups present their materials in a written or oral form to the whole group. Each presentation is discussed with a researcher, in order to highlight beliefs and convictions, and tackle doubts, difficulties and unclear content regarding both the mathematical content and the analysis of the related learning processes. Another setting for this final phase requires each sub-group to prepare a written presentation that is exchanged, so that each sub-group presents orally the material of another sub-group to the whole group. The final discussion based on the oral presentations is carried out with the same characteristics described above and aims at investigating pre-service teachers' subjectification.

We have implemented the model in a pre-service teacher professional development program that involved 140 students of the Faculty of Education of two Italian universities in 2019-2020. The use of standardized assessment data via Gestinv allows pre-service teachers' process of subjectification and their ensuing positioning in the social and cultural world of education as prospective mathematics teachers.

Concluding remarks

Our aim is to acknowledge the need for operative interventions informed by standardized assessment and a critical approach that addresses its informative and developmental potentials. We focussed our attention on pre-service teachers' professional development as one of the possible routes to realize educational interventions informed by standardized assessment that take advantage of its informational and developmental potentials.

We therefore devised a model for pre-service teachers' professional development that rests on two interacting pillars:

- a theoretical framework cast in Radford's Theory of Objectification that conceives teacher training as a subjectification process;
- Gestinv database, which realizes the entanglement of Di Martino and Baccaglini-Frank's *informational and developmental potentials* of standardized assessment.

Our model, based on processes of subjectification, is particularly appropriate to frame pre-service teachers'

professional development, since prospective teachers can be seen as individuals who are transitioning from the identity of a student to the identity of a teacher. Thus, the notion of subjectification as a dynamic positioning in a particular social space (the school system) appears effective to frame the development of teachers' professionalism. The model brings a novel understanding of teacher professional development based on the production of new subjectivities according to the dialectical materialist philosophical stance advocated by Radford.

As a final remark, we can say that the need for a critical and operational approach towards standardized assessment quantitative data has opened a possible new trajectory in pre-service teachers' professional development that ascribes a prominent position to cultural-institutional features intertwined with the production of new subjectivities.

Notes

[1] <http://www.gestinv.it>

[2] Quadro di riferimento delle prove di INVALSI di matematica. <https://tinyurl.com/FLM43-2-1>

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False friends in mathematics education: Implications for meaning making in multilingual classrooms

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In this short communication, I focus on the linguistic barriers that make teaching and learning mathematics challenging for students who are neither native speakers nor are communicatively competent in the medium of instruction. When learning a new language, learners may encounter