Under the Banyan Tree

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I start with three thought experiments.

To begin with, imagine yourself teaching mathematics to a group of children. They can be young or old as you choose, but whatever their age they are eager students who attend your lessons regularly and seem to enjoy them. Your well-designed classroom is warm and airy; it is equipped with all sorts of teaching aids including enough computers. The children have their own textbooks and calculators. They work well whether on their own, in small groups or as a whole class. You follow a very sensible pre-determined syllabus but have plenty of opportunities to explore lots of other topics. You enjoy every minute of your work and are rewarded by the obvious pleasure your students have in mastering mathematics as well as by their success in public examinations. Think about the style and content of your lessons ....

The second experiment requires some change in the variables. Imagine yourself teaching mathematics to a group of children. They may be young or old as you choose, but note that there are likely to be a few who are one or two years older than the average age. You can place yourself in the sort of classroom you are familiar with, but note that the walls are bare, a few window panes are cracked and some light bulbs are missing. The children sit in desks that are arranged in rows and columns, but note that the room is very crowded—there may well be more than sixty children in the room. Some of them may not have been with you the day before, some of them will not be there the day after. There are no books, but the students have exercise books and pens and you have a chalkboard and some chalk. You follow an extremely outdated pre-determined syllabus which you never adequately complete. Neither you nor your students enjoy the time you spend together which on your part is chiefly devoted to trying to maintain order. Almost all your students will leave school without any qualifications as soon as they can. Think about the style and content of your lessons.

The final experiment is taken to another extreme. Imagine yourself teaching mathematics to a group of children. They may be mainly young or old as you choose, but note that there are likely to be some considerable variations in age including some adults. This class always meets outdoors, perhaps under the shade of a tree. The students sit on the ground, crowded together. Sometimes there may be as many as a hundred of them, sometimes just a few. Some of them may not have been with you the day before, some of them will not be the day after. They have no books or paper or writing materials. You have no pre-determined syllabus and can teach what you like, but there are possibly some sort of public qualifications that can be obtained. Your students are very keen to learn, and you and they enjoy the mastery they sometimes achieve. Think about the style and content of your lessons.

Some questions

Do these experiments reflect any sort of reality? Well, I suppose some readers will have occasionally come across something like the classroom suggested in the first experiment. Some may even teach, or have taught, in something like it themselves. Certainly many teachers would want to be able to work in such a classroom: it is often the one implicitly pre-supposed by writers in journals such as this. But it is also certain that this is an ideal which only a relatively privileged few could aspire to, let alone achieve. Something like the working conditions described in the second experiment may well be the global norm, whether in the inner-city schools of post-industrial societies or in the schools of those societies that have only relatively recently developed a system of universal education.

Whereas the atmospheres invoked in the first two experiments were based on observations of actual classrooms I have been in, that of the third is based on myth and personal fantasy. There are various associations which idiosyncratically cluster for me around the image of a group of people sitting under a banyan tree. This is the Indian figtree (*ficus religiosa* or *indica*) whose branches drop shoots into the ground to take root and support the parent branch. In many villages in India it provides an impressive focal point, a place where people meet and settle things. I recall stories I read as a child in which magical things happened under such trees. I do not know whether children or adults ever now gather under a banyan tree to be taught, but I link the idea with an image from some film about a teacher whose disciples gather round him under a tree and I have often summoned this image when I read authors like Paulo Freire. I know that the actual conditions in which people nowadays meet to decide things or to learn from each other or from a teacher may be quite different. Neverthless, I continue to symbolise the enterprise as taking place under a tree, and I often silently invoke this image when I myself am teaching.

My main purpose in proposing the three thought experiments is to raise some questions about the mathematics curriculum. For example, are there any invariants to be found among the diverse topics that one might find being taught in the three situations? Are there some topics that seem so quintessential to the mathematical enterprise that it would be surprising if they were not included in the work done in any mathematics classroom ? Are there any topics that you would expect to be taught—even perhaps over and over again—whatever the age and stage of the students? Putting it slightly differently, what topics would

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you choose to teach as an absolute priority, whatever else happened? Passing to the limit: is there just one thing you would go to the stake for?

A township classroom

It seems to me that such questions need to be addressed urgently in the context of the second situation, whereas too often they—and questions like them—are discussed in terms of the first. This was particularly brought home to me personally a few years ago when I was invited to sit in and and observe some lessons in a township school in South Africa. This was a dramatic experience for me who had spent most of my working life involved with schools in a predominantly rural county in the south-west of England, though much of what I observed was not so very different from some of the classrooms I had known. I need to set the scene a little more carefully in order to emphasise the sort of context in which I think it is important to consider the above questions.

One classroom comes to mind. It was much as I have described: there were more than sixty black boys and girls in the room, mainly thirteen-year-old but with a sprinkling of older students. The lesson began with the teacher marching up and down the aisles inspecting open exercise books with a ferocious expression on her face, and wielding a *sjambok*. [1] It turned out that she was checking on whether homework had been done. Where it had not been done there was a summary punishment on the spot with the sjambok. I soon realised that the ferocity was but a mask, that she was neither angry nor upset, merely carrying out something that had to be done.

After those who had not done their homework had been dealt with, it was time to go through it with those who had. Three exercises from the textbook had been set: these involved the multiplication and division of three unbelievably complicated signed rational numbers all expressed in mixed fraction form and bracketed off when negative. Each example would be written up on the board and worked through, the class being asked first to state the procedure at each stage and then to carry it out. The rule or procedure would be chanted by the whole class. What sort of fractions are these?, the teacher would ask. Mixed fractions, sang the class with drawn-out inflection. What do we put them into? Proper fractions. Then someone would be asked to do this. Usually this would be done correctly, but any mistake automatically earned a thwack with the sjambok. What do we do next? Remove brackets. But someone was heard to say "change brackets" and earned another thwack. Removing brackets involved an accumulation of negative signs; divisions involved the interchanging of numerators and denominators—words which permitted a drawn out musical phrasing; the multiplication rule involved a particularly lengthy chorus: mul-ti-ply-ing num-er-at-ors and mul-ti-ply-ing de-nom-in-a-tors; and the class had then to steer through the quite complicated cancellations and final reduction to mixed fraction form.

There followed—almost unbelievably—carbon copies of this lesson with two other classes. The sheer repetition brought home the automatic nature of this method of teaching. The sjambok was not wielded for bad behaviour but for making mistakes: the learning was thoroughly behaviourist, based entirely on heavy negative re-inforcement. This was the way most of the teachers had been taught. They were the successful products of the system. They had apparently successfully internalised an image of what it was to be in authority even though this involved what to other eyes might seem like gratuitous violence. I felt sad about this cycle for it was clear that these children were eager to learn and deserved something more worth their while. It was this sense of waste rather than a horror of corporal punishment that moved me. It was not surprising to learn that in the circumstances many of the students play truant for long periods and are uninterested in any form of higher education. [2]

A lot for a little

It is all too easy for bystanders to comment on what happens in other people's classrooms. In describing a township classroom I am well aware that I have been fortunate enough to work in more flexible and much more lavishly supported contexts. And I should add that I ended with considerable respect for the teachers whose lessons I observed. Sitting in their classrooms was an eye-opener and I now find it difficult to see educational issues except through this opening. So I need to embed the sort of questions I would like to raise about classroom style and content in the sort of situation that these teachers have to work in. On the other hand, I do not want to particularise the context too much. After all, behaviourist psychology, covert violence, and a meaningless curriculum are not unknown in schools in affluent suburbs. So this is where I invoke my mythical lesson under the banyan tree. My proposal is that in order to identify what basic activities you would want to see in *any* classroom, you try to imagine them being enacted without any of the usual resources, without even a roof overhead.

My own answers would invoke a principle of economy according to which teachers work, whenever they can, with whatever powers students already own. [3] One such power (in all except very exceptional cases) is the ability to conjure up and recall images. This provides a particularly economic access to mathematics. This is clearest in the case of geometry which can hardly be studied without some form of visual imagery. But there are various images derived from other senses than sight that may also be called upon. For example, almost all children have a sense of rhythm and some aural imagery which enables them to chant the number words of a first language in the right order. This is the power that I would want to invoke over and over again under the banyan tree or wherever else. "Them as counts counts moren than them as dont count." [4] Thus I would expect *any* group to be able to chant communally through various arithmetic progressions: for example, the even numbers, the multiples of nine and so on .... "Starting at 1,089 let's count backwards saying every 7th number." What, I would want to know, would be the point of doing any other number work with students who couldn't do *that*?

I have of course already begged a question by referring to such chants as counting. This is ordinal counting, namely counting in the intransitive sense—saying certain number-names in order. Current educational practice prefers to emphasise cardinal counting, namely counting transitively—counting a set of objects and finding the number-name for the set. It is relatively easy to count in the ordinal sense, but cardinal counting is another matter involving some quite sophisticated mathematical ideas. [5] It can take a lot of time and attention for children to learn that the number of objects in a (finite) collection is independent of their nature, their position, or the order in which you count them. But by "just counting" you get what Caleb Gattegno used to call a "lot for a little". Thus the cyclic structure of the number system to base ten enables you to count up to a million knowing just the names of the first nine numbers and the powers *-ty, hundred, thousand*. This assumes you say things like "one-ty three" for 13; to use the correct English expressions requires you to know a few special anomalous forms like *thirteen* . You certainly get a lot from as little as twenty or so number-names.

The customary cardinal emphasis leads you to read 3 plus 2 = 5 as a statement about the cardinal number of a union of disjoint sets. But you could read it as "counting on" two places from the name "three" in an ordinal chant. You get a lot for a little by milking this second reading for all it is worth: in fact you get the four rules of arithmetic, as Philip Ballard pointed out seventy years ago.

> The four fundamental processes in arithmetic are merely four different ways of counting. Adding is counting forwards, and subtracting counts backwards. In multiplying or dividing we count forward or backward by leaps of uniform length. [6]

Counting forward or backwards in uniform leaps might be seen as a natural and early example of linearity. Who would not be happy to work with a class which had mastered that, even if nothing else?

"Starting at 1,089 let's count backwards saying every seventh number." What, I would want to know, would be the point of doing any other number work with students who couldn't do that?

Notes

[1] The Afrikaans word *sjambok* is related to the Urdu *chabuk*, Persian *chabouk*, and English *chawbuck*, a strong and heavy whip made of rhinoceros or hippopotamus hide, used for driving cattle and "sometimes for chastisement"; the one the teacher wielded appeared to be a length of hosepipe.

[2] It is going to take a lot of time and patient work to shake off the legacy of apartheid in the form of the large classes in these schools and the manifest under-resourcing of them. Verwoerd had asked, "What is the use of teaching a Bantu child mathematics when it cannot use it in practice?" When both child and mathematics are referred to as "it", you know what was meant when he went on to say that "education must train and teach people in accordance with their opportunities in life". No doubt there may soon be some changes in the syllabus even though it may take longer—perhaps a few generations—to see real changes in life opportunities for a large number of black South Africans.

[3] See various books by Caleb Gattegno: for example, *The common sense of teaching mathematics* (Educational Solutions, New York, 1974), or *The science of education, part 2B: the awareness of mathematisation* (Educational Solutions, New York, 1988).

[4] In the words (and spelling) of the eponymous spokesman of Russell Hoban's novel, *Riddley Walker.*

[5] The distinction between ordinal and cardinal number was originally only of interest to grammarians. Apart from the Egyptian legacy of naming denominators with ordinals, mathematicians did not distinguish these two types of numbers before Cantor explored ways of counting infinite sets. The issue then became important in foundational studies, and there was considerable disagreement as to which was the more fundamental, as indeed there had been among the grammarians and was to be among psychologists. Piaget claimed that ordination and cardination were inextricably linked, but his careful study of children's understanding of cardinality (which was based on the work of mathematical logicians) led to a pedagogical emphasis (some might say an obsessive one) on conservation, one-one correspondence and so on. There are alternative approaches to number and the one proposed by Lebesgue (who took ordinality to be the basic notion) has been very fully developed by Caleb Gattegno (in the second book listed in note 3 above).

[6] P Ballard, *Teaching the essentials of Arithmetic*, University of London Press, 1928, p. 59

Live with your century, but do not be its creature; render to your contemporaries what they need, not what they praise.

Schiller
