Reflections on a TIMSS Geometry Lesson

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The notion of the reflective practitioner, and its impact on teacher education programmes worldwide, has been prevalent in the literature for the last two decades. However, the term 'reflection' is used almost exclusively in the sense of self-reflection, often as an implicit assumption. To take just one example, Loughran (1996) defines reflection as:

> the purposeful, deliberate act of inquiry into *one's thoughts* and actions (p 21; *our italics*)

The popular use of reflective journals also lays emphasis on the 'self'. Perhaps this is as it should be, since the ultimate aim of any reflection is presumably to improve one's own practice. Nevertheless, the idea of reflecting on *others'* practice is also a legitimate and important activity and is, of course, the rationale behind peer observation and analysis.

In this context, Lerman's definition of reflection is interesting:

> developing the skills of sharpening attention to what is going on in the classroom, noticing and recording significant events and 'working' on them in order to learn as much as possible about children's learning and the role of the teacher (1994, p 52)

Although Lerman himself goes on to discuss teachers' self-reflections, his definition does not restrict itself to this interpretation. In this article, we discuss our reflections on a videotaped geometry lesson from a Japanese classroom. Apart from considering some key elements of the lesson, we also focus on what, significantly, does *not* happen. In relation to this aspect, we contrast the lesson with the work of a group of teachers on a B.Ed. programme who were given the same problem-solving task as the Japanese pupils. But first we place the videotaped lesson in its context.

The Third International Mathematics and Science Study (TIMSS) was completed in 1996. While that study largely focused on pupil performance at selected grade levels in different countries, one part of the project focused explicitly on classroom instruction. This was the TIMSS Videotape Classroom Study, which was the largest international study of a qualitative nature in mathematics education ever undertaken. In all, 231 eighth-grade mathematics lessons were taped in the U.S.A., Germany and Japan.

Two of the aims of the study were to provide a rich source of information about what actually took place in the lessons and to develop objective measures of classroom instruction as indicators of teaching practices in the three countries. One of the reasons for the focus on teaching is well described by Stigler and Hiebert (1999) in *The Teaching Gap*, a book that reflects on the results of the study:

> Standards set the course, and assessments provide the benchmarks, but it is teaching that must be improved to push us along the path to success. (p. 2)

The final report (prepared by Stigler *et al.*) of the TIMSS videotape study was published in 1999. It was a condition of the study that the videotaped lessons of the teachers who took part would be kept confidential. Nevertheless, a number of teachers volunteered to produce a series of taped lessons which would be available for public consumption. These lessons are considered to be representative of those actually used in the study. Six taped lessons were produced, two from each country, on algebra and geometry, and they form the basis of materials produced by the U.S. Department of Education (1997) intended to generate discussion and analysis among teachers and teacher educators.

In this article, we discuss one of these taped lessons, a geometry lesson taught in a Japanese school. Among the suggestions given for viewing the tapes, and for providing a focus for one's reflections, the following questions are listed:

> What do you think is the teacher's goal?
> Are there key moves or moments in the lesson?
> Are there crucial missed opportunities?

We want to stress that it is in the spirit of *reflection*, and not of negative criticism, that this analysis of the lesson is offered. In fact, it is worth stating at the outset that we find many aspects of this lesson to be highly encouraging in terms of a problem-solving approach and pupil involvement. Nevertheless, there are also aspects that we have found worth exploring further, particularly in relation to the questions listed above.

A description of the lesson

For those readers not familiar with the videotape of this lesson, or with the lesson transcript, it is necessary to give a brief outline of the main features and we shall try to capture the flavour of the lesson as best we can.

In the introduction to the lesson, the teacher makes a brief recap of an important result established with the pupils in their previous lesson, namely that all triangles with the same base and between the same parallels are equal in area. The result is illustrated with a simple dynamic software illustration on the computer at the front of the class. The teacher then presents the 'problem of the day'. This concerns the land owned by two people (the teacher uses two pupils in the class named Eda and Azusa) whose boundary is a bent line. The teacher draws a diagram on the board, similar to that in Figure 1a.

What is required by the problem is to replace the bent line by a single straight line so that land is neither gained nor lost for either Eda or Azusa. The pupils are asked for suggestions about how to achieve this. The teacher initiates the discussion by placing a long pointer flat on the diagram and asking if this is a good position. The ruler is then moved

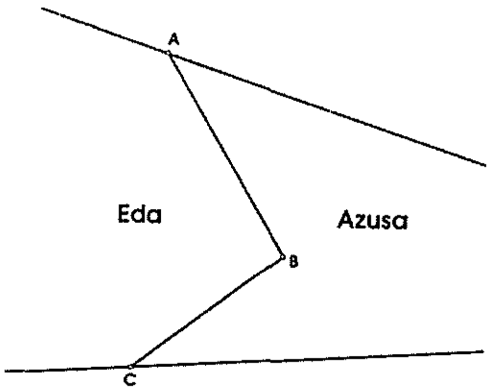

Figure 1a Original problem

across the diagram and there is some good-hearted banter about some of the extreme positions the teacher indicates.

Then another pupil is invited to show the best position on the board. She juggles the ruler around the middle of the diagram and places it approximately as shown in Figure1b.

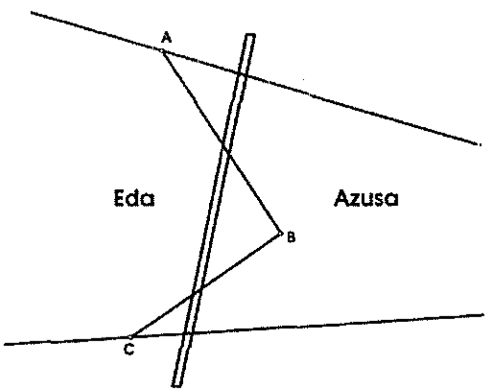

Figure 1b Approximate solution with pointer

The teacher comments that "we got an estimate" and then asks the pupils to think about the problem for themselves for the next three minutes A sign is also placed on the chalkboard that reads:

> Think about a method of changing the shape without changing the area.

As the teacher circulates around the class, we hear him making suggestions and posing questions to various pupils, such as:

> First of all, draw a figure [.] that of last time. Is there a method that uses the area of the triangles?
>
> You were able to make this a triangle, right? [.] Would you get triangles with the same area? Would you make this the base?
>
> Somewhere there are parallel lines, okay?

After the three minutes are up, the teacher makes some suggestions to the pupils on how to continue. If they have come up with an idea for a solution, they can either discuss it with another teacher (who is at the back of the classroom) or they can discuss it in groups with their friends. Otherwise, they can refer to some hint cards that have been placed at the front of the class

Another twelve minutes is spent on this activity during which the teacher selects two pupils to explain their solution on the board. The videotape shows the first pupil trying to explain his solution and, although he is not always very articulate, the main idea comes through clearly. After the second pupil has shown her method, the teacher emphasizes the main concept involved in both methods and asks the pupils which of them used the different methods. It appears that all pupils were successful using either or both methods. The two solutions are shown in Figures 2a and 2b.

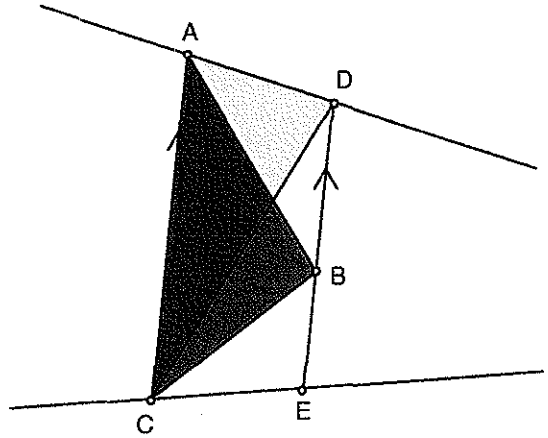

Figure 2a Area ABC replaced by ADC

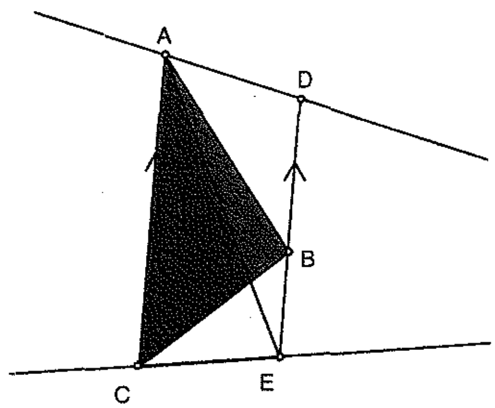

Figure 2b Area ABC replaced by AEC

The teacher now places a cut-out quadrilateral on the board and poses another problem to the pupils, namely to change the shape of the quadrilateral into a triangle without changing its area. The pattern of the first part of the lesson is then repeated, with the pupils working on the problem individually for three minutes and then working in groups, or with the teacher, or using the hint cards

The next section of the tape is some twenty minutes later and clearly, in the intervening time, the teacher has asked some pupils to draw their solutions on the board The teacher discusses some of these diagrams, particularly illustrating how alternative solutions have been constructed and checking which pupils were able to come up with the various solutions. There are ten diagrams on the board but two of these appear to duplicate other diagrams, so in fact eight different solutions are shown. The teacher then illustrates two solutions once more, using the computer, dragging a vertex along a line parallel to a diagonal (see Figures 3a and 3b).

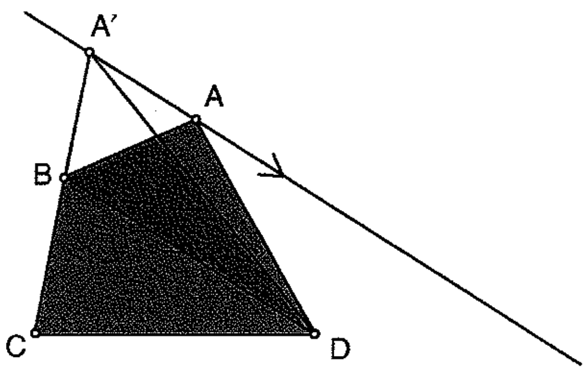

Figure 3a A′CD replaces ABCD

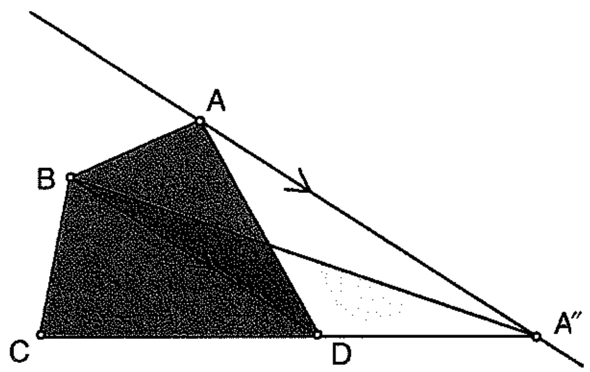

Figure 3b A″BC replaces ABCD

The teacher explains that as A is dragged along the parallel line, a point will be reached where the angle ABC disappears and becomes a straight line. He further explains that a similar situation occurs when A is dragged in the other direction: hence, there are two solutions arising from a line drawn through A, parallel to diagonal BD. During the discussion of the diagrams on the board, the teacher has also made clear that drawing a parallel to BD through C will produce other solutions and similarly the diagonal AC can be used with the vertices B and D. Finally, the teacher asks what they would like to try next and, after a brief discussion about pentagon or hexagon, it is decided they will try to change a pentagon into a triangle of the same area. This is left as a homework task

Analysis of the lesson

We have shown the videotape of this lesson to many of our teachers on both in-service and pre-service programmes. The reaction has invariably been the same. They are all deeply impressed by how well-structured the lesson is, how involved the pupils appear to be in the problem-solving tasks, the level of communication between pupils and teacher and also the level of mathematical thinking required. Many are surprised by what they see, in that it clearly does not fit their 'stereotyped' image of a Japanese mathematics classroom. Nearly all the teachers also commented on the nature of the first problem posed, saying that it was an interesting, realistic and motivating problem that was an excellent example of applying the mathematical principle that was the basis of the lesson. We would agree with all these comments, but we now want to look beyond this initial appraisal

Even with a videotaped lesson, there are still many questions one would want to ask that cannot be answered and so we have to speculate a little. For example, to what extent are the solutions produced by the pupils their *own* constructions? We shall return to this a little later, but first, notwithstanding the positive comments made above, what are the most striking features of this lesson? For us, probably the most striking of all is the relationship between the initial discussion of the boundary problem and the subsequent solutions. Or perhaps we should say the *lack* of relationship

There is a curious disjunction between the discussion that takes place before the pupils start working on their own and the solutions that are presented later. Initially, the discussion centres on the idea of moving a straight-line boundary across the whole diagram. This carries the implicit concept that at some point the amount of land lost will balance the amount of land gained. In fact, if this problem were presented 'cold' (that is, without the introductory reminder about triangles of equal area), this would seem to be a very logical approach.

However, during the period when the pupils are working on the problem, this approach seems to be completely ignored. We cannot be sure of this without seeing what every pupil tried to do, but certainly there is no evidence on the tape and the teacher never once refers to the idea again. This is all the more surprising, since it is the teacher himself who first provides the idea of a moving straight-line boundary with the use of the pointer.

Why is this idea not developed, given that it seems such a promising one? One of the problems, of course, is that it is *theoretically* very difficult to determine where this type of straight-line solution will be. However, it is ideally suited to investigation with a computer and two such solutions are shown in Figures 4a and 4b (overleaf).

In Figure 4a, a line parallel to AC is dragged along one of the boundary lines until the loss and gain of areas is balanced. The position of the point D is then determined. Figure 4b shows a similar process, where the line being dragged is perpendicular to one of the boundary lines.

This immediately draws our attention to another question concerning the lesson. What is the function of the computer? It is very clear that the whole lesson is structured around a problem-solving context and yet the computer plays no part

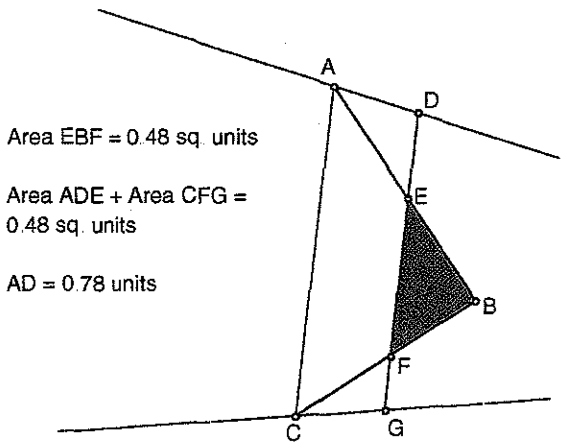

Figure 4a DG parallel to AC

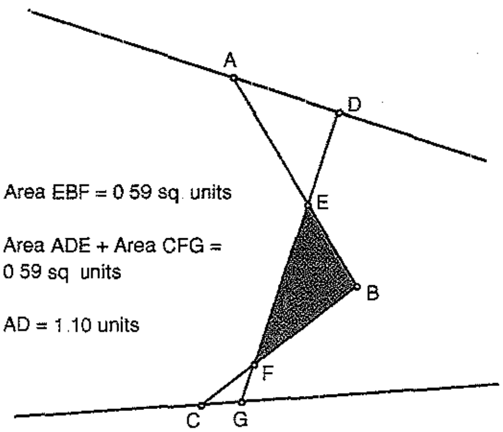

Figure 4b DG perpendicular to AD

in this aspect of things It is essentially used as a dynamic blackboard, either to remind pupils of previous work or to illustrate the solutions established during the lesson. It certainly does not contribute to the problem-solving *process* in any way. This is true of both the initial boundary problem and the later quadrilateral problem Given the rich possibilities inherent in both these problems, this seems a great pity and we return to explore these possibilities later.

We raised a question earlier concerning how far the pupils had constructed their own solutions It has been a common reaction among our own teachers, when viewing the tapes, that they are very impressed by the performance of the pupils in this lesson Their success with these quite difficult problems is rather striking, especially when one considers that Japanese classes are not streamed or set in any way: this is a mixed-ability group.

But if we look back to the teacher's suggestions and hints that we listed in the description of the lesson, we must be struck by the very strong guiding hand that is at work here. Even the sign that is put on the board, just prior to their working individually on the boundary problem, refers to changing 'the shape', but at this stage no particular shape has been identified. Which shape is being referred to? The subsequent repeated mentions of 'triangle' in the teacher's comments are a strong indication of the shape in his mind, and consequently on the direction the solution should take.

In addition, we do not know how explicit the hints given on the cards are, to which the pupils are encouraged to refer However, to judge by the successful outcomes, and by the fact that all the pupils produced the same *type* of solutions for the problems, it seems likely that the hints are at least as explicit as the hints given orally by the teacher And all these hints, suggestions and questions clearly guide the pupils in a very particular direction.

None of this is intended as a criticism of the teacher or the lesson. It is perfectly reasonable to assume that the teacher's goals for this lesson are precisely those of 'guided problem solving' and 'applications of a geometrical theorem'. Nevertheless, there are definite consequences arising from this approach The first is that what is ostensibly an open problem-solving approach to teaching may, in reality, be much closer to a traditional textbook format. We do not mean here traditional teaching *with* a textbook Clearly, a textbook *per se* played no part in this lesson, and it is a remarkable result of the TIMSS video study that textbooks were used in only 2% of all Japanese lessons studied!

However, the *structure* of typical chapters in many mathematics textbooks is that of a section of exposition and explanation of new concepts or techniques, followed by practice examples, followed by applications to problems Pupils seldom have to think about which concept or technique to apply in a problem, since it is obvious from the placement of the problems within a given chapter Given the degree of guidance evident in this lesson, one must wonder just how different it is from the structure just described.

A more serious consequence of the approach described in this lesson is that it effectively closes off certain avenues of exploration. And this, in turn, leads to a perception that the solutions obtained represent *the* solutions In other words, we are left with the distinct impression that the eight solutions found for the quadrilateral problem represent the complete set of solutions Even with the boundary problem, despite the initial discussion about the moving straight-line boundary, this is later ignored as we have remarked earlier and again there is the impression that the two solutions found somehow represent the complete set We have already seen that this is certainly not the case

Also, for the quadrilateral problem, even if we keep within the confines of using the 'equal triangle areas' concept (same base, same parallels), by investigating the problem with a computer we may extend our vision For example, Figure 5a illustrates what happens when we put in both possible parallels to one of the diagonals instead of concentrating on just one (as we had in Figures 3a and 3b). Now, by drawing any straight line EBF through B, and joining ED and FD, we can easily generate an infinite number of solutions by dragging E or F along the parallel lines.

Moreover, it also becomes very clear that the eight solutions which were generated in the lesson (which appeared to constitute a complete solution set) are now just special cases of general solutions that can be constructed in a similar way to that shown in Figure 5a. One such special case is illustrated in Figure 5b

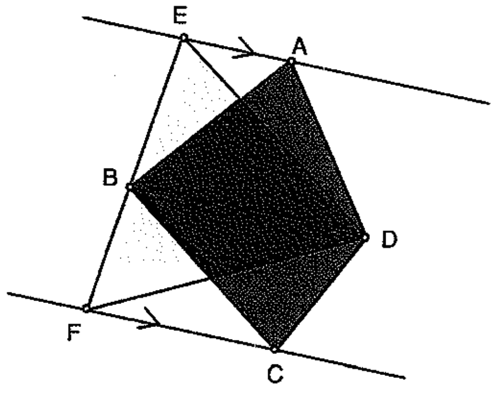

Figure 5a A generalised solution

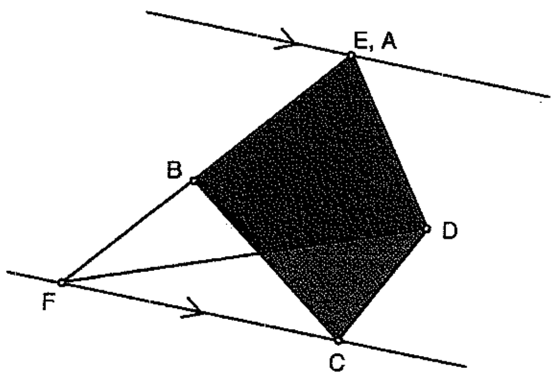

Figure 5b A special case

Of course, it is quite possible to find these general solutions without the aid of a computer. Indeed, there is no logical reason why working on a computer should be any more likely to lead to these solutions than working on paper. Nevertheless, there is evidence in the literature, and indeed our own experience also suggests, that the ease with which one can try out ideas in a computer environment does lead to more opportunities arising for alternative solutions (Arzarello, 2000; Leung and Lopez-Real, 2000; Noss and Hoyles, 1996). And this is particularly true in a dynamic geometry environment such as *Geometer's Sketchpad* or *Cabri-Géomètre*

Thus far, the possibilities we have described of using the computer have been our own suggestions and so it is only speculation that these kinds of solutions would occur in a classroom context. We were interested to see how some of our teachers would respond to the same problems which were presented in the Japanese lesson. We therefore gave the problems to a group of in-service teachers who were in their second year of study on a part-time B.Ed. programme Clearly, this is a very different group from a class of secondary school children and we are not making any direct comparisons here. Nevertheless, we think the outcomes are very illuminating and we discuss them in the following section

Results of an open investigation

Before looking at some of the solutions produced by the teachers, we need to make the context clear Although this was a group of serving Hong Kong teachers, many of them teach in primary schools and, in fact, have little confidence in their own knowledge of and ability in mathematics. There are also some other very important differences when compared with the Japanese lesson.

First, we presented the boundary problem *without* any reminders of mathematical concepts or theorems that might be helpful. However, the initial presentation was deliberately very similar to the Japanese lesson, with a ruler being used to explain how the boundary had to be changed to a straight line.

Second, the teachers were working in a classroom where computers were readily available to all of them. They were not *told* to use a computer, but were simply informed that they could use them if they wanted to. In the event, all the teachers used a computer for at least some of the time during the session and many of them spent the whole time on a computer. Some of the teachers worked in pairs on a computer whilst others chose to work alone

For the boundary problem, there were some teachers who did indeed produce solutions based on the 'equal triangles' concept shown in Figures 2a and 2b However, the most striking thing to us, though not unexpected, was that most of the teachers worked on the moving straight-line principle and developed dragging solutions the same as, or similar to, those previously illustrated in Figures 4a and 4b Two other solutions along these lines are shown in Figures 6 and 7a.

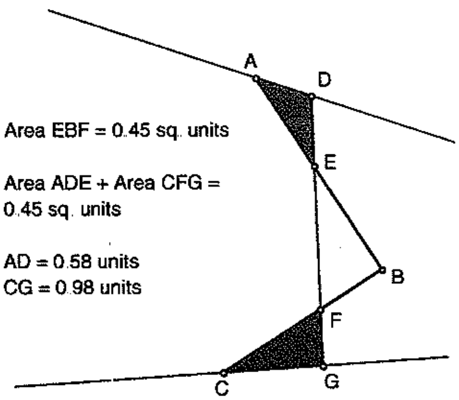

Figure 6 Points D and G dragged independently

In Figure 6, two arbitrary points are selected on the two straight-line boundaries and then dragged until a balance of areas is achieved. It is very apparent with this solution strategy that an infinite number of coupled positions are possible and the diagram shows just one of these.

The solution shown in Figure 7a is particularly interesting in the context of this discussion. It is based on a very simple dragging strategy achieved by marking any point E on the lower boundary and joining that point to A. The point E is then dragged along the lower boundary until the two triangular areas ABD and CDE are equal. So in this case only two areas are required to be balanced compared with the three areas in the previous dragging strategy solutions.

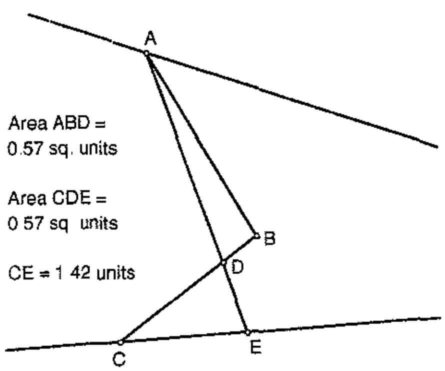

Figure 7a Balancing just two areas

A follow-up to any solution should involve some discussion of *why* it works With this particular boundary problem, this will naturally lead to the question of whether there is any clear relationship between the final solution and the original situation. For example, the solution in Figure 4a might lead us to hypothesise that the point D is the mid-point of AH, say, where H is the point of intersection of the upper boundary and a line through B parallel to AC In this case, a check on the lengths of AD and DH will quickly show us that the hypothesis is false

Conversely, the nature of the solution in Figure 6 would not suggest any definite relationship exists, since the points D and G give us an infinite number of solutions. However, the beauty of the solution shown in Figure 7a is that further discussion could easily lead to considering the diagram in Figure 7b, and consequently the fact that triangles ABC and AEC are also equal in area. In other words, this solution can lead us directly to the concept of 'same base, same parallels' that served as the introduction to the Japanese lesson.

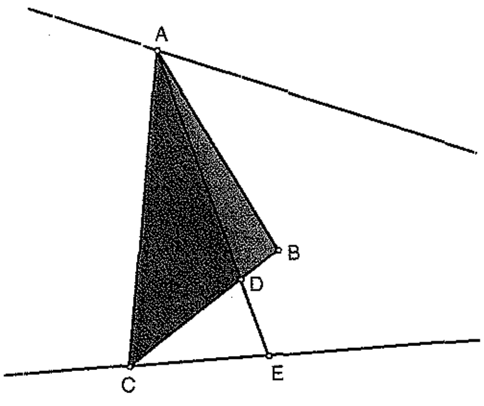

Figure 7b Why is Area ABC = Area AEC?

We turn now to the solutions produced by the teachers for the quadrilateral problem. As with the boundary pro-blem, some did, in fact, produce solutions based on using equal-area triangles as shown in Figures 3a and 3b. However, whether they were influenced by their previous approach to the boundary problem, or perhaps because it is in any case a legitimate general approach, many other teachers tackled this problem by again using the moving straight-line approach and balancing areas One such example is shown in Figure 8. A line is drawn through C parallel to the diagonal BD This line is then dragged across the quadrilateral until the shaded triangular areas balance The required triangle equal in area to ABCD is then AGH.

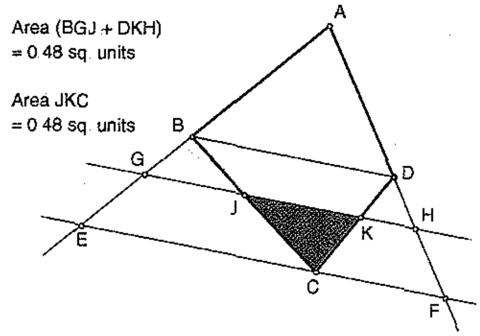

Figure 8 Balancing-area solution to quadrilateral problem

Apart from the solutions just described, two other solutions were produced which we consider both elegant and simple, shown in Figures 9a and 9b.

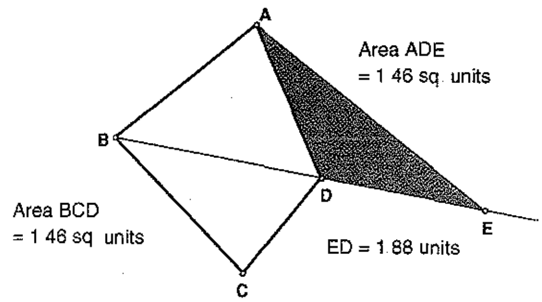

Figure 9a Balancing just two areas

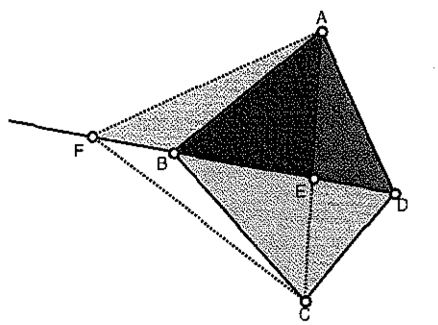

Figure 9b Why is Area ABCD = Area AFC?

In Figure 9a, the diagonal BD is extended and a point E placed on the line and joined to A. The point E is then dragged along the extended diagonal until the area of triangle ADE becomes equal to the area of BCD. Triangle ABE is then the final solution Rather like the eight solutions obtained in the Japanese lesson, it is clear that this method can also generate eight different solutions.

In Figure 9b, both diagonals BD and AC are drawn first, intersecting at E. Diagonal DB is then extended and the length BF is constructed equal to ED. The point F is then joined to A and C The required final solution is the triangle AFC It is interesting to note that this method is not a dragging strategy but a pure construction. Also, the concept used is effectively the one used in the Japanese lesson, except that here the bases of the triangles are equal by construction, rather than being on the same base It is also worth noting that this type of construction can generate four different solutions

Conclusions

In the preceding sections, we have tried to identify the most salient aspects of the video lesson and, in particular, we have concentrated on the possible directions in which the pupils' problem-solving activity could have led. Perhaps the first point we should make here is how very valuable such lesson analysis can be, especially for pre- and in-service teachers This is precisely what the 'Attaining excellence' materials are designed for and the study of the TIMSS videos allows the opportunity for in-depth and comparative study of mathematics lessons.

We have used this particular Japanese lesson in two different ways and the comparison is informative Normally, the video lesson is observed and then the teachers discuss and evaluate the lesson However, in the case of the B Ed. teachers referred to in our solutions above, they first went through the problem-solving exercise for themselves, as we have described. Only after this did they view and discuss the video.

In the first type of situation, most of our teachers, after their first viewing of the video, are left with the strong impression that this is an excellent example of an open problem-solving lesson It is only later, on closer analysis, that it becomes apparent the situation is not as 'open' as it first appears and is, in fact, quite a tightly controlled problem-solving environment This is evident in the features we have identified earlier: for example, the teacher's questioning and the lack of follow-up of certain avenues of exploration In addition, we may note that the eight solutions to the quadrilateral problem had already been prepared on the computer prior to the lesson

Now, of course, this can be seen as a very positive aspect of the lesson. After all, surely one quality of a good teacher is being well prepared and anticipating the outcomes that are likely to emerge in a lesson This is perfectly true and, as we acknowledged earlier, all this may legitimately form part of the teacher's goals for this lesson However, we would argue that there is an important difference between *anticipating* and *determining*. A teacher's knowledge of the likely errors that pupils may commit, in algebra for instance, is a valuable resource in the preparation of their teaching On the other hand, if truly open problem-solving opportunities are to be given to pupils, then the teacher needs not only to be aware of possible solutions or strategies that may arise, but also to be flexible enough to follow up on unexpected directions. This is, of course, a far more challenging role

In the case of our B.Ed. teachers who first went through the problem-solving workshop, when they viewed the tape afterwards their immediate reaction was to point out the difference between their experience and that of the Japanese pupils: that is, in not having been given any reminders of a particular geometric result before attempting the problem Because of the comparison with their own workshop experience, they were able to identify readily the points in the lesson where the pupils are guided into particular solution strategies and where other possibilities are not exploited. They were also quick to comment on the fact that they themselves were able to work with computers, whereas the Japanese pupils clearly had not had this opportunity And this in turn, they felt, had been a powerful factor in the rich variety and range of solutions that their group had generated.

It is the points in the previous paragraph which we want to stress as a final conclusion. As we have been at pains to make clear, it has not been our intention in this article to make negative criticisms of the TIMSS video lesson that we have analysed. What we have tried to show is that the particular problem situations used in the lesson have a rich potential for a genuinely open problem-solving experience and also that they lend themselves to exploration using appropriate computer software.

It is not only the range of possible solutions that is so impressive in this case, but also the *type* of solution When looking at these solutions, one very fruitful comparison that can be discussed is the difference between the 'dragging' solutions and the 'construction' solutions. This, in turn, can lead to discussion of approximate and 'conceptually exact' solutions and the relationship between a solution and the real-life problem it models.

So finally, despite the well-structured nature of the video lesson we have analysed, what seems a pity to us is the potential problem-solving and learning experience that the pupils have missed and are unlikely, at least for these particular problems, to return to. The large number of figures we have had to include to illustrate this article is, in itself, a testament to the richness of the possibilities opened up by a computer-enhanced exploration

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