Communications

How ethnomathematics can bridge informal and formal mathematics in mathematics learning process at school: a framework

MULIA PUTRA

An important issue in mathematics education in many countries has long been how to make use of students' daily life mathematics (informal mathematics) in teaching mathematics at school (formal mathematics). This communication proposes an alternative framework of mathematics education that bridges informal and formal mathematics using ethnomathematics.

Ethnomathematics can be defined as mathematical techniques or styles in explaining, learning, knowing and coping with natural, social, cultural and even imaginary environments (D'Ambrosio, 2006).

> Ethnomathematics is a research programme of the way in which cultural groups understand, articulate and use the concepts and practices which we describe as mathematical, whether or not the cultural group has a concept of mathematics. (Barton, 2004, p. 214)

Vithal and Skovsmose (1997) argue that there are some difficulties in understanding the nature of ethnomathematics, since adequate detail of practices is lacking.

Concerning the role of ethnomathematics in the school curriculum, Rowlands and Carson (2002) criticise four possible ways in which an ethnomathematics curriculum could occur: replacing academic mathematics, as a supplement, as a springboard for academic mathematics and in preparing learning situations. They conclude that ethnomathematics is an inadequate basis and that a universal mathematics should be central to a curriculum that is intended to help students meet the challenges of modern society. However, these arguments were countered by Adam, Alangui and Barton (2003) who answer the question, "What would an ethnomathematics curriculum look like and where would formal, academic mathematics fit in such a curriculum?", by stating:

> Debate on cultural issues in mathematics [...] must be based on contemporary writing in the field and should not focus on extreme views within the political justification for ethnomathematics. [...] The role of ethnomathematics in mathematics education is now predominantly an empirical matter, and [...] some preliminary results from recent studies [...] indicate a positive role for culturally-based curricula. (Adam *et al.,* 2003, p. 327)

Horsthemke (2006) rejects Adam *et al.*'s shift of attention away from philosophical discussion of ethnomathematics towards empirical research, arguing that fundamental issues concerning the relationship between ethnomathematics and academic mathematics remain to be considered. Here I move this discussion forward by describing ethnomathematics as a tool to bridge formal and informal mathematics in schools.

The Indonesian Context

Below I show how ethnomathematics can act as a bridge between formal and informal mathematics in an Indonesian context, using a dialogue between three personifications of ethnomathematics, formal mathematics and informal mathematics. First it is important to introduce relevant parts of the Indonesian context.

Indonesia is a country with an enormous population and cultural diversity; each area is unique in terms of cultures and characteristics (Miftah, 2016). Previous research has explored ethnomathematical examples from particular tribes in Indonesia such as the Baduy tribe (Karnilah, Juandi & Turmudi, 2013) and the Sundanese (Abdullah, 2017). Here, I consider two of the many cultural aspects in Aceh province that can be explored to enrich mathematical learning: the *Pinto Khop* structure and the *Pinto Aceh* jewellery motif.

The *Pinto Khop* (or *Khop* gate) is a heritage site from the reign of Sultan Iskandar Muda, in the 16th century. It is a small dome-shaped gate located in the capital city of Aceh, Banda Aceh, in the Putroe Phang Park (see Figure 1).

Figure 1. Pinto Khop.

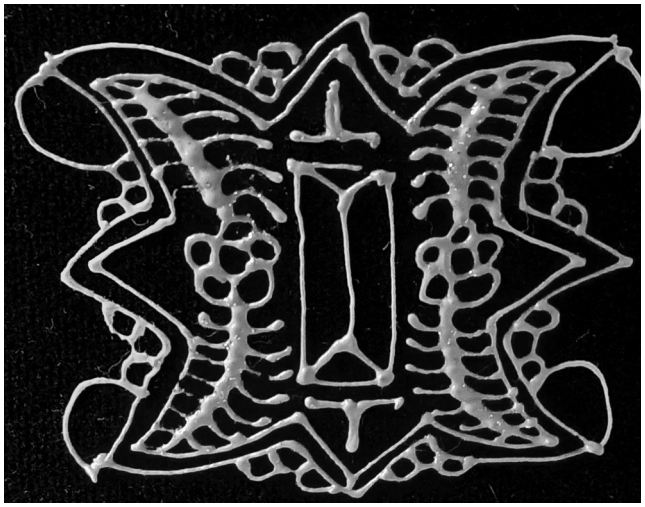

Figure 2. Pinto Aceh.

Pinto Aceh is a motif used originally in jewellery, but now also found in other contexts (see Figure 2). The motif is said to come from the *Pinto Khop* (Annisa, Selian, and Zuriana, 2016).

A conversation

The following conversation is an example of how ethnomathematics can act as a bridge between formal and informal mathematics. It takes place between Ethios (a personification of Ethnomathematics), Meukhop (a personification of Informal Mathematics) and Geom (a personification of Formal Mathematics).

Once upon a time

Meukhop Hi, Ethios!

Ethios How do you know me?

Meukhop Well, of course, we must be related. Don't you look like me?

Ethios I suppose in one way, yes.

Meukhop I have a question for you, Ethios. I see this design, the *Pinto Aceh*, everywhere, and people say it comes from the *Pinto Khop* but I don't know why they say that.

Ethios Do they look different to you?

Meukhop Well of course. The *Pinto Khop* is a building. It has a front and a back and sides, and above and below. The *Pinto Aceh* is flat. You can print it on a T-shirt. And it looks the same if you turn it on its head. The top of the *Pinto Khop* has points, but the bottom is flat, as it sits on the ground.

Ethios Well, I think my friend Geom might be able to explain. I will ask him.

Later, elsewhere

Ethios Hi Geom, I want to ask you something.

Geom Of course, mate, how can I help you?

Ethios Do you know the *Pinto Khop* right? For my friend Meukhop it is like his body, it is a physical form of his culture and history. And he sees the *Pinto Aceh* everywhere. It is practically the symbol of his culture. What do you think about them?

Geom Well, I think they match.

Ethios How can you say that? Any reason?

Geom Well, I think by using some tools of mine like symmetry and reflection, they seem to fit together.

Ethios Are you sure?

Geom I am pretty sure, of course.

Ethios If so, what about getting together to talk about this, because I don't think Meukhop can work that out by himself. He doesn't have tools like yours.

Geom OK.

When they meet

Ethios Well, here we go, Meukhop! Geom said that your *Pinto Aceh* symbol fits with your body, the *Pinto Khop* and he has some instruments to explain it!

Meukhop Is that true, Geom?

Geom Of course. Why these things are so famous, I don't know, but they are certainly connected. By the way, how come you said that the *Pinto Aceh* symbol doesn't fit with your *Pinto Khop*?

Meukhop Well, my *Pinto Khop* is fatter and larger, and has sides, and top has points, but the bottom is flat. The *Pinto Aceh* looks the same if you turn it on its head.

Geom Well, consider the reflection of the front of the *Pinto Khop* over its base [see Figure 3]. Or of the side. After it is reflected it has the same two perpendicular reflection axes as the *Pinto Aceh*, and that means that it also has a 180° rotational symmetry.

Ethios That means it looks the same if you turn it on its head.

Meukhop But how could it be reflected? It is a building!

Ethios Think about what happens when the lake around it floods. You can see the reflection of the *Pinto Khop* in the water, and it looks like the *Pinto Aceh*.

Geom You do not even have to reflect it to see the resemblance to the *Pinto Aceh*. From the front and side it has only a vertical symmetry axis, but from above it has two. Have you ever seen it from above? I used a drone to take a picture. Look! [see Figure 4]

Meukhop Wow!

Geom If you consider symmetry, and use reflection or change your point of view, the *Pinto Khop* can look like the *Pinto Aceh.*

Ethios Let me see. I can understand now, why the *Pinto Aceh* looks like that. Maybe someone was inspired by the reflection in the lake, or maybe they imagined being a bird and flying over the *Pinto Khop*, and made the *Pinto Aceh* to show what they saw. Or perhaps someone familiar with Geom's culture used his tools to transform one into the other.

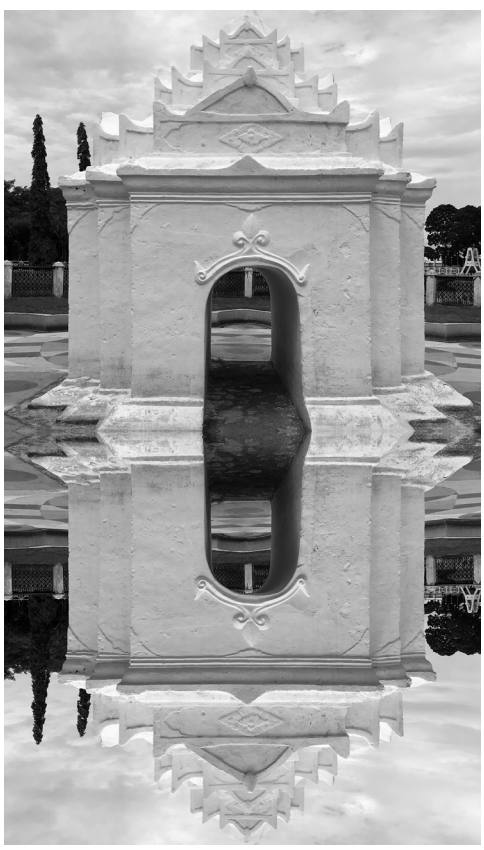

Figure 3. The Pinto Khop reflected.

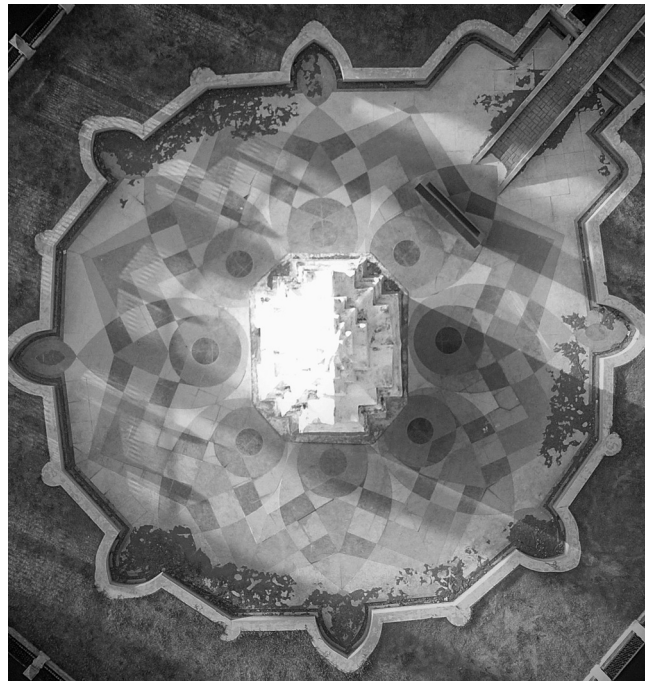

Figure 4. The Pinto Khop from above.

Meukhop I never thought the *Pinto Khop* looked like that.

Ethios Yes, it was eye opening for me too. So, thank you, Geom, for a good discussion today.

Geom It is a pleasure to be helpful.

Based on the conversation above, the role of ethnomathematics as a bridge in this context is showing that the tools of informal mathematics (describing and measuring) are not enough to understand Meukhop's culture. One reason for this is that Meukhop's point of view is limited, but by using tools or a point of view from formal mathematics (reflection or the view from above), Meukhop sees other aspects. But the connections between formal mathematics and everyday culture are not immediately obvious. Ethnomathematics can serve as a bridge, for example by connecting the formal idea of 180° rotational symmetry to the informal idea of turning it on its head.

Proposed Framework

Figure 5 diagrams the process and elements involved when ethnomathematics bridges informal and formal mathematics.

The primary focus of this framework is the process of communication between ethnomathematics and informal mathematics and between ethnomathematics and formal mathematics. This process of communication occurs in the interaction between teachers and students and aims to create a common understanding between informal and formal mathematics. It makes use of several tools that permit connections to be made, specifically universal mathematical activities (Bishop, 1991) and symbolising activities

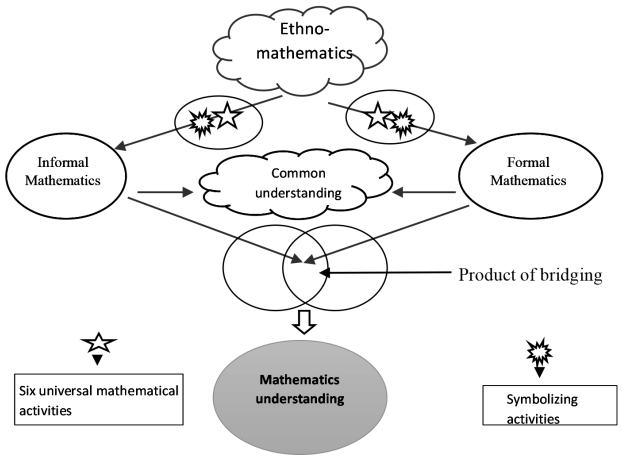

Figure 5. Scheme of bridging formal and informal mathematics.

(Tillema, 2010). Through these activities a new understanding can emerge between informal and formal mathematics which I call a *product of bridging*. It is the result of *conceptual blending* (Fauconnier & Turner, 2002) between the universal mathematical activities and symbolising activities which creates a new space of knowledge (Zazkis, 2015).

Here the explanation of the process is abstract, but it is the same as the process of bridging illustrated in the dialogue above, using personifications of ethnomathematics, informal mathematics and formal mathematics. It remains to be explained how this process looks in a classroom interaction.

In the classroom, communication between informal and formal mathematics occurs through both universal mathematical activities and symbolising activities. The core activity is producing a *product of bridging,* a common understanding between teachers and students. This is the goal of learning mathematics. Ethnomathematics in this process is used by teachers and students to understand informal mathematics and compare this explanation to formal mathematics.

Conclusion

This framework for bridging informal and formal mathematics in mathematics learning process at schools is not meant to replace other frameworks for the process of learning. Instead it aims to complement others to improve mathematics education in the future. This framework focuses on ethnomathematics as the primary technique to explain culture as it occurs in mathematics learning processes, and teacher enculturation is a critical point in implementing this framework. What is presented here is not intended to be a fixed framework, but rather a starting point that invites improvement and criticism by other researchers. Furthermore, I hope this framework can also enrich mathematics education by enlarging the space of ethnomathematics.

Acknowledgment

I thank professor Takuya Baba for the motivation, and the Indonesia Endowment Fund for Education for supporting my study of mathematics education.

References

Abdullah, A.S. (2017) Ethnomathematics in perspective of Sundanese culture. *Journal on Mathematics Education* **8**(1), 1-16.

Adam, S., Alangui, W. & Barton, B. (2003) A comment on: Rowlands and Carson 'Where would formal, academic mathematics stand in a curriculum informed by ethnomathematics?' A critical review. *Educational Studies in Mathematics* **52**(3), 327-335.

Annisa, A, Selian, R.S & Zuriana, C. (2016) Perkembangan Bentuk dan Motif pada Kerajinan Tas di Gampong Dayah Daboh Kecamatan Montasik Aceh Besar. *Jurnal Ilmiah Mahasiswa Program Studi Pendidikan Seni Drama, Tari dan Musik* **1**(3), 181-191.

Barton, B. (2004) Mathematics and mathematical practices: where to draw the line? *For the Learning of Mathematics* **24**(1), 22-24.

Bishop, A.J. (1991) *Mathematical Enculturation: A Cultural Perspective on Mathematics Education,* Vol. 6. Dordrecht: Kluwer Academic Publishers.

D'Ambrosio, U. (2006) *Ethnomathematics: Link between Traditions and Modernity.* Rotterdam: Sense Publishers.

Fauconnier, G. & Turner, M. (2002) *The Way We Think: Conceptual Blending and the Mind's Hidden Complexities.* New York: Basic Books.

Horsthemke, K. (2006) Ethnomathematics and education: some thoughts. *For the Learning of Mathematics* **26**(3), 15-19.

Karnilah, N., Juandi, D. & Turmudi (2013) Study Ethnomathematics: Pengungkapan Sistem Bilangan Masyarakat Adat Baduy. *Journal Online Pendidikan Matematika Kontemporer* **1**(1), 1-15.

Miftah, M. (2016) Multicultural education in the diversity of national cultures. *QIJIS: Qudus International Journal of Islamic Studies* **4**(2), 167-185.

Rowlands, S. & Carson, R. (2002) Where would formal, academic mathematics stand in a curriculum informed by ethnomathematics? A critical review of ethnomathematics. *Educational Studies in Mathematics* **50**(1), 79–102. https://doi.org/10.1023/A:1020532926983

Tillema, E. (2010) Functions of symbolizing activity: a discussion. *For the Learning of Mathematics* **30**(1), 2-8.

Vithal, R. & Skovsmose, O. (1997) The end of innocence: a critique of 'ethnomathematics. *Educational Studies in Mathematics* **34**(2), 131-157.

Zazkis, D. (2015) Monsters, lovers and former friends: exploring relationships with mathematics via personification. *For the Learning of Mathematics* **35**(1), 33-38.

Conversation

DAVID A REID, MELLONY GRAVEN, JEAN-FRANÇOIS MAHEUX

David We often make the comment that submissions are not 'conversational'. Sometimes we refer to "the ongoing FLM conversations", sometimes to "FLM issue-driven conversational style" and sometimes to "articles that stimulate conversation". I suspect that long-time readers of FLM will recognise the quality that we are referring to, but I think those new to the journal find the term 'conversational' a bit mysterious. So, I'd like to invite the two of you to engage in a conversation with me about 'conversation' in FLM.

JF You know I like etymology to get my thinking going. The word conversation comes from the Latin verb for 'converse', *conversari* which mean 'keep company (with)', made from *con,* 'with' + *versare*, 'to turn'. So a conversation is the repetitive turning to something or someone we are in conversation with, so that we both are in company. I

think this is a first, simplistic of course, interpretation of what we mean.

Mellony Yes, to me a conversation engages others in responding rather than, for example, talk in the form of 'telling' which requires listening. So while FLM readers are not able to respond directly to the authors the writing style should elicit reader engagement with ideas.

JF We often read submissions that feel like monologues, authors simply exposing their views or results. Interactively engaging with a reader on paper is certainly not the simplest thing. For me, a good way to start is by asking questions, inviting readers to look for things we talk about in what they do, and wonder how other perspectives could respond to what is offered.

Mellony We are not saying, however, that research reports cannot also be written in conversational style and I have read many that are—indeed Anna Sfard and Jill Adler's articles (among others) always draw me into conversation with their ideas—however research reports are usually judged against the findings of a study and whether the author has convinced readers of the validity and reliability of the findings based on a comprehensive and rigorous explanation of the theory, literature and methodology guiding the study. While tensions or dilemmas faced in the research may be welcomed by reviewers they are not required for acceptance of a research report. I believe FLM readers are particularly interested in these tensions and dilemmas and want to enter into 'conversation' about these within the rigorous research authors have conducted.

David I am glad you brought up research reports. I fear that some readers will read the content and form of this communication and assume that what we mean by 'conversational' is that articles should be written as dialogues. We do print dialogues (and poems, and narratives, and plays, and essays, and so on) but 'conversational' isn't about the form. I could imagine writing a conversational research report, and as you say the best writing in that genre is conversational; it connects to past work, not superficially through a list of related publications, but substantially, building on and reinterpreting the results of past work from a new perspective.

Mellony Involving others in a conversation tends to require a compelling issue and telling a good story (based on rigorous research) is an excellent way to ground the conversation and make the issue visible in tangible ways. In FLM we value authors opening up their vulnerabilities on an issue or within a story—I recall a colleague telling me her most cited work is one where she opens up to readers how she became aware of her role in co-constructing a story of mathematical failure for the student she was researching.

JF It can also mean being in conversation *with yourself*. Discussing how you come to ask some questions, expressing doubt or conviction, when that is the case, can be interesting way to be present as an author so people reading the article clearly not only engage with ideas, but with you as well. Needless to say, there can always be too much of something: what matters here is the spirit of telling a good story to someone you care about. Exposing yourself, taking risks, are ways to show people you consider them good company, and you want to preserve that.

David Some authors assume that by asking them to engage in "the ongoing FLM conversation" we mean they should pepper their article with references to past FLM articles. But I always have Dick Tahti's words in mind as well: "please, could it have shorter book lists?"(in **4**(2) p. 32).

Mellony Long lists of references to show one has read all there is in the field certainly detracts from the conversational flow of an article. I find it particularly problematic when such references are dated and newer grapplings or conversations with the research issue being addressed are overlooked. In this respect, it is important to search recent FLM articles and other leading journals to find out whether, and if so how, others might be grappling with related or similar issues.

JF Turning to the journal itself shows you are listening (an important part of conversing!), and then the piece does not read like your ideas were never discussed before. But simply referencing is not making conversation: it is 'talking about'. Conversing with a published article (within FLM or not) means addressing it (and its authors), and not merely addressing such and such topic through it.

David In spoken conversation we manage to refer to things that have been said before without using (Name, Date) references. Why not do the same in print?

JF I am not sure what to suggest to help people do that, except—take time to read the journal, and get acquainted with the sort of ideas people discuss, and how they do it.

David Can either of you, having had this conversation, formulate a succinct answer to the comment "I am not entirely sure what the editors mean by conversational, but would be happy to work on it given clarification"?

Mellony I see conversational style as requiring thought-provoking issues that invite readers to grapple with the unfolding arguments and questions raised in the article. Authors need to ask themselves, "What

issue am I raising in this article?" "Why should readers grapple with this issue?" "How can I bring readers into grappling with this issue as they read my article?" "How have others grappled with this issue (if at all) and what am I contributing that is new to this?" Thinking about engaging *others* while writing, perhaps, is what it comes down to.

JF At this point, I think I'd say: "Have a look at **38**(3); there is a short communication piece just about that. And then please feel free to ask more precise questions". We do like to talk with our authors, but it is also very good to have this little piece here, because repeating yourself is not very, well, *conversational*.

Response to Culligan and Wagner's *This is not mathematics*

ANNA SIERPINSKA

In issue **34**(2) Karla Culligan and David Wagner quote me distinguishing between the 'mathematical layer' and the 'didactic layer' in my article *Formats of interaction and model readers* (in issue **17**(2)). They later suggest that "when mathematics education researchers try to separate mathematical communication from other communication in the classroom, they are in effect saying 'This is mathematics' or 'This is not mathematics'" (p. 17). They imply that there is some value judgment underlying these distinctions. I am a poor sinner like everybody else, but I do not admit to the particular iniquity that the authors are accusing me of [1].

In my article, the terms 'mathematical layer' and 'didactic layer' of the text in a mathematical textbook served to distinguish between sentences such as 'Theorem. A square matrix A is invertible if and only if $\det(A) \neq 0$' and 'Read this section carefully. You will see its importance when we deal with linear transformations in Chapter 5.'. The first would belong to the mathematical layer of the textbook, the second to its didactic layer. I am not saying that the 'didactic layer' is 'not mathematics'. Implying that this is what I mean is applying a two-valued logic to a context where it does not belong. The different layers of a textbook overlap and interact with each other; one does not make sense without the others; just like the different 'storylines' the authors talk about in their paper. There is also no value judgment attached to these layers. Textbooks can be compared according the relative size of their didactic layers, but we cannot say *a priori* whether textbooks with larger didactic layers are more or less useful to the learners. The point that I was trying to make in the article was that it depends on the format of interactions that develops between the learner, the text and the teacher.

The authors say that the phrase they use, 'This is not mathematics' alludes to Magritte's picture of a tobacco pipe, with the inscription 'Ceci n'est pas une pipe'. But if I were to create an analogous picture with the inscription, 'This is not mathematics', I would choose a text from the mathematics layer, because the point is to distinguish an object from its representation, and not between two different objects. In fact, since the term 'mathematics' is very broad and it would be difficult to choose a representation giving justice to all the different areas and aspects of mathematics (and also fitting within the frame of a small picture), I would draw something like Figure 1.

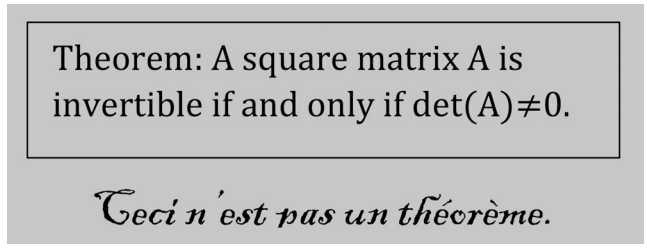

Figure 1. An analogous picture.

There are other ways to make the same point that Magritte made in his pipe picture, and one I like very much is the following joke: "Comment passer Jacques par le trou de la serrure? — Simple, on écrit 'Jacques' sur un bout de papier et on le passe." [How can you fit Jack through a keyhole? — Simple, you write 'Jack' on a piece of paper and put it through.] The joke has to be told, not written—in writing the element of surprise vanishes. I heard it in the late 1970s from Josette Adda, a French mathematics educator, and a student of Hans Freudenthal. She told it in the context of a lecture on the shortcomings of the New Math reforms in France, in the 1960s, where there was so much focus on representations (*e.g.,* graphical ones, such as Venn diagrams, for sets; algebraic ones, such as equations, for geometric objects) that there was a risk of completely eliminating the objects they were meant to represent and of forgetting the arbitrariness of their representations; the risk of confusing Jacques with 'Jacques'.

Note

[1] They also single out two other authors for similar distinctions, but here I will speak only for myself.

Reference

Culligan, K. & Wagner, D. (2018) This is not mathematics. *For the Learning of Mathematics* **34**(2), 14–18.

Metaphor and problem-solving among English language learners

ROD E. CASE, GWENDOLYN M. WILLIAMS, PETER COBIN

Many students experience difficulties with mathematical problem solving, and these difficulties can be compounded for language learners. In this communication, we describe a research/teaching experience conducted with a class of third-graders most of whom were English language learners.

The aim was to identify a technique which would allow us to analyze problem solving in a context where linguistic expression was limited. We draw on work in cognitive linguistics (*e.g.,* Lakoff & Núñez 2000), specifically the concept of metaphor. Metaphor, according to Lakoff and Núñez, is often used to express complex or abstract ideas (the target) by linking them to a more familiar or concrete notion (the source). They describe many metaphors that help elucidate the connections between language and mathematical problem solving; we focus here on the Source, Path, Goal Metaphor (SPGM). The SPGM describes how directed motions or movement, be it with the body or described in mathematics, are goal-directed and patterned. The SPGM, according to Lakoff and Nuñez, is a basic schema. As such, it functions as a building block of cognition that is developed in early childhood and used without conscious attention by the individual.

The examples discussed here are taken from a lesson on two-step word problems that asked the students to solve a problem about three children playing a board game. Although there was some evidence that some of the students drew on a strategy to solve the problem, few could express how they solved the problem. Unable to fully articulate their problem-solving process in writing, they gave explanations of how to play the game described in the problem or no explanation at all rather than how to solve the mathematical problem. This communication explores how we used the SPGM as a guide for understanding how our students moved in and out of the SPGM as they explained their problem-solving attempts. The insights we gained from our analysis of the students' work are shared below.

Background on class

In this lesson the teacher presented a word problem on the overhead together with the accompanying game board (See Figure 1).

After receiving a worksheet with the word problem, students read the directions on their own and could confer with each other to answer the question. The question asked students to write an explanation of how to solve the problem below.

> Question: Olivia, Oscar, and Omar are playing a game called Rocks and Shells. Olivia moved 3 spaces. Oscar moved twice as far as Olivia. Omar moved twice as far as Oscar. But after they moved, they got a card that said the leader has to go back two spaces. Where did they all end up?

We did not give further clarification of the procedure because the format of the directions was familiar to the students, and there were many previous lessons on multi-step problem solving. During the lesson, teachers asked students to solve the problem, write an explanation of how they solved it, and draw a picture depicting their process.

Working in and outside of metaphor

While this task seemed like a potentially motivating context to set a math problem in, it turned out to be source of confusion. Many students (19 out of 35) read about children playing and assumed that the problem was to give strategic advice on how to win the game rather giving an

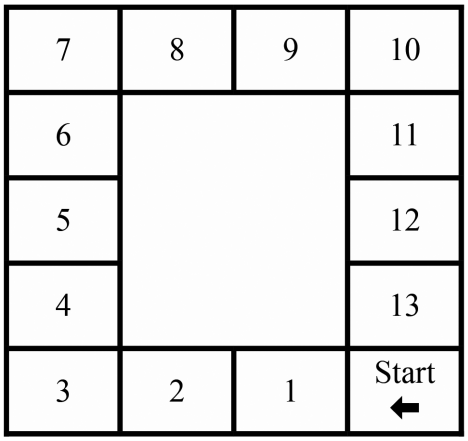

Figure 1. The Rocks and Shells game board.

explanation of how they solved the problem. Still others gave responses that were partially rooted in an analysis of the game but also showed some evidence and awareness that the task was to solve a math problem. Use of the SPGM was informative here because it provided an analytical tool that we could use to separate these efforts.

The first example is one in which the student anchors her response entirely within a framework of playing the board game and outside an SPGM framework. All three sentences tell how to prepare the pieces for playing the game and neglect a discussion of how to solve the problem.

> *Student 53* So first you get paper and you measure. Then cut where the line is. Finally, you make blue rock and red is shells

The next example was representative of a smaller group of students. Unlike their peers who construed the problem entirely as how to play a game, these students provided an explanation of how to solve a math problem within the framework of playing a game. It represented a hybrid response in which traces of mathematical thinking and problem solving could be identified and used to reveal important clues into their thinking and problem-solving processes. The SPGM analyses are included as comments.

> *Student 12* So, first Olivia and her friends were playing shells *[Not in SPGM]*
> Then, Omar moved as far as Oscar *[Position of Omar is incorrect]*
> Finally, Omar had to go back.
> *[Direction of Omar is correct]*

> *Student 33* The problem is about *[Not in SPGM]* that Olivia went two more *[Source location is incorrect]*

Our initial response was to dismiss these students' work as a misreading of the problem, no different from their peers who read the problem as an exercise in how to play a game. The SPGM was useful here though because it allowed us to fine tune our analysis and draw out bits of information in the response that demonstrated mathematical thinking. Our analysis began with student 12. Student 12 opened with a

statement that was outside of the SPGM, but in the following lines the student identified two points of information. She correctly described the direction of Omar but misidentified his position. Student 33 similarly opened her response outside of the SPGM. She gave the source, which was just one piece of information and incorrect. Nevertheless, both revealed that the game, like the actual problem, involves movement and thus located their thinking within the arithmetic as motion along a path metaphor.

An alternative analysis of any of these responses might suggest that, because English is their second language, they simply did not understand the problem. The assumption is that the teacher can help the student by identifying specific vocabulary items, unique sentence structures within the problem or deficiencies within the student's language lexicon (*e.g.,* Adams, 2003; Chan, 2015). Beyond the awkward phrasing in the third line in which the student writes, "you make the blue rock," there is little evidence to guide the concerned teacher on where the problem resides. A review of the wording in the problem similarly offers little guidance, as the problem is written within what was intended to be a familiar and motivating genre for the students.

The push and pull of language use in problem-solving

Because a number of students were still developing proficiency in English, their responses were necessarily short. When these same students gave incorrect answers, the problem became how we can determine if the problem is in difficulty with mathematics or second language proficiency. The SPGM revealed that students could often provide a correct answer, despite their still-growing proficiency in English. In these responses, students provided clues in their use of language and mathematical notation. The first example is from a student who solved the problem correctly but wrote very little.

Student 32 Olivia 3 *[Source is correct]*
Oscar 6 *[Position of Oscar is correct]*
Omar 12–2=10 *[Goal, position of Omar, is correct]*

In this example, the student used simple mathematical notation rather than a written explanation to indicate his understanding of the problem. The trade-off suggests that the student is able to draw on his receptive skills in English, probably during class instruction, to communicate his understanding of the content. Within an SPGM, there are at least five bits of information (Source of Omar, Position of Oscar, Position of Omar, and Goal). While there were just three written words, the SPGM traced the student's problem-solving work from goal to path and then to source.

A second student with a higher level of productive skill in English gave the response below.

Student 47 So, first Olivia moved 3 *[Source is correct]*
then Oscar moved twice *[Direction of Oscar indicated]*
3+3=6 *[circles '6' and labels 'Oscar moves']*
[Position of Oscar indicated correctly]

This particular student's proficiency in English was still developing, and, like her classmate, she gave responses in which she moved between using mathematical notation and written English to communicate her problem-solving skills. In the first sentence, she establishes Olivia's position (Source) using both mathematical notation and written English. The answer is correct.

Next, she moves on to determining Oscar's position (Path). She correctly indicates this using mathematical notation and uses the word 'moved' in her written response. While it's possible that use of the word 'moved' is a restatement of the problem, it is more likely that the use of the word indicates an understanding of how language is used in the problem itself.

More specifically, the problem relies on understanding that initially the children are in the same place. To calculate Oscar's position, the reader has to make an inference across sentences two and three that a movement in sentence three has taken place which is double that which is indicated in sentence two. The clue is in the repetition of the word 'moved'. Student 47's use of the word "moved' and her subsequent calculation is a strong indicator that she has a proficiency in English that is sufficient to understand the problem. While she did not complete the final calculation (Goal), it is likely that this may be a limited understanding of the math rather than her proficiency in English.

Some conclusions on math, language and English language learners

While setting the problem within the motivating and familiar environment of playing a game initially seemed like a good idea, it turned out to be surprisingly difficult for the class. Use of the SPGM allowed us to draw out multiple points of information from a limited amount of writing that is characteristic of young learners in general and English language learners in particular. The analysis that we did was quick and could be used by any math teacher. While use of SPGM may be limited to either certain problem types or assignments which require students to give a written explanation of their work, we also found that casting students' attempts within problem-solving metaphor offered a deeper explanation for their struggles than what the research on language and English language learner instruction could provide.

References

Adams, T.L. (2003) Reading mathematics: More than words can say. *The Reading Teacher* **56**(8), 786–795. Retrieved from http://www.jstor.org/stable/20205297

Chan, S. (2015) Linguistic challenges in the mathematical register for EFL learners: Linguistic and multimodal strategies to help learners tackle mathematics word problems. *International Journal of Bilingual Education and Bilingualism* **18**(3), 306–318. doi: 10.1080/13670050.2014.988114

Lakoff, G. & Núñez, R.E. (2000) *Where mathematics come from: How the embodied mind brings mathematics into being.* New York: Basic Books.