

History in Mathematics Education – as I Read It

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*In memory of Shmuel Avital (1915-2001)
and John G Fauvel (1947-2001)*

The International Commission on Mathematical Instruction (ICMI) was first established at the International Congress of Mathematicians held in Rome in 1908 and its first President was Felix Klein. ICMI was reconstituted in 1952, after an interruption of activity between the two World Wars, becoming an official commission of the International Mathematical Union (IMU)

ICMI decided to invest effort in the identification and investigation of issues or topics of particular significance to contemporary mathematics education and to encourage the implementation of concrete studies on them. The emphasis of a given study may be on analytical or action-oriented aspects, but some analytical component should always be present. Built around an international seminar, each study culminates with the publication of a volume intended to promote discussion and action at an international, national, regional or institutional level.

The tenth of the eleven ICMI studies which have been completed by 2001 (and which are listed in the appendix at the end of this piece) is the focus of this article.

An ICMI study – a process

Let us look at the processes involved in the elaboration of an ICMI study through the examination of a specific case. In general, an ICMI study is composed of a series of steps:

- a. A theme is decided upon by the Executive Committee (EC) of ICMI.

In this case, the theme chosen was 'The role of the history of mathematics in the teaching and learning of mathematics'.

- b. The EC appoints an International Program Committee (IPC), which is responsible for conducting the study on behalf of ICMI. Usually, a country which is willing to host the corresponding study conference is identified concurrently with the appointment of the IPC.

John Fauvel, the former chair of the *International Study Group on the Relations between the History and Pedagogy of Mathematics* (HPM), together with Jan van Maanen – the then-current HPM chair during the period of the study – were appointed to direct this ICMI study.

The IPC consisted of: Abraham Arcavi (Israel), Evelyne Barbin (France), Jean-Luc Dorier (France), Florence Fasanelli (USA), John Fauvel (UK, co-chair), Alejandro Garcadiago (Mexico), Ewa Lakoma (Poland), Jan van Maanen (Netherlands, co-chair), Mogens Niss (Denmark, *ex officio*), Man-Keung Siu (Hong Kong).

- c. The first task of the IPC is to produce a discussion document that sketches out some of the concerns to be addressed in the ICMI study, in the hope that many people across the world will contribute to the international discussions and the growing understanding reached in and about this area. In this document, a number of key issues and sub-themes related to the theme of the study are identified, presented and described in a preliminary way.

The discussion document for this ICMI study was prepared by specialists like John Fauvel and Jan van Maanen with the help of Abraham Arcavi, Evelyne Barbin, Alphonse Buccino, Ron Calinger, Jean-Luc Dorier, Florence Fasanelli, Alejandro Garcadiago, Torkil Heiede, Victor Katz, Manfred Kronfellner, Reinhard Laubenbacher, David Robertson, Anna Sfard and Daniele Struppa.

- d. The discussion document is circulated internationally as widely as possible in journals, magazines, newsletters, etc. Readers are invited to respond in writing to the IPC which then organizes an international study seminar with a limited number (50–100) of invited participants.

The Discussion Document for this ICMI study appeared, for example, in the Newsletter of the HPM, the ICMI *Bulletin* 52 (June 1997) and the website: <http://elib.zib.de/IMU/ICMI/bulletin/42/icmiHistoricalStudy.html>

- e. The ICMI study seminar forms a working forum for investigating the theme of the study. This forum is to be composed of experts in the field and newcomers with interesting ideas or promising work in progress, as well as representatives with a variety of backgrounds from different regions, traditions and cultures

Between seventy and eighty participants were invited to attend the study seminar held in Luminy, France, during April 1998.

- f. The final outcome of an ICMI study is a study volume, a carefully structured and edited book, not solely conference proceedings.

As always, Kluwer Academic Publishers published the ICMI study volume: its title was *History in Mathematics Education* and it was first presented at the International Congress of Mathematics Education in Japan in 2000

The volume (1)

History in Mathematics Education (Fauvel and van Maanen, 2000) is the product of the work of sixty-two contributors working together on eleven topics, which then became the following chapters:

1. The political context
2. Philosophical, multicultural and interdisciplinary issues
3. Integrating history: research perspectives
4. History of mathematics for trainee teachers
5. Historical formation and student understanding of mathematics
6. History in support of diverse educational requirements: opportunities for change
7. Integrating history of mathematics in the classroom: an analytic survey
8. Historical support for particular subjects
9. The use of original sources in the mathematics classroom
10. Non-standard media and other resources
11. Bibliography for further work in the area

It is clear already from the title of the chapters that the book covers a diversity of issues concerning the relationship between history and pedagogy of mathematics. As expressed by Fulvia Furinghetti (2001), the now-current chair of HPM:

the book contains a plurality of voices and is a unique fresco of opinions and experiences. (p. 2)

The topics are connected to each other and one of the book's achievements is precisely *not* to try to break these connections. For example, the reader interested in mathematics teacher education may find important ideas in Chapter 4 – the chapter explicitly dedicated to this issue – but also in Chapter 7 in which there is an exhaustive survey of reasons for integrating history of mathematics in the classroom and in Chapter 10 in which there is a list of web-based historical resources for the mathematics teacher

The main purposes of this study volume were to:

- (i) survey and assess the present state of the whole field;
- (ii) provide a resource for teachers and researchers, and for those involved with curriculum development;
- (iii) indicate lines of future research activity;
- (iv) give guidance and information to policy-makers about issues relating to the use of history in pedagogy. (p. xvi)

Towards the first purpose and the last one, Chapter 1 eclectically summarizes the experience of sixteen countries via their outlines guiding the inclusion of the history of mathematics:

- (a) in the school mathematics curriculum;
- (b) in textbooks;
- (c) in teacher education colleges.

The final section of this chapter, entitled *Policy and politics in the advocacy of a historical component*, is oriented toward different groups involved in the decision-making process of integrating history of mathematics in mathematics education: political authorities, teacher associations, professional mathematicians, teacher educators, textbook authors and parents.

Reading this section reminded me of the aphorism:

For those who believe, no proof is necessary.
For those who don't, no proof is possible.

Chapter 2 introduces philosophical issues concerning mathematics and mathematics education, as well as cultural and interdisciplinary aspects. I strongly recommend reading the opening pages (pp 39–45) as an initial assignment in a teacher-training course on philosophy of mathematics.

For example, teacher educators may find the following paragraph from the beginning of the chapter inspiring:

There is a *history of documents* and a *history of ideas*. The latter needs the former, but didactics and epistemology need the latter. This means that we must avoid the identification of philosophy of mathematics with mathematical logic. Our philosophy must guide and explain educational choices; it must help in a better planning of teaching. It must be open to new reflections (p. 39)

This chapter may encourage reflection on key issues like the nature of mathematics, the development of mathematical knowledge in the individual and society, as well as the (false) dichotomy of invention versus discovery concerning mathematical ideas.

Another of the purposes of this ICMI study is to indicate lines of future research activity. This volume indeed invites further research, not only of the history of mathematics as a subject matter area in its own right, but also research on the impact of the use of the history of mathematics in teaching and learning. In that sense, this book may be of particular interest to graduate students or young researchers looking for a topic for further research.

Specifically, Fauvel and van Maanen offered what I believe is appropriate advice for future research in this area:

The question of judging the effectiveness of integrating historical resources into mathematics teaching may not be susceptible to the research techniques of the quantitative experimental scientist. It is better handled through qualitative research paradigms such as those developed by anthropologists (p. xvii)

I would like to agree with the contributors to Chapter 3:

When we learn about the historical development of mathematics, it affects how we think about the time our students spend in developing mathematical understanding. (p. 65)

But I did not find any reference to research intended to justify this assertion. Although it is a rich chapter, I consider its title *Integrating history: research perspectives* misleading, because almost no research in mathematics education is mentioned, simply research on the history of mathematics itself and anecdotal reports of teachers interested in the use of history of mathematics in the classroom.

Chapter 5 presents an analysis of the historical formation of mathematics and student understanding. It provides a much-needed link between ideas from the epistemology of mathematics, psychology and philosophy. Though the profound ideas discussed maybe difficult to grasp, the reader will be well compensated by her/his enriched understanding of the learning process of mathematics.

Chapters 7 and 8 provide a closer look at different experiences of the integration of history of mathematics in the classroom. Chapter 7 emphasizes the 'how' and Chapter 8 the 'what'. They are very well integrated and the reader may benefit from the interconnected, marked references.

I especially enjoyed reading the chapter on the use of original sources in the mathematics classroom. It includes an enlightening analysis of hermeneutics and language (pp. 298-299). As a teacher educator, I used to present medieval Hebrew mathematical texts to my students to develop their 'cultural understanding' (p. 292) and to make them think about their own views of the subject. But this chapter, and the entire book in fact, lacks sufficient reference to Arabic mathematics. I think original texts written in Arabic need to be highlighted and the research community has to bring them to the attention of Arabic-speaking mathematics learners.

Following the discussion of the use of original texts, the motivations, the aims and the presentation of possible strategies for integrating original sources in the classroom, the editors evidently found it important to include a chapter presenting what seems to be the opposite pole: the integration of history of mathematics in the classroom through the use of new technologies like computer programs and the world-wide web, as well as dramatization. This chapter includes two pearls: the use of ancient instruments in the modern classroom (pp. 343-350) and web-based historical resources for the mathematics teacher (pp. 362-370). I consider them pearls in two senses: first, because of the originality of the idea and the information included and, second, because of the way it is presented.

In that sense, this book may be of great interest for secondary school mathematics teachers interested in new teaching strategies and for teacher educators willing to encourage *didactical reflection* among their students through their exposure to historically-based teaching ideas.

As John Fauvel points out in the closing chapter:

A considerable amount of work has been done in recent decades on the subject of this study, which is here [in Chapter 11] summarised in the form of an annotated bibliography, for works appearing in eight languages of publication. [Chinese, Danish, Dutch, English, French, German, Greek, Italian] (p. 371)

It was surprising to me to find that works written in Spanish or Portuguese were missing

Concluding remarks

The purposes of this ICMI study as expounded were, in general, attained. However, in my opinion, the most important quality of this book is its humanity: it is a book written by people, thinking of people and respecting them. I want to join Jan van Maanen who, in a tribute to the recently deceased John Fauvel [2], wrote:

It confirms my belief that a regard for the history of mathematics can generate in one a *warm and humane attitude* that will show up not just in the intellectual commitment in the discipline but also in other aspects. (p. 7)

Francis Bacon once wrote:

Some books are to be tasted, others to be swallowed, and some few to be chewed and digested.

I believe the reader will agree that *History in Mathematics Education* is a book of the final variety.

Note

[1] In this piece, unless otherwise specified, every citation is from this volume.
[2] John Fauvel, at the time of his tragically early death, was preparing a piece for FLM 21(2) about the way David Wheeler had significantly encouraged this area in this journal. [ed.]

References

- Fauvel, J. and van Maanen, J. (eds) (2000), *History in Mathematics Education*, Dordrecht, Kluwer Academic Publishers
Furinghetti, F. (2001) 'Remembering John Grant Fauvel', *HPM Newsletter* 47, 2-3.
van Maanen, J. (2001) 'John Fauvel - a tribute', *HPM Newsletter* 47, 6-7.

Appendix - ICMI study volumes

- 1 *The Influence of Computers and Informatics on Mathematics and its Teaching* (Richard Churchhouse et al. (eds), 1986)
- 2 *School Mathematics in the 1990s* (Geoffrey Howson and Bryan Wilson (eds), 1986)
- 3 *Mathematics as a Service Subject* (Geoffrey Howson, Jean-Pierre Kahane, Pierre Lauginie and Elisabeth de Turckheim (eds), 1988)
- 4 *Mathematics and Cognition* (Pearla Neshet and Jeremy Kilpatrick, (eds), 1990)
- 5 *The Popularization of Mathematics* (Geoffrey Howson and Jean-Pierre Kahane, (eds), 1990)
- 6 *Cases of Assessment in Mathematics Education* (Mogens Niss (ed.), 1993)
Investigations into Assessment in Mathematics Education (Mogens Niss (ed.), 1993)
- 7 *Towards Gender Equity in Mathematics Education* (Gila Hanna (ed.), 1996)
- 8 *Mathematics Education as a Research Domain: a Search for Identity* (Anna Sierpinska and Jeremy Kilpatrick (eds), 1998)
- 9 *Perspectives on the Teaching of Geometry for the 21st Century* (Carmelo Mammana and Vinicio Villani (eds), 1998)
- 10 *History in Mathematics Education* (John Fauvel and Jan van Maanen (eds), 2000)
- 11 *The Teaching and Learning of Mathematics at University Level* (Derek Holton (ed.), 2001)