MATHEMATICS AND THE STEEL DRUM

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Trinidad and Tobago is a twin island Republic at the southernmost end of the Caribbean archipelago. It has a multicultural population, a consequence of a history of various agricultural labor experiments – namely slavery and various forms of indentureship during successive waves of Spanish, French and British colonization. The mathematics curriculum, however, continues to be dominated by a Eurocentric view of mathematics. This perspective is exemplified in terminal examinations that model the English school-leaving-examinations, in mathematics textbooks that give little insight into the socio-historical development of mathematics and that offer no connections to the varied histories of students, and in pedagogy that is aimed primarily at helping students to be successful in formal examinations. Such is the enduring legacy of colonialism.

Like other small developing nations, Trinidad and Tobago is concerned about the mathematical competence of its population, in the face of high failure rates on terminal examinations, low rankings in international comparisons, and a pervasive attitude that mathematics cannot be done by most people even though it is deemed important (Republic of Trinidad and Tobago, Ministry of Education, 2002). In attempting to address some of these issues, Trinidad and Tobago has adopted a reform oriented national curriculum via the Secondary Education Modernisation Programme (SEMP). The curriculum aims to provide opportunities for all students to "develop an understanding and appreciation of the diversity of our culture [and] for beauty and human achievement in the visual and performing arts" (Republic of Trinidad and Tobago Ministry of Education, 2002, §1:5–6). In mathematics, these aims underscore the desire for students to experience the structure, elegance and power of mathematics while connecting mathematics to their interests and experiences and appreciating its role in aesthetics (Republic of Trinidad and Tobago Ministry of Education, 2002, §2:3). These sentiments express a desire to expand the scope of what is considered relevant to mathematics education in the 21st century, to loosen old colonial ties, and to increase student enjoyment and achievement in mathematics. However, there are as yet no examples of what might constitute the subject matter and how such a programme might be enacted. The exploration undertaken in this paper is a preliminary attempt to theorize one possibility – the steelpan/drum – that, while particular to the Trinidad and Tobago context, might resonate in other locations and with other learners.

Steel pan history

The steep pan/drum has "a haunting, ethereal, sweetly ringing tone that sounds like nothing of this earth". [1] More affectionately known as *pan,* it is the national musical instrument of Trinidad and Tobago. The instrument emerged from the discarded accoutrements of an industrializing society in the depressed urban areas surrounding the capital city some 60 years ago. Although drums play an important part in the cultures, religious expressions and celebrations of both Africans and East Indians, drumming was banned in 1884 by the then-British government out of a fear that it could be used to communicate uprisings. In the poverty-stricken areas of Laventille and John John, however, drumming continued in open defiance as individuals sought to retain their religious rites. In the 1930s these urban youths began to experiment with the cast offs of industrial society – metallic trash cans, paint cans and biscuit tins. From these humble origins, a new instrument was born.

The development of the steel pan was a non-linear, discontinuous, evolutionary process to which many persons contributed. As one website puts it, the "early development of the steel pan was not the outcome of a stroke of genius of a single individual, but the product of socially outcast communities groping for self-expression". [2] Initially these metal percussion instruments produced only rhythms. However, it is thought that someone, probably Winston "Spree" Simon, discovered while pounding to restore the shape of a drum that different pitches could be produced if the surface were shaped in different ways.

The earliest pans had as few as four notes, but the range was gradually expanded through experiments involving heating and pounding. Later discarded oil drums were used, because their larger surfaces could accommodate more notes (Brown, 1990). More and more notes were produced and eventually complete scales were formed, ranging from the lowest bass to the highest treble. By 1946, the lead pan had evolved to 14 notes but was not tuned chromatically. [3] Other subsequent innovations involved the discovery that more notes could be produced if the surface were concave rather than convex, the introduction of double drums (two drums tuned together to form one tuned double pan instrument), and the development of chromatic tuning that extended the tonal range to five octaves. Bore pans, in which small holes are bored to separate the tonal zones or notes from one another, are another recent innovation that allows for brighter, more resonant tones (Johnson, 1998).

Steelbands or steel orchestras represent the greatest expression of the diversity and versatility of the pan family. Large conventional steelbands have an upper limit of 120 players in the Panorama competition [4], which according to Johnson (1998) is "half again larger than a fully constituted orchestra" (p. 66). Johnson also notes that as "pan acquired more notes, which were in turn tuned more accurately steelbands acquired more pans, which gave them a wider and ultimately symphonic range" (p. 63). This range, I think, is one of the surprises that international audiences have on hearing the pan, that it is possible for entire symphony pieces to be played on a single type of instrument. Johnson captures this surprise, much like the mathematician's first appreciation of a proof, when he reminisces that, "the shock of recognition and pleasure can never be repeated" (p. 69).

Socio-culturally, pan and pannists have had a difficult journey in gaining acceptance and legitimacy as a new musical art form. According to Brown (1990), "[n]ot only were they offensive to the British on aesthetic grounds but on moral ones too … the early steelbands were perceived as lower-class gangs, not as musical ensembles" (p. 92). Violent inter-band street rivalries were part of the early evolutionary history of the instrument, and this past has occasionally dulled the luster of both the instrument and pan pioneers. However, as intimated by its name, pan is now a global phenomenon, played and enjoyed by persons of all ages.

In light of the rich historical heritage of the steel pan movement and the goals espoused in recent curriculum documents, I have often wondered about the potential of pan as a domain for mathematical investigations. It is a visually inspiring mathematical masterpiece with its elliptical and circular notes on a chromed sonorous parabolic surface and its deformed cylindrical shape bounded by the circle, that classic image of geometric perfection. But what can we learn from the instrument itself and the way it is played that might be mathematically rich, yet accessible to an elementary or early secondary school child? I ask these questions from the position that the origin and expressions of pan are rooted in the human, cultural and the social. A mathematical exploration of this sort should attempt to honor this legacy.

I find the following problem interesting and potentially rich. How is it that many of the early pannists who could not read formal sheet music were able to play complex scores? Consider historian and cultural critic Kim Johnson's (1998) comment that

> steelband music … is highly orchestrated, and the method of rote learning requires weeks of practice. Thus, even after a tune has been mastered, that lasts only as long as the player's recall. A year later it is gone. (p. 67)

The playing and enjoyment of steelpan is not merely an aural experience but is an extremely visual and embodied one as well. Thus, it is my conjecture that learning to play a score involves not only tonal awareness but also some degree of spatial cognition and spatial memory. [5] The movement of the pan sticks, understood as extensions of the drummer's hands and fingers, is not linear as in most traditional orchestral instruments (*e.g.,* piano, flutes). Rather, this movement creates ephemeral 'shapes' in the hollowed out space over the surface of the instrument – a space into which the player descends, a space into which the listener is invited, a space in which a community is formed and perhaps can be renewed.

In attempting to theorize mathematical engagement around pan, I draw on ethnomathematics and embodied mathematics perspectives. I believe they provide suitable positions to begin to work towards achieving the goals set out in the SEMP curriculum.

An ethnomathematical perspective

There is no single ethnomathematical perspective and no simple answer to the question, "What is ethnomathematics?" (Rowlands & Carson, 2002). This lack of a singular definition is a reflection not merely of the differences of opinion among the many advocates of ethnomathematics. It also represents a healthy diversity of interests, foci, methodologies and agendas of various researchers and practitioners. Several definitions are offered in the literature (*e.g.,* Adam, Alangui & Barton, 2003; Eglash, 1997. I align my own opinion with that of Adam *et al.* (2003) who stress that "ethnomathematics is not a pedagogic philosophy. Rather it is a lens through which mathematics itself can be viewed" (p. 329). This conception incorporates D'Ambrosio's (1997) view that such a lens must also be turned to look on other socio-cultural constructions and phenomena, such as pan.

Barton (1996) lists and describes four tasks that are considered relevant to ethnomathematical projects: descriptive activity, archaeological activity, mathematising activity and analytic activity. The previous section on pan's history and development is an attempt to meet the first two criteria. However, Pimm (2001) asks that we consider the very important and relevant question when mathematizing art:

> [I]n what sense are we mathematising the work and in what way … are we remathematising it? In other words to what extent did mathematics consciously play a role in the creation of the piece in the first place; to what extent is mathematics designed in? (p. 32)

Given that the first reaction to much of what is considered ethnomathematics is an aesthetic and affective response to the objects as a whole – and not as a mathematical work – Pimm's comments are a necessary caveat. Ethnomathematical activity can involve both mathematization and remathematization of the artefacts and practices of different cultures. However, it is the latter – the discovery or creation of correspondences between the original intent of diverse cultures of peoples with our own mathematical language and culture – that respects the integrity of both sets of traditions. While the ethnomathematics literature says very little explicitly about instruction, what it offers educators is a sensitizing concept with a potential for developing increased cultural sensitivity and respect for cultural diversity – goals laid out in the national curriculum.

Embodied perspectives

Playing pan is more than a cerebral affair. The musician's entire body is involved, and this engagement serves as an interruption to the belief that mathematics is a universal, abstract and disembodied construction – what Lakoff and Núñez (2000) call the "Romance of (Western) Mathematics". This romance infuses many school pedagogical practices and it supports conceptions of mathematics in which symbolic forms of reasoning are privileged over other types of reasoning. Rooted in the Cartesian mind/body dualism, this romance is rejected by theories of embodied cognition and, more specifically, embodied mathematics (Anderson, 2003).

Embodied perspectives hold the view that human cognition is body-based and thus question the extent to which sensorimotor processing is implicated in cognition. Anderson (2003) argues that it is specifically this concern for the bodily grounding of cognition that distinguishes embodied cognition from related situated perspectives. Investigations of the way in which the body and cognition interact is a growing area of research. For example, Picard *et al.* (2004) claim that, through using the Logo Turtle,

> children learn important geometric ideas in a more 'body syntonic' way … thus leveraging their intuitions and experiences of their own bodies into more formal knowledge and into a more personal relationship with mathematics. (p. 262)

They refer to other projects at the MIT Media Lab that

> have contributed to expanding the range of ways in which the body can be morphed into mathematics … in which the body in motion can support intuitive, emotionally deeply interconnected conceptual realms. (p. 262)

Such research demonstrates at a very practical level the importance of the body in doing and learning mathematics.

Embodied perspectives, such as ethnomathematical perspectives, also draw attention to the fact that much of human activity that might not be seen as formal academic mathematics probably involves implicit mathematical ideas – for example recursion and pattern formation, especially evident in various drawing traditions (Ascher, 2002; Demaine, Demaine, Taslakian & Toussaint, 2006; Gerdes, 1996; Stathopoulou, 2006). As such, these sorts of bodily and sensory experiences, already familiar to learners, could be used to establish connections to more formal mathematical ideas. I conjecture that the playing of complex scores by pan musicians, untrained in traditional music theory, is partly accomplished through memorization that depends on the close relationship between their hand movements, aural feedback and simple geometrical motifs.

Emerging from the perspectives on ethnomathematics and embodied mathematics discussed previously I attempt a mathematisation of the situated cultural practice of playing pan, and suggest possible correspondences in need of further empirical and phenomenological investigation.

Mathematising pan

I begin with one of the simplest of cases: A single stick played by a single player on a single drum. Later I will describe how this could be extended and suggest that the situation that arises is complex. I begin by searching for patterns or motifs in a song used to instruct beginning pannists, *Joy to the World,* the notes for which are:

> CBAG / FEDC / GA / AB / BC/ CCBAGGFE / CCBAGGFE / EEEEFG / FEDDDDEF / EDC / C# / AGFE / FEDC

I have decided to illustrate the sequence of notes by shaded vectors and to illustrate on the most common pan, the C-lead pan. Further I limit this foray into notes arranged only on the outer whorl, to avoid complication at this early state. Consider the first eight notes played as two sets of four.

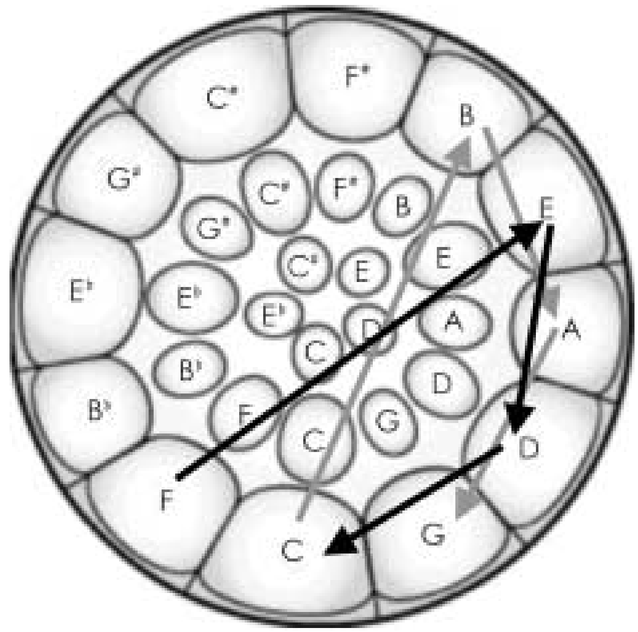

Figure 1: The pattern of CBAG / FEDC

One sees that the first four notes (CBAG) produce a structure resembling an open trapezium. Further the second four notes (FEDC) repeat this structure, with the original trapezium being rotated or displaced through one note. The rotation preserves size and shape – but unlike a strict geometric rotation, there is a change in a qualitative dimension when one considers that what is being rotated is not merely a figure but a sequence of sounds/notes. What qualities does this transformation preserve musically and why? The next sequence of notes GA / AB / BC reverses the first set of notes played (CBAG) and returns us to the starting note. We might also begin to think of the trapezium as a one-to-one mapping or function as each note under the mapping maps to another note and every note has an inverse mapping back to the original note. We can also see that, since the relation maps to the note adjacent and to the right, repeated application of this trapezium mapping will take us through every note in an anticlockwise fashion. We could set it up as a permutation.

$$\text{Trapezium 1} = \begin{pmatrix} C\ F\ B^{\flat}\ E^{\flat}\ G^{\#}\ C^{\#}\ F^{\#} B\ E\ A\ D\ G \\ G\ C\ F\ B^{\flat}\ E^{\flat}\ G^{\#}\ C^{\#}\ F^{\#}\ B\ E\ A\ D \end{pmatrix}$$

This would represent one possible permutation of four notes from the 12 available. Other permutations would produce different structures – and one might, with more advanced learners, begin to explore some relationships with group theory. One might also be interested in which structures produced the most appealing sounds or in identifying all the permutations given by specific structures.

Within the next sequence of eight notes the original sequence of four notes is repeated (see Fig. 2). Visually the structure contains 2 copies of the trapezium motif, with the side AG being common to both and the second running in the reverse direction.

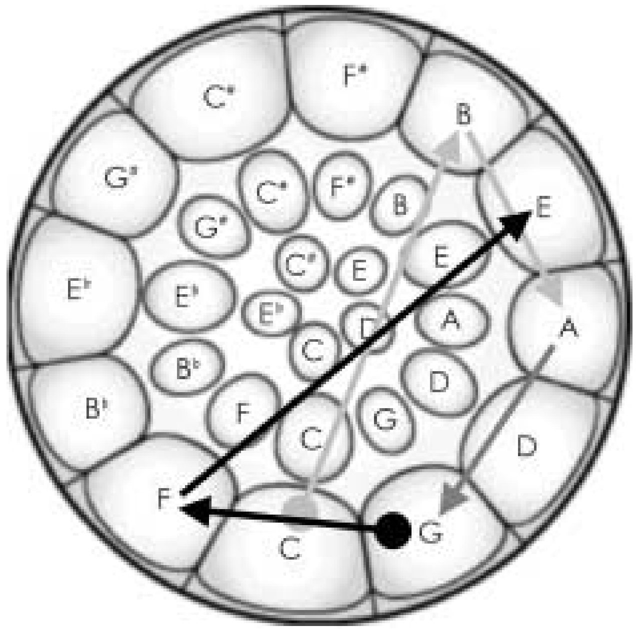

Figure 2: The pattern of CCBAGGFE

The fifth sequence of notes produces the pattern illustrated in Figure 3. This again has part of the basic trapezium motif that we have been seeing though the side GA is absent. It might be considered an open triangle.

The sequence of notes depicted in Figure 4 is palindromic and could be used as a jumping-off point for investigating these beautiful and mathematically rich patterns. Once again part of the trapezium motif can be identified (the side DC is absent). In Figure 5, illustrating the final two sequences, we observe the same trapezoidal structure. However they are related to each other by rotation and reflection. This is a more complicated situation.

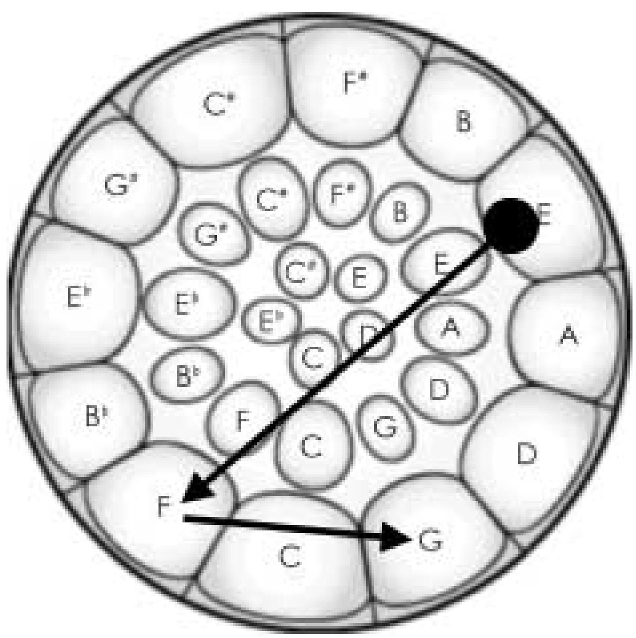

Figure 3: The pattern of EEEEFG

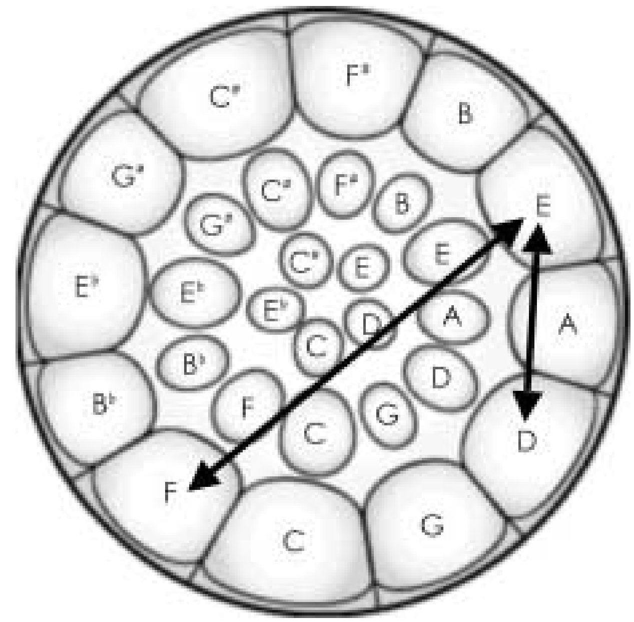

Figure 4: The pattern of FEDDDDEF

In this simple musical composition a trained musician might spot the recurring motifs. However this visual exposition, I argue, demonstrates one conceptualization that might enable individuals with no formal musical training to overcome the limitations of working memory. One might go further to analyze common patterns found in some of the pieces written specifically for pan – for example longer sequences that require more complicated movements and that would result in more complex patterns, or categorizing patterns that produce clearly identifiable and desirable sounds. One might also attempt to mathematise these patterns further.

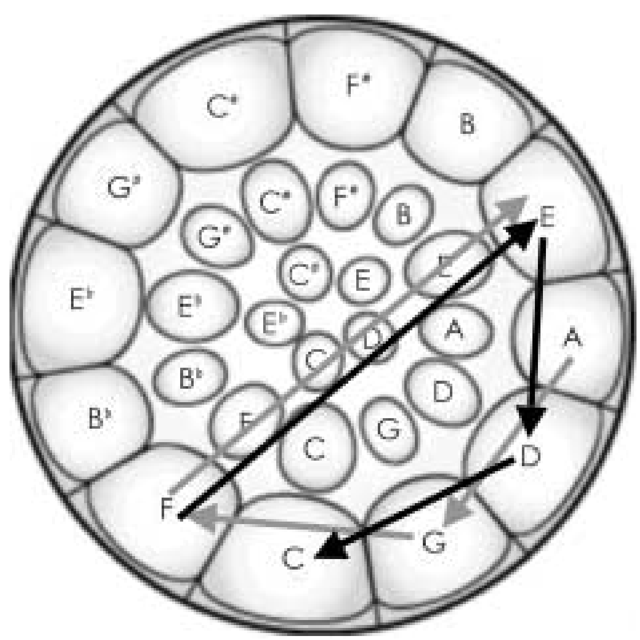

Figure 5: The pattern of AGFE / FEDC

Complication and complexification

So far, my exposition has been limited to a single stick on a single instrument. I will now complicate matters by admitting that pannists generally play with two sticks and virtuosos may play with up to four. To further complicate matters, the double pans involve two instruments and afford a wider range of notes. In some steel orchestras an individual may move between as many as four or more pans. My analysis also ignores the fact that there are two other whorls of notes. Thus the motif I have described may not be the actual way that a pannist organizes his/her movements. An interesting question for research would be to examine how pannists (who are not trained to read music) actually go about remembering complex sequences of notes. Further, given the availability of copies of the same note, it is possible that different pannists may play the same sequence of notes by minimizing movement across the surface of the drum by utilizing the inner whorls – giving us an optimization-type problem. Again, these are conjectures in need of investigation and validation or rejection. Not only do these complications make the problem more difficult, they make the problem more complex.

My final claim is that steel orchestras might be understood as embodied complex systems, and this claim has potentially interesting implications for pedagogy. The evolution and development of the instrument itself shows some of the elements of complexity: diversity, redundancy, selection, and a lack of central control (Davis & Simmt, 2003). It is interesting that the development of pan involved many individuals working co-operatively with innovations spreading quickly and without a central organization. Johnson (1998) describes a similar process at work in the arrangement and composition of steelband pieces. The process relies not on formal symbolic representation but on a dynamic evolutionary principle in which the arranger

> is there in the panyard with the band, and the tune *grows* as it is practiced, with players making contributions, the arranger tailoring his piece to suit the skill of the band and the taste of the players … the tune changes according to … comments and the way it *feels* when it is performed before the crowd. (Johnson, 1998, p. 67, italics added)

Substitute 'teacher' for 'arranger', 'classroom' for 'panyard', 'learners' for 'players' and 'mathematics' (or another subject) for 'tune', and a potentially powerful and radically different type of instruction emerges. What more might we learn from exploring embodied, culturally situated, mathematical pedagogical performances that are oriented by similar evolutionary and pantisocratic dynamics?

Like other complex systems, these performances exhibit what Johnson calls "the production of aesthetic transience" (p. 69) – an ephemerality and irreproducibility that is appreciated (and perhaps exists) only in the moment it is heard. Johnson argues further that such transience derives its value from its temporality and the relationship between performers and audience as

> the momentary communion of a musician and his audience is intrinsic to the aesthetic Africans brought to the New World…. A call-response structure involves both parties in the creation of the music, removing the barriers between them. (p. 72)

This value system stands in contrast to the need for formal notations in European music. Such notations emerged with the increasing complexity of scores and the desire to preserve beautiful forms. In the Western world, they contributed to the eventual dominance of composers over performers. More broadly, these notations contributed to the dominance of written scores over the production of inventive, complex, improvisational artifacts intended for immediate consumption or destruction by non-Western cultures.

The embodied mathematics that occurs in a steel orchestra might be considered a call or invitation to community – a call that might be more appealing to some learners than the traditional call of mathematics, with its emphasis on re-creating over-formalized eternal (Western) forms. It is a call to celebrate the aesthetic of the present, while recognizing the historical rootedness of our embodied actions. What if this value of aesthetic transience were brought in to mathematics educational practices? What would it mean to really value the ephemerality of the mathematics produced by students and their teacher in their inter-actions in the irreproducible pedagogical moment? Perhaps, as Henle (1996) suggests,

> mathematics should be taught as music is taught. Students should make mathematics *together*, not alone … students should perform mathematics; they should *sing* mathematics and *dance* mathematics. (p. 28)

What happens in a steel orchestra, I argue, can be understood as a type of this culturally situated embodied mathematics. As such it could be used as a vehicle for the exploration, creation and performance of inter-cultural, mathematical, musical, aesthetic and ultimately human ideas among students of all ages, particularly in the primary and early secondary school.

There are numerous mathematical studies of music (*e.g.*. Henle, 1996; Toussaint, 2005) and several involving drumming (Belcher & Murrell, 2006; Hall, 2004; Sharp & Stevens, 2007). [6] However, as Henle (1996) notes, such studies "do not explain the real affinity between mathematics and music…. It is the cultural context that matters not abstract principles" (p. 19). Mathematical projects involving the steel pan should strive to be true to the cultural context in which the pan emerged and was nurtured. Such studies might contribute to achieving some of the desired outcomes in the SEMP mathematics syllabus as it relates to mathematics and the visual and performing arts. This sort of program is necessarily collaborative and distributed, continuing to exhibit some of the properties of complex systems that gave rise to the pan movement.

There is also an ethical dimension to this proposal for a sustained program of mathematical investigations, based on and inspired by the pan as a contributor to peace, justice and security in a land historically and currently marred by physical, psychological and structural violence. The first is reparative in undoing the historical neglect of the genius of pan pioneers, players and pedagogues by recognizing, understanding and appreciating their achievement through the social lens that is formal mathematics. The other is restorative and, perhaps, transformative. Trinidad finds itself struggling to come to terms with a statistically incomprehensible level of crime and brutality – more than a murder a day – much of it perpetrated by rival gangs of school age young men from impoverished areas not unlike those from which pan emerged. A newspaper reports that unaffiliated youth in these 'war-zones' sought solace and sanctuary in the Panyard of their community. [7] The pithy picture accompanying the report, shows a young man "concealing himself behind a steelpan". If this is where the learners are, this is where mathematics education (in Trinidad and Tobago) must courageously go – if it is ever to be "for all". [8]

Acknowledgements
I would like to thank William (Bill) Higginson for providing some of the initial stimulus for this paper a version of which was presented at CSSE 2006 at York University, Canada, and the thoughtful comments of the two anonymous reviewers.

Notes
[1] Reich (1999) 'Steelpan alley', *Chicago Tribune* August 15, pp. 3–6.
[2] Steel Island. (2000) 'History', http://www.steelisland.com/history.asp, accessed 11/06/07.
[3] Ibid.
[4] Panorama is the National competition for steel orchestras. It is held during the Carnival period.
[5] I have not yet been able to locate any reports that have examined the memorization strategies of pannists.
[6] See also: Belcher, J. and Murrell, J. A. (2006) Group theory, L-systems and African Rhythmic Structure [Abstract], Available at http://www.math.bu.edu/people/ep/AFRAMATH06/belcher-p.pdf, accessed 11/06/07.
[7] Connelly, C. (2008) 'Youths take shelter in panyard', *Trinidad Guardian* February 16, p. 5.
[8] I have begun to explore these ideas with undergraduate teachers and art/music educators in Trinidad and look forward to reporting on this aspect in the future.

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Anyone for tennis?

Take eight players, ranked in order of merit, and organise a tournament with 1 playing 2, 3 playing 4, 5 playing 6, and 7 playing 8. Then the winners of the first round will be 1, 3, 5, 7, and those of the second round will be 1 and 5; the final will then be won by player 1, defeating player 5 who wins the second prize but actually started in the lower half of the ranking. To avoid this difficulty, devise a method for re-scheduling all the rounds so that the first three prizes go to the best three players.

[from Lewis Carroll's "Lawn Tennis Tournaments: The True Method of Assigning Prizes with a Proof of the Fallacy of the Present Method" (1883)]