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The cover illustration shows a closed curve generated from Fourier descriptors. These are the coefficients of the Fourier series representing the difference between the angular direction of the curve and that of a circle, where both are parametrized by arc length. In our example the amplitudes are 1 and 2 in the fourth and eight harmonics, respectively, with the two amplitudes 90° out of phase.

This representation of curves is useful for shape discrimination in the recognition of printed characters. For details see C. T. Zahn and R. Z. Roskies, "Fourier Descriptors for Plane Closed Curves", *IEEE Trans. Comp.*, v. C-21:3, 269-281 (1972).

The computer drawing is by Eric Regener of Concordia University.

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Toward a Philosophy of Computation*

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* The text of a talk to the members of the Canadian Mathematics Education Study Group at Queen's University, Kingston, June 1982

Ladies and Gentlemen:

I am delighted to be here this evening and I am greatly honored by your invitation to speak. When the arrangements committee asked me to talk about the role of the computer in mathematical education I drew back. Although for the past fifteen or twenty years I have been propagandizing for more computer involvement in the college mathematics curriculum, my current concern is less about the specifics of courses, of hardware and software, than about the philosophical implication of the increasing mathematization and computerization of the world. At any rate, the committee was agreeable to a broader and fuzzier address, and out of it, hopefully, may come something of use in education.

The word "toward" in the title of the address emphasizes that at the present moment there is hardly such a subject as "the philosophy of computation"; it also stresses my own inadequacies in the philosophical line. When I have done speaking, I shall not have presented a systematic and fully developed position, but only a few groping remarks on what I consider to be the important issues that such a philosophy should deal with.

This Cartesian age

I need hardly tell this audience that computers and computer science is where the action is now. In this area there is activity, ideas, motivation, energy, enthusiasm, money, jobs, prestige. The accomplishments of computers span the range of human concern. And, as if to make the whole enterprise even more characteristically human, there are now computer millionaires and computer criminals.

Will the field saturate? The present signs read: not in the forseeable future. But, of course, all this might have been said of the railroads 125 years ago.

There is considerable evidence that the brightest and the best students who in my generation would have elected a career in mathematics or physics are now selecting computer science. Mathematics is losing to computer science in a number of ways; gaining in others; and some computer imperialists assert that mathematics can not only be exploited by computers, but can also be created by them.

When did this computer revolution which sweeps us all along in its stream begin? Shall we say it began in 1944 when Howard Aiken and the Harvard group designed the Mark I? Shall we push the date back to 1889 when Herman Hollerith patented a machine for tabulating population statistics? Shall we move the date further back to 1833 to Charles Babbage's Analytical Engine or to 1805 with the automation of the Jaquard Loom?

Let me suggest that the computer revolution began on November 10, 1619 in a well-warmed room in a small German village. On that date, René Descartes, a Frenchman, then twenty three years old, experienced a revelation. We may, with some justification, date the modern world from that moment.

The experience, Descartes tells us, was preceded by a state of intense concentration, excitement and agitation. His mind caught fire. He was possessed by a Genius and an idea was revealed to him in a dazzling and almost unendurable light. Later, in a state of exhaustion, he went to bed and dreamed three dreams that had been predicted by the Genius. He was so struck and so bewildered by all of this that he began to pray. He vowed that he would go on a pilgrimage. He vowed that he would put his life under the protection of the Blessed Virgin.

What was the idea that Descartes saw in a burning flash? It was no less than the unification and the illumination of the whole of science, the whole of knowledge even, by one and the same method. And this method would be applicable in an almost automatic fashion.

As with all good programmers, Descartes provided us with uneven documentation.

Eighteen years passed before he gave the world more details of this grandiose vision; such as he gave are contained in the famous "A Discourse on the Method of properly guiding the Reason in the Search for Truth in the Sciences; also: the Dioptrics, the Meteors and the Geometry, which are Essays in this Method". In this "Discourse" Descartes reveals that his method for "properly guiding the reason" is the method of mathematics. Mathematics, that is, the science of space and quantity, with the logical underlay provided by the Greeks, was the simplest and surest of all the conceptions of the mind. Why should it not form the proper basis for a universal method?

Descartes was a brilliant mathematician. He thought of himself first and foremost as a geometer and claimed that he was in the habit of turning all things into geometry. Ironically, though Descartes felt himself to be a geometer first, his method, by its very success, reduced geometry, in the visual and humanistic sense, to a minor role. The drive to quantification denigrates all that can not be thus treated. It splits the world into what is numerical and formalizable and what is not, and often compels the latter to masquerade as the former.

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As we know, the reduction of geometry to algebra by use of Cartesian coordinates represents a turning point in the history of mathematics and in the history of ideas. As the French poet and essayist Paul Valéry observed, "It won him the most brilliant victory ever achieved by a man whose genius was applied to reducing the need for genius".

In geometry itself, the culmination of the victory occurred in 1931 when the mathematical logician Alfred Tarski proved – in the theoretical sense at least – that all problems of elementary geometry can be "automated out".

The vision of Descartes became the new spirit. Two generations later, the mathematician and philosopher Leibnitz, with even worse documentation than Descartes, talked about the "characteristica universalis". This was the dream of the universal method whereby all human problems, e.g. those of law, could be worked out rationally, systematically, by logical computation.

In our generation these visions of Descartes and Leibnitz are implemented on every hand.

If we can locate a flow of information that we want to process in some logical sense, then we are inclined to call it computation. The view of computer science, currently in a state of extreme euphoria, is that practically everything is computation. This provides a unitary view of the universe. The slogan of the neo-Cartesians is

"Computo ergo sum".

I compute, therefore I am. This is neat. But it is also dangerous.

It confronts us with *loss of meaning*. Loss of meaning in our personal lives. Loss of meaning in how we relate to our fellow humans. Loss of meaning in how we relate to and interpret the cosmic processes. In science there are no theoretical truths, only models from which conclusions may be inferred with some reliability. In mathematics there are no truths, just paradigms of non-contradictory systems. In our social outlook, we are afraid to specify preferred behavior. We live by cultural relativism in the name of tolerance.

Against this background which has been developing for several hundred years, against the "Cogito ergo sum", the assumption that ultimate individual identity and meaning lies in the intellectual process itself, the computer scientists, the artificial intelligencers, the mechanical brain people, now stand up and assert that what we previous thought was uniquely human, our thought, our intelligence, is easily simulated by a silicon chip and a bit of electric current. We feel debased and diminished by this view. Though religions had assured us that we are dust and will return to dust, we imagined for ourselves a moment of intellectual glory in between when we thought, we reasoned, we computed. No wonder we shriek to the heavens when this uniqueness is threatened. We seek other avenues to meaning; through utility, through aesthetics, through ethics, through assertions of free will, through religious values, all of which seem momentarily free from the processes of digitalization. Or we allow ourselves to be coopted by the process, and assert that the medium is the message, the transformation of formal symbols is itself the only and ultimate meaning.

The philosophy of computation and its pursuit

What is the philosophy of computation? The subject hardly exists except as a subset of the philosophy of mathematics or as a label for interminable discussions as to whether the computer thinks. Perhaps we can help to create such a philosophy.

The word "philosophy" has a popular sense. One says: my philosophy of life is early to bed, early to rise makes one healthy, wealthy, and wise. Or, in computer science, one says: my philosophy of programming is never to use a "go to", or: my philosophy of ordinary differential equations is always to use the Runge-Kutta method. This is practice, strategy; and this is *not* what I want to talk about.

What, then, should the philosophy of computation talk about? Well, classical philosophy in the hands of Aristotle, discusses the true, the good, the beautiful. So we may ask, as starters: what is true about computation; why should I believe a computation? What is good about computers; why should I allow the computer revolution to continue in its course? What is beautiful about computation; where and in what way does the computer create and enhance aesthetic values? One knows also that classical philosophy discussed many other things; for example, the nature of knowledge and its relation to perception, the idea of perfection, the idea of the divine. These discussions, too, should be capable of admitting computer extensions.

Another philosophical question is *Does the computer think*? Is the brain a computer? What, in fact, is thinking? This question has preempted the bulk of the philosophical writings, and the infant philosophy of computation in many minds is synonymous with it.

There are many ideas, then, which the philosophy of computation ought to discuss and one should insure, especially in its early days, that it does not focus on some few at the expense of wide coverage. We can learn from the philosophy of mathematics. In the last seventy years, although this subject might profitably have discussed a wide variety of things, the philosophy of mathematics has come to be almost synonymous with discussion of the *foundations* of mathematics. The focus has primarily been on one question: why is mathematics true and how do we know it is true?

I believe there are two reasons for this preoccupation with an ultimately unprofitable question. The first reason is to be found in Bertrand Russell, in his work and personality. One of the very great figures in the modern philosoplhy, Russell lived an unusually long life and colorful life. Courageously eccentric, an extraordinarily brilliant writer when he chose to popularize, he was influential for more than two generations. Why did Russell stress the question about the truth of mathematics? We know the answer, because he tells us in his autobiographical writings. The intellectuals and scientists of his generation experienced a loss of faith in the revealed truths of religion. Religious truth having vanished for them, they sought to find a firm basis for truth in mathematics. Hence the question: is mathematical knowledge indubitable?

The second reason reflects a change in philosophers of science. A hundred years ago, the philosophy of science was pursued by the scientists themselves, men like Peirce, Maxwell, Mach, Whitehead, Russell, Poincaré, Hilbert; but around fifty or sixty years ago, philosophy of science became a subject in its own right, pursued by people who were not scientists, and who had limited personal experience with what it meant and felt to create and discover new things. In mathematics this second group latched onto the principal topic: foundations, and never let go. There is some evidence of a turn around. We have begun to see articles such as "Restoring the Mathematician to the Philosophy of Mathematics".

Some topics for the philosophy of computation

I have mentioned how the true, the good, and the beautiful might profitably be discussed within the context of world computerization. Let me now suggest some topics with a more technical aspect.

What is the relationship between mathematics and the computer? As mathematicians and educators, we ask "How has each enhanced the other?" "How has each conflicted with the other?" "What is the relationship between computable structures and existential structures, between the finite and the infinite?" For example: how is it that we are able to use the computer productively in mathematical analysis considering that the statements of real variable theory make no sense on a real world computer?

There are currently three distinct formulations of calculus which are lobbying for exclusive rights in the classroom. There is the traditional way of limit theory with its deltas and epsilons. There is the way of nonstandard analysis with its hyperreals, restoring infinitesimals at the cost of installing filters, ultrafilters, or ideals and quotient rings. There is the way of constructive, algorithmic, computable mathematics. Will the real calculus please stand up? All three are correct in context, but none of them provides an accurate description of what really goes on in the practise of numerical analysis.

What are the logical limitations of computation? How does the programming environment, sensibility and intuition, compare with that of mathematics? What is the relationship between language, symbols, mathematics and computation?

What is the relationship between computation and physics?

Should a theory of physics be computable? What reasons do we have for believing the Church thesis and the physical Church thesis? These theses assert that all computation is reducible to that which can be carried out on a Turing machine and its physical realizations. What are the limitations on computation implied by quantum physics, by relativistic physics, by thermodynamics? In view of the fact that in a model one part of the universe is modelled by another part (physical or mental), why should it all reduce to a very special kind of mechanism known as the digital computer? Is the computer just a big pencil and a big wad of paper or is it more than that? Does quantity and speed of computation change quality? Can quantity of computation go "critical" and become "supercomputation"? What is the relationship between time and mathematics and computation? Didier Norden has pointed out that "Mathematics has been defined as the one scientific subject in which time is irrelevant". Norden says this is an inadequate view, and so do I, but explication is required.

Finally, is the whole universe a computer? This is not a recent view put forward by computer enthusiasts; on the contrary, it goes back as far as Pseudo-Aristotle (*De Mundo*, 300 B.C.). According to this writer, God is a mathematician. He makes his will known through computations which drive the world in a mechanistic fashion. His will can be known if we study mathematics and make the proper models. Give or take God, this is still a popular view of the "unreasonable effectiveness" of mathematics in physics.

The way of dichotomy

There is yet another way in which the philosophy of computation might be developed. Open the fundamental document of Cartesianiam, the "Discours de la Méthode" to its title page. There, beneath the title has been placed a colophon which depicts a man cultivating his garden and acting under the divine inspiration of Jehovah. We may take this little picture to be symbolic of the split of the universe into the world of mind and the world of matter, or, in a later but parallel terminology, into software and hardware.

Philosophy is confronted with numerous dialectical dichotomies or splits. There is the mind/matter split (the distinguished philosopher of science Sir Karl Popper has recently found it useful to split this into three). There is the subjective/objective split. There is the finite/infinite. There is the deterministic/probabilistic, the spatial-kinaesthetic/symbolic-linguistic, the time/space, the temporal/eternal, the freedom/constraint splits, and numerous others. Do these dichotomies lead only to pseudoproblems or are they the *fundamental tensions* out of which the creativity of mankind flows?

At any rate, a description and interpretation of these splits as they operate within computation is a program of the first importance.

Why philosophize?

Speaking jocularly, one might say that all decent subjects need a philosophy. Art has a philosophy as does literature. Science has a philosophy, mathematics has a philosophy. Computer science needs a philosophy. In it would lie the cachet of respectability.

As a mathematician, living in a world that is being increasingly mathematized and computerized, I want to know where my subject stands with respect to this trend. I want to know what meaning I can give to the symbols that are created by humans and increasingly processed by machine, or created by the machine and consumed by humans.

Finally, there is the area of social concern. I want to know whether there is salvation or damnation in the computer. Or is it just one more thing that we play with? Claims have been staked out on either side of the line. The brilliant mathematician I.J. Good sees in the computer, and in the "superthought" of the "supercomputer", the only way out of the mess that mankind finds itself in. Here is the computer as messiah.

On the other side, the humanistic writer on technology, Lewis Mumford, worries that raw technology, computer driven, will destroy the human elements of civilization. Joseph Weizenbaum points to the instabilities created by the automation of decisions. A generation ago, the religious convert Simone Weil, sister of the Princeton mathematician André Weil, wrote "Money-mechanism-algebra. The three monsters of contemporary civilization". Money, mechanism, algebra. Put them together and they spell – not mother – but computers.

I began my talk with the vision of Descartes of a universal, automatic method. Cartesianism – rationalism – judges men by their activities. Systems of exclusive salvation, which the age of enlightenment overthrew, judged men by their beliefs. I live in the Cartesian age and am glad that I do. But actions, when formalized and automatized tend to become devoid of meaning. They lead to formalism and emptiness.

It is great to be a Cartesian; but one shouldn't push it too far.

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DISCOURS

DE LA METHODE

Pour bien conduire ſa raiſon,& chercher
la verité dans les ſciences.

PLUS

LA DIOPTRIQVE.

LES METEORES.

ET

LA GEOMETRIE.

Qui ſont des eſſais de cete METHODE.

A LEYDE

De l'Imprimerie de IAN MAIRE.

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Auec Priuilege.