ASSESSING UNDERSTANDING IN MATHEMATICS: STEPS TOWARDS AN OPERATIVE MODEL

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In recent years significant contributions have been made in the field of the assessment of understanding in mathematics, tackling questions such as how and under what conditions understanding could be assessed and what methods and techniques could be used Especially relevant among these contributions are the proposals that seek to assess understanding in terms of how mathematical knowledge is represented and what internal connections it involves (Hiebert and Carpenter, 1992; Castro *et al*, 1997), either by taking into account the way epistemological obstacles are overcome (Sierpinska, 1990, 1994) or in accordance to relationships with pre-established institutional meanings (Godino, 2000).

Also worthy of note are the methods and techniques that focus on creating profiles of understanding, both general (Pirie and Kieren, 1989, 1994) and specific (DeMarois and Tall, 1996), as well as recommendations that advocate analyzing the range of situations associated with different mathematical knowledge for purposes of selecting appropriate situations for assessment and achieving an acceptable level of validity in instruments for the diagnosis of understanding (Godino and Batanero, 1994; Niemi, 1996).

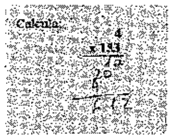
However, while most existing approaches tend to acknowledge the idea that observable manifestations are the most appropriate means of obtaining relevant information regarding a subject's level of understanding (Duffin and Simpson, 1997), the underlying complexity of this phenomenon makes the comprehensive interpretation of such intentional external actions an essential and ongoing difficulty in assessment. The dimensions of this problem can be illustrated, at least partially, in the following example involving elementary arithmetic.

Spheres of understanding in the standard [1] written algorithm for multiplying natural numbers

In the teaching and learning process of elementary arithmetic, there exists a well-reported gap between students' abilities to reproduce and mechanically apply calculation algorithms on the one hand, and their ability to recognise and be aware of the fundamental properties that justify them on the other. Some approaches even make use of the classical dichotomy between procedural (*technical*) and conceptual (*formal*) knowledge to establish appropriate characterizations and differences among learners in terms of their understanding. However, by observing and analyzing the way students use algorithms to deal with different situations that give such algorithms meaning, it becomes possible to identify a series of nuances and variations that paint a more complex picture of students' understanding.

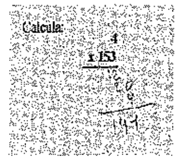
For the standard multiplication algorithm, the technical application includes three different facets, which can all be interpreted in terms of understanding:

1. the *recognition* of the algorithmic sequence
2. *skill*, understood to mean the appropriate and regular use of the algorithm
3. the *independence of the placement of factors* [2] in solving the problem, requiring students to be able to apply the algorithm even when the factors are not placed in columns in the traditional manner.



Tomás (14 years old)

Miguel Ángel (14 years old): Well, I put this number [153] which is the bigger one, up here on top and then I multiplied. [...] Because... I didn't know how to do it the other way. It looked strange. I've never done it that way



Matías (12 years old)

Figure 1: Independence of the placement of factors as indicator of technical understanding

For example (see Figure 1), Tomás (14 years old) gives indications of a more developed technical use and therefore of greater and more sophisticated technical understanding in independence of placement than Miguel Angel (14 years old) and Matías (12 years old).

In the formal sphere, it is possible to perceive significant differences among students in terms of their use and understanding of the algorithm.

An example is seen in the answers given by three students (Figure 2) to the formal question: *Why do you leave a 'gap' in the row for the second partial result of the multiplication problem?* In the explanation given by Clara (14 years old), which remains in the sphere of external relationships, no indications are found that she possesses formal understanding of the algorithm (Example A). In contrast, Carlos's response (14 years old), gives evidence of the formal use, albeit initial and eventual (Example B). Specifically, his unstable intuition that the 'gap' is a zero constitutes an aspect of formal understanding that is not manifested in the first example. On the other hand, the answer given by María Dolores (14 years old), which uses internal properties of the

[To provide visual support for this question, the multiplication problem 146 x 23 was presented to the subjects, using the standard written algorithm for multiplying natural numbers]

Example A
Clara (14 years old): I don t know [..] When you do the second one [the second partial product], you have to put it below its column and you can't keep on putting it below the column with the 3. That would mean you're still multiplying by 3. You leave a space I don't know how to explain it **[absence of formal use]**

Example B
Carlos (14 years old): *[..]* It's as if there was a 0 here, that is, as if you were multiplying by 20. **[indication of formal use]**
Researcher. I understand And why do you have to add the digits in the columns to get the final answer?
C. Because the top row is the result of multiplying 3 by 146 and the bottom row is the result of multiplying 146 by 2 or by 20 **[doubts]**
R: So is it by 2 or by 20?
C: Well, it s . Because 146 times 3 gives one result and 146 times 2 gives the other result **[contradiction to the previous answer]**

Example C
Mª Dolores (14 years old): [...] It s as if it were a 0 It s the same thing as if you multiplied 20 times 146; that s why there's a gap [ ] If you multiplied it just by two and put it directly underneath, since there's a previous calculation, the answer wouldn't be the same And you're multiplying by 23, not by 2 [ ] And the place that each number occupies changes the result and makes it different **[recognition of place value: indication of formal use]**

Figure 2: Different manifestations of formal understanding of the algorithm

algorithm informally but with regularity and intention, puts her ahead of her classmates as far as formal understanding is concerned (Example C).

The intentional use of links on the external level among the algorithm's different parts, which goes beyond the usual relationships derived from the original procedure, makes it possible to identify a third *analytical* sphere which can be differentiated from the technical and formal spheres. More specifically, one of the indicators of analytical use involves the act of isolating and treating specific steps in the algorithmic sequence separately and analyzing and establishing possible links between them This is shown in the examples in Figure 3, where Carlos (14 years old) is unable to determine the number of digits in the multiplier and fails to show indications of analytical understanding comparable to that of Example B, in which María (12 years old) gives an answer that includes non-usual relationships between different parts of the algorithmic sequence.

Lastly, another sphere of understanding can be identified, one which is different from the fundamental sphere of understanding involved in the obligatory application of the algorithm in its technical, formal or analytical variations It becomes apparent when one considers situations that can be solved using more than one method or item of mathematical knowledge, when the algorithm in question represents just one problem-solving strategy out of several different options. In such cases, the student's ability to identify the situation as being appropriate for using the algorithm and their subsequent ability to select the algorithm from among other possible options constitutes another aspect where clear differentiations can be made among subjects, and which therefore should be interpreted in terms of their understanding.

In the following multiplication problem the multiplier and the multiplicand have been hidden:

```
  [hidden]
x [hidden]
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   37035
  24690
 12345
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 1518435
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(a) Can you determine how many digits are in the multiplier? Explain your answer

(b) Can you determine how many digits are in the multiplicand? Explain your answer.

Example A
Carlos (14 years old): In the multiplier one
Researcher: Why?
C: There's no way to determine it You just don t know
R: And in the multiplicand?
C: I think there could be four in the multiplicand and one in the multiplier

Example B
María (12 years old): But it can have more than. It can t have. *There s more than one option.* right? To know how many digits it has.
Researcher: Which one?
M: Well, it could have four digits. And come out 37 in the last number **[indication of analytical use]** Do you understand? [...] I already told you: *there could be four or five*
R: And the one on the bottom the multiplier?
M: *Three* [ ] *Because there are three rows* **[indication of analytical use]**

Figure 3: Differences in the analytical understanding of the algorithm.

In Example A (see Figure 4) the student does not use the algorithm because she does not recognize it as an instrument for solving the problem, thus demonstrating a more restricted use of the algorithm in comparison to the students in Examples B and C, who use it in its technical and analytical facets.

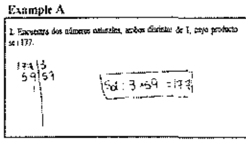
A theoretical and methodological framework for the assessment of understanding in mathematics

Assessment in mathematics is an extremely complex task due to several factors, including its inferential nature, as well as the difficulties involved in designing mechanisms that provoke useful and observable manifestations and the use of a wide range of different tasks and situations. The algorithm described here serves as an example which helps to bring to light a series of questions, such as:

- the role that mathematical knowledge plays in assessment

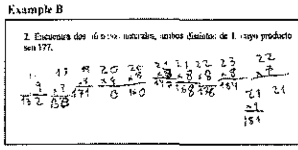
[Find two natural numbers neither of which are 1 whose product is 177]

Example A



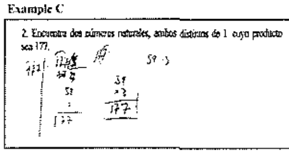
Rosa (14 years old): By factoring the number.
Researcher: Factoring. And what if you had thought of doing it a different way?
Ro: Maybe it's possible but right now this is the only idea I can think of.
Re: Only this one and no others?
Ro: Yes, only this one. But there might be other ways, of course.

Example B



Miguel (12 years old): By multiplying.
R: Multiplying numbers...
Mi: Natural numbers. [ ] This one gets you closest.
R: Which one?
M: This one 8 times 21. Then 9 times 21. .181.
R: But what happened? You can't find the number?
M: No. 21 times 7 .. no. it's too small: 147

Example C



David (19 years old): [..] I tried to do it backwards. I looked for a number which. multiplied by another. would end in 7 **[indication of analytical use]** first. which gave me 3 times 9. I knew that I already had a 7 here, so then I had to look for another number which multiplied by would give me 17. or an approximation. so if I add two **[another analytical use]** so then 5 was the best choice It gave me 15 plus 2 17.

Figure 4: Choice of the algorithm as manifestation of their understanding.

- the theoretical reference points which should be used to guarantee objectivity in the interpreting process in assessment
- the methodological references which are necessary to justify the situations or tasks which are selected and used to create efficient data-gathering instruments

All these aspects evidence the need to continue developing increasingly elaborate and operative approaches and methods for the diagnosis and evaluation of understanding. Such procedures must guarantee useful, valid and reliable information, taken from real data obtained from specific instruments, and must also guarantee that the assessment is appropriate for the learners' cognitive realities. This was precisely the purpose of the study we carried out earlier (Gallardo, 2004) and is the purpose of the proposal outlined herein

General assumptions and principles

We are working towards a proposal or theoretical-methodological approach that is an operative path towards diagnosing and assessing different aspects associated with mathematical understanding, using an epistemological and phenomenological analysis of the mathematical knowledge under study. The operativity and potential applications of this proposal are demonstrated through the above-mentioned multiplication algorithm. In this section we explain the general assumptions and principles guiding the theory behind this proposal, as well as their main characteristics.

The process of assessment is based on the following assumptions and principles:

Mathematical knowledge, as an object of understanding, is considered to be a previously established and specific entity with two basic, specific and exclusive structures that define its nature and existence. These structures arise from relationships to other areas of mathematical knowledge (epistemological structure) and from the situations that give the knowledge meaning (phenomenological structure).

Students show understanding of a specific item of mathematical knowledge when, faced with situations of cognitive imbalance, they decide to voluntarily tackle, work out and give satisfactory answers that are appropriate for the situation and that involve the use of such knowledge. In this sense, we believe that *what an individual uses and how he or she uses it to voluntarily work out and give situation-appropriate answers provides specific information as to what the individual understands and how he or she understands*. On the other hand, if the individual does not answer or provides an incorrect answer, we cannot make any conclusions with regard to his or her understanding, as we do not know the real reasons explaining why he or she acted in this manner. We consider it to be more feasible and appropriate to limit assessment to the external or observable manifestations of understanding, leaving formulations and conclusions involving internal characteristics to the side. Moreover, affirmations should be made exclusively from and based on the *observable uses* (conscious and intentional) that individuals give to mathematical knowledge. Such uses may be different in each case and will depend on the circumstances of each problematic situation being dealt with, so it is convenient to carry out preliminary tasks to analyze and characterize the full set of situations associated with the mathematical knowledge under study. This set of situations, organized and typified in accordance to specific classification criteria, constitute a necessary reference for creating appropriate data-gathering instruments and subsequently interpreting actions and answers in terms of understanding and competence.

We believe that all mathematical knowledge has significant phenomenological and epistemological characteristics that are essential for understanding. In contrast to other less important factors, these characteristics are responsible for the cognitive differences between students, among other things, and they mark the limits of the levels of preciseness in studies on understanding.

We consider it appropriate to take the issue of mathematical knowledge itself as the starting point from which to develop the assessment process. The epistemological and phenomenological analyses associated with a given piece of knowledge constitute a certain guarantee of objectivity [3] in the interpretations of what is observed, thereby helping to support the validity of the conclusions obtained with regards to a student's understanding

We consider it appropriate to adopt a multi-faceted assessment strategy where the use of different tasks with diverse formats is essential; such tasks must be designed with the intention of obtaining indicators of students' understanding of particular aspects of the mathematical knowledge under study. To achieve this, it is important to make the learner feel involved in the situation, so it is convenient for the situations to be specific, simple to understand, without distracting elements and with controlled answers

Explanation of the proposal and application to the algorithm

The aforementioned theoretical foundations have been used to create a specific methodological procedure for determining situations which can be used to design and put into practice operative, valid and reliable data-gathering instruments to obtain data on the understanding of specific areas of mathematical knowledge.

The process (Figure 5) consists of the following phases:

Phenomenological and epistemological study (theoretical level): This phase seeks to establish a theoretical classification, albeit provisional and dependent on subsequent comparison to empirical data, for the situations associated with the mathematical knowledge under study. To obtain such a classification, which serves to organize the corresponding set of situations and make it more manageable, the following stages must be carried out:

> *Stage 1:* Analysis of available information on the mathematical knowledge under study. This stage involves reviewing a representative sample of mathematics textbooks, carrying out specific background research on any works that focus exclusively on any phenomenological and epistemological aspects related to the mathematical knowledge under study, and a review of publications aimed at student mathematics teachers at all different educational levels.

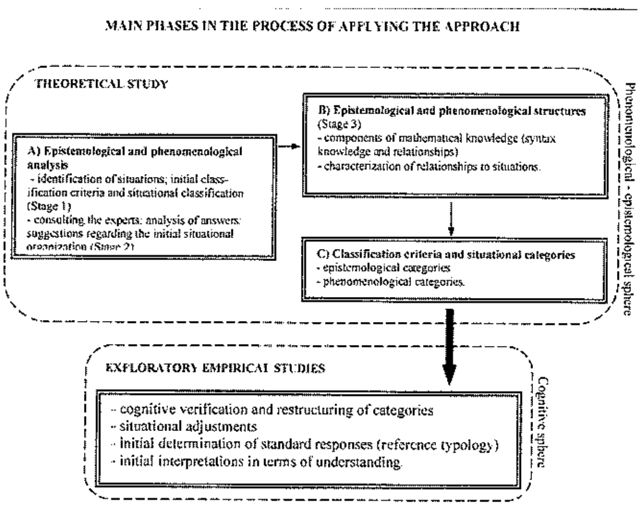


Figure 5: Fundamental phases involved in the assessment process.

This initial review provides an initial collection of situations and an initial set of characteristic elements of the epistemology and phenomenology of the knowledge under study. These are used to make a rough organization of the set of situations, involving an initial proposal for classification, which is then put to the test in the second stage of the study

Stage 2: Experts in mathematics and mathematics education, as well as active mathematics teachers were consulted to determine the appropriateness of the initial typology arising from the analysis of the material gathered in the previous stage. In such consultations, we considered any suggestions to modify or possibly expand both the set of situations and the categories themselves

Stage 3: Using the results obtained in previous stages, the epistemological and phenomenological structures associated with the mathematical knowledge under study were characterized. Using these structures as reference, phenomenological and epistemological classification criteria were defined, thus making it possible to establish a polished theoretical classification, with different categories and possible relevant situations to be used in activities for the diagnosis and evaluation of understanding.

To give an example, the epistemological and phenomenological structures of the algorithm are established below:

Epistemological structure: The multiplication algorithm is considered in this work to be a well consolidated and widely accepted item of mathematical knowledge in the scientific community, so its existence is independent of the learner trying to understand it. It is an algorithmic procedure with the following particular features:

1. *Syntax*: The multiplication algorithm is considered to be a calculation method with a clearly defined syntax. At present its written expression has been reduced to the classical and widespread representation in columns. Additionally, the initial placement of factors in the algorithm's ordinary presentation constitutes (see Figure 1) a relevant syntactical element that conditions the way students use and understand the algorithm.

2. *Previous basic knowledge*: The algorithm is based on the structure of the decimal number system, on positional value, on numerical factorization, on multiplication tables and on the distributive property of the product with regards to the sum (Gómez, 1999), which together constitute the foundation of previous knowledge that the algorithm is based on.

3. *Relationships*: The different pieces of previous knowledge involved in the algorithm are related to each other in the same system of symbolic representation. Analyzing these connections makes it possible to establish three groups of clearly differentiated relationships:

- *Group 1, External relationships (technical level)*: This group consists of the usual relationships between the algorithm's basic elements that make it possible to go through the established procedural sequence in the correct order (for example, the relationships which are carried out as a link between two contiguous steps).
- *Group 2, External relationships (analytical level)*: This group is made up of the external relationships not included in Group 1. They are non-usual relationships, such as: the total number of partial results depends on the number of digits in the multiplier, while the number of digits in the partial results depends on the digits in the multiplicand; the product of a digit in the multiplier and multiplicand, in addition to giving a result, also provides information on the relative position that the result must occupy among the rest of the digits that make up the space containing partial results.
- *Group 3, Internal relationships (formal level)*: This group is made up of the relationships that support and validate the algorithm's underlying mechanism. Mainly, it involves breaking down the factors into linear combinations in the power of ten (positional decimal number system), as well as the distributive property of the product with regards to the sum, and the sum of each one of the resulting summands

Phenomenological structure: This structure is laid out by defining the relevant aspects characterizing the specific relationship between the algorithm and the set of situations that give it meaning. It is convenient to limit this set to situations that are directly linked to the algorithm, which will allow us to reduce the size of the set considerably, thereby providing a greater guarantee of the approach's operative usefulness. Another aspect has to do with the fact that some situations necessarily require the learner to use the algorithm and cannot be solved unless this method of calculation is applied in one way or another, while in other situations, in contrast, the algorithm provides just one of several alternatives for solving the problem, even though other elements of mathematical knowledge could also be used to lead to the same solution.

		Phenomenological categories	
		Exclusive situations	Non-exclusive situations
Epistemological categories	Technical	Calculate 12 x 11	- John has 12 boxes of chocolates. Each box has 11 chocolates inside How many chocolates does John have altogether? - Find a rule to multiply any natural number by 11 quickly and easily.
	Analytical	Find the digits that complete this multiplication: * * x 1 1 1 * * 2 * * *	The product of two consecutive natural numbers is 132. Find them
	Formal	Why do we move the result of the 2nd partial product one space to the left?	Use a calculator to do the following multiplication problem, explaining the process you followed: 2 2 2 4 4 4 9 9 9 x 6 4

Figure 6: Table showing examples of situations organized by categories.

Exploratory studies (empirical level): The model of assessment recognizes the existence of a strong relationship between the phenomenological and epistemological characteristics of mathematical knowledge on the one hand, and the learners' understanding in terms of satisfactory uses of such knowledge on the other In this sense, exploratory empirical studies were carried out to demonstrate with empirical data the nature of this relationship in each case and the validity of the methodological proposal in general Specifically, it is desirable for the categories of situations arising in the phenomenological and epistemological sphere to be reflected in the cognitive plane, something that is a necessary condition to guarantee its usefulness as an operative instrument for the diagnosis of understanding. This phase involves:

- Refining the categories of situations extracted from the epistemological and phenomenological analysis by comparing them to the cognitive sphere and to students' observable behaviours and answers.
- Extracting a group of possible appropriate situations for assessment from the set of situations.
- To provisionally identify and set out some initial standard references of behaviours and answers associated with the different categories of situations and which are interpretable in terms of understanding.

The epistemological and phenomenological structures described in the case of algorithm lead to a set of classification criteria that make it possible to configure a specific categorization of situations

The epistemological criterion takes into consideration the possible relationships detected between each of the algorithm's internal components as factors that condition the way students use the algorithm, and leads to the following categories or dimensions:

1. *Technical category*: consists of situations where the algorithm is used mechanically as an instrument for making calculations The relationships that must be established are external on a technical level (Group 1, Figure 1 shows an example of a situation that belongs to this category)

2. *Analytical category*: includes situations that require a reflexive use of the algorithm to lead to a solution In this category it is not enough simply to apply the algorithm as mentioned in the technical category; it requires an additional analysis of the external structure and function of the method of calculation, involving a conscious, explicit and permanent consideration of the external relationships on a technical level (Group 1) and the intentional use of non-usual external relationships on an analytical level (Group 2, Figure 3 shows a situation that belongs to this category).

3. *Formal category*: composed of situations that require learners to use the basic principles that the algorithm is based on to reach the solution. The situations included in this category (such as the one presented in Figure 2) require the use of internal relationships on a semantic level (Group 3)

The phenomenological criterion considers the possibility that the algorithm can intervene in a situation as a necessary requirement or as one alternative from among several items of mathematical knowledge, and leads to the following types of situations:

a *Exclusive situations*: the algorithm must necessarily be used, a fact which is obvious to the person solving the problem. In this case, the individual can plainly see that the problematic situation he or she is faced with can be solved if he or she uses the algorithm, or in other words, if he or she immediately identifies the situation as part of the algorithm's set of situations In these tasks, the algorithm proves to be the 'exclusive' procedure for solving the problem in comparison to other mathematical knowledge.

   Fundamental understanding is thus characterized based on students' responses and behaviours when faced with exclusive situations. It constitutes an initial sphere of knowledge of the algorithm with three separate and distinct facets, as seen in the examples provided at the beginning of the article, which we have called *technical*, *analytical* and *formal* in order to maintain the similarities with the epistemological terms used herein.

b. *Non-exclusive situations*: can be solved in different ways, one of which includes using the algorithm. In such situations, the student must first identify the situation as appropriate for being solved with the algorithm, and then decide to use this method of calculation instead of other alternatives of mathematical knowledge

It thus becomes possible to identify a sphere of extended understanding, which broadens the scope of fundamental understanding through non-exclusive situations. This sphere, which to date has not been explicitly considered in other approaches, is also derived from the characterization adopted here. The use of the algorithm in such situations is interpreted in a positive light as an indication of the student's understanding of the same, thus representing a first order differentiating aspect [4]. The example presented in Figure 4 provides an illustration of this facet

In sum, as seen in the examples included in the table (see Figure 6), this set of classification criteria makes it possible to discriminate most situations reasonably well, thereby providing an operative reference point on which to base the assessment of understanding

Conclusion

The present study provides a clear example about the complexity of the diagnosis and assessment of understanding in mathematics and the potential and practical applicability of the methodological procedure suggested herein. This has been shown in the new nuances that the study contributes to didactical knowledge of the standard written algorithm for mutiplying natural numbers, as well as in the evident utility of the procedure described for the curriculum design and the development of the educational work in the classroom.

To finish, we should indicate the need for verifying the operativity of the model in the case of other more complex mathematical knowledge and developing systematic proposals to effectively include the current information obtained in regular curricular design

Notes

[1] Here, the word 'standard' refers to the procedure of multiplying using columns (see Figure 1), as established by Spanish curricular programs for education in Primary Schooling (6-12 year olds)

[2] The ability to overcome the influence exercised by the standard placement of factors in columns when using the algorithm, specifically: they are placed in rows with the greater number (multiplicand) placed on top of the lesser number (multiplier), both of which are aligned vertically with the 'ones' digit placed on the right

[3] We acknowledge that the methodological option of carrying out an epistemological and phenomenological analysis of specific mathematical knowledge does not guarantee total objectivity in assessment, but neither do the methods of other existing approaches.

[4] It is worthwhile highlighting the fact that a student s preference for using one algorithm over another is not considered to be a criterion of understanding, but rather as a recognition of the connection between the algorithm and the non-exclusive situations that give it meaning. In this sense, we understand that individuals who do not recognize a situation as appropriate for being solved by using the algorithm manifest a more limited understanding when compared to those who are able to establish a connection between the situation and the algorithm, all of which is independent of the procedure that is ultimately used to solve the problem.

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