

DIALOGUE BETWEEN DISCOURSES: BELIEFS AND IDENTITY IN MATHEMATICS EDUCATION

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Compatibilities and incompatibilities between theories have troubled many in the field of mathematics education, yet nowhere are the theoretical divides stronger than in the field of affect, where discursive frameworks question the basic conceptual apparatus of the field. Relying on Wittgenstein's (1953) late philosophy, concepts such as 'beliefs', 'attitudes' and even 'emotions' have been declared by discursive psychologists such as Harré and Gillett (1994) and Sfard (2008) as inaccessible to operational inquiry and therefore in need of replacement. Authors in the growing field of research on discourse in mathematics education, though perhaps not always aware of it, are consistently avoiding these affective concepts in favor of more publicly observable constructs such as narratives, routines, norms, positions, storylines and discourses. Between these two camps lies the field of identity studies. Its theoretical roots are mostly in discursive frameworks, yet it examines phenomena that are very similar to those that interest researchers of affect (Hannula, 2012). Here I examine the theoretical and philosophical roots as well as the implications of the divide between research on affect and research on identity. I do so by first reviewing the discursive critiques against 'cognitivist' frameworks, relating those specifically to the field of affect. I then move to compare and contrast two particular studies that have some very similar characteristics. One, from the cognitive camp, is that of Beswick (2018) on teachers' beliefs. The other, from the discursive camp, is my study with my colleague Galit Shabtay (Shabtay & Heyd-Metzuyanin, 2017; 2018; Heyd-Metzuyanin & Shabtay, 2019) on teachers' identity and pedagogical discourse. Through this comparison, I aim to examine what is foregrounded and backgrounded in each of the studies, deriving from this analysis strengths, as well as limits of each theoretical framework.

Critiques of the cognitive-individual lens

Authors such as Sfard (2008) and Harré and Gillett (1994) have named as 'cognitivist' or 'cognitivism' those frameworks of psychology that assume the existence of 'mental constructs' that causally effect behavior. Harré and Gillett trace this assumption to the Cartesian mind-body dualism: "Although the idea that there were two distinct kinds of 'stuff' quickly lost its appeal, the idea that the mental life was 'inner', as distinct from behavior, which was 'outer,' lingered on" (p. 4). Harré and Gillett claim that although the 'cognitive revolution' eschewed the behaviorists' idea that

mental life was a 'black box', the leaders of this revolution still held firmly to the idea that mental constructs were affecting behavior causally. The only difference was that the 'hypothetico-deductive' account of scientific explanation was now enabling the examination of these mental constructs empirically, even though they were inaccessible to the observer, much like electrons were hypothesized in physics despite being inaccessible to the human observer via machines at the time.

Sfard (*e.g.*, 2015) has been particularly active in promoting a non-Cartesian, discursive view in the field of mathematics education. She points to the division between theories of cognitivism and 'participationism', claiming that cognitivism held a view of learning as an 'acquisition' of knowledge whereas participationists viewed learning as becoming a participant in a certain discourse. Her critique of cognitivism is quite fierce:

Cognitivism suffers from all the usual consequences of excessive objectification [...] Because of its objectification-engendered assumption that individual minds are the principle source of their own development, cognitivism is ill-equipped to deal not just with interpersonal and cross-situational differences, but also with those changes in human processes that transcend a single life span. (Sfard, 2008, p. 70)

Sfard locates the problem of cognitivism, much like Harré & Gillett (1994), in the mechanistic view of mental entities (which she terms the 'objectification of mental processes') as affecting behavior. Much of this critique is based on her reading of Wittgenstein (1953):

For Wittgenstein, meaning was neither a thing in the world nor a private entity in one's mind: It was an aspect of human discursive activity and, as such was public and fully investigable (Sfard, 2008, p. 73).

From this reading, Sfard derives certain operational recommendations for researchers: "To disobjectify the discourse on human doings, one needs to define the keywords of this discourse in operational terms." (p. 74). Wittgenstein's rejection of mental objects as explanatory of human behavior is often linked to his famous 'beetle in the box' analogy:

Suppose everyone had a box with something in it: we call it a "beetle". No one can look into anyone else's box, and everyone says he knows what a beetle is only

by looking at *his* beetle.—Here it would be quite possible for everyone to have something different in his box. One might even imagine such a thing constantly changing.—But suppose the word “beetle” had a use in these people’s language?—If so it would not be used as the name of a thing. The thing in the box has no place in the language-game at all; not even as a *something*: for the box might even be empty.—No, one can ‘divide through’ by the thing in the box; it cancels out, whatever it is. (Wittgenstein, 1953, §293).

Though Wittgenstein originally dealt with the concept of pain to illustrate the paradoxes that could be arrived at when relating to subjective feelings, others (such as Harré, 1986) took this analogy to the realm of other mental experiences, claiming all objectified words that signify human private sensations suffer from the same flaws. Wittgenstein, in Harré’s (2009) reading “pointed out that words for subjective feelings could not have been learned by pointing to exemplars, which always remain essentially private” (p. 295).

The cognitivist-discursive debate in the domain of affect in mathematics education

Unlike the domain of general mathematics education, where the discursive point of view has been increasingly adopted, the domain of affect has seen less of the impact of these theories. An exception can be found in the work of Jeppe Skott (*e.g.*, 2015a), whose research stems from the domain of teachers’ ‘beliefs’, yet is inspired by the writings of sociocultural theorists such as Sfard as well as Holland and others (Holland, Lachicotte, Skinner & Cain, 1998). Skott (2015b) points to two major drawbacks of over-reliance on reifications of beliefs: one is that researchers often attribute beliefs to teachers and students, rather than quote these beliefs directly from the participants’ mouths—which he terms, like Sfard, as ‘ontological collapses’. The second disadvantage is that participants’ description of their ‘beliefs’ in reified terms is unquestionably taken to mean these reifications impact practice. In Skott’s (2015a) words: “The questions (of the field) are based on the premise that beliefs are the default explanation for classroom practice and explicitly address the corollary that observed incongruities require further explanation” (p. 38).

Skott’s solution is to offer an alternative framework to the study of beliefs (specifically teachers’ beliefs), namely ‘Patterns of Participation’. Here I take a different approach. I wish to examine how issues similar to those dealt with as ‘beliefs’ are studied under the term ‘identity’—a field of study that has been growing steadily in the past two decades (see Darragh, 2016; Radovic, Black, Williams & Salas, 2018; Graven & Heyd-Metzuyanin, 2019, for recent reviews of the field). To address this goal I compare and contrast two particular studies, that of Beswick (2018) on “systems perspectives on mathematics teachers’ beliefs” with our study of teachers’ pedagogical discourse as drawing on a particular ‘figured world’ or Pedagogical Discourses (Shabtay & Heyd-Metzuyanin, 2017; 2018; Heyd-Metzuyanin & Shabtay, 2019). The question I ask is: Are these two studies looking at the same thing, only using different words? Or are there incommensurabilities between the frameworks that cannot be dismissed?

A comparison of two studies

The two particular studies have been chosen because of some striking similarities, namely in participants, overall focus of study, and means of data collection. In what follows, I first briefly report on the studies, and then move to discuss what each of the studies highlights and what it backgrounds or fails to explain.

Beswick—Teachers’ beliefs

In a plenary talk at PME 42, Beswick (2018) presented her study of teachers’ beliefs as being clustered within ‘complex systems’, building on complexity theory from the writings of Davis and his colleagues (*e.g.*, Davis & Sumara, 2006). By this, Beswick points to work on belief systems that answers critiques of beliefs as having simple cause-effect relationships with behavior. She states “an individual’s beliefs are in some sense separate, but in terms of their implications for attitudes and behaviors they do not operate alone but rather as parts of systems [...] from which attitudes and actions can be considered to emerge” (p. 6). Still, it is very clear that within this framework, beliefs are talked about as objects: “Individual beliefs can be thought of as the agents that comprise belief systems” (p. 6). These objects are ‘held’ by teachers, for example, in statements such as “Teachers who *hold* such beliefs are likely to [...]” (p. 6, my italics). Thus, although Beswick answers, in some way, the critique about the too-simplistic relation between beliefs and actions, she does not attempt to de-objectify beliefs. On the contrary, she makes use of the metaphor of beliefs as objects to connect to systems theory and analogize them to ‘agents’.

The study itself included interviews with eight teachers with various years of experience. Interviews were semi-structured, audio recorded, ranging from 30-60 minutes. The study’s goal was “to explore teachers’ beliefs relevant to their mathematics teaching” (p. 9) and interviews included questions related to teachers’ views about what it means to do mathematics and to think mathematically; their personal use of mathematics; the reasons for which some students experience difficulty with learning mathematics and more. In addition, teachers were also asked to respond to each of seven short paragraphs that all start with “You believe that...” or “You see...”. For example:

Students’ achievement is predetermined: You see the students’ maths achievement as a function of factors largely beyond the teacher’s control. You believe that students are either good at maths or not and that if they do poorly in one year they are likely to do poorly in subsequent years. (p. 10)

This paragraph ends with a statement: “Some students are not capable of achieving conceptual understanding of the mathematics they are studying” (p. 10). Other paragraphs similarly mix between statements of the sort “You believe that...” (*narratives about the self*) and propositions about the state of affairs in the world (“Students are...” or “It is part of the teachers’ responsibility to teach...”).

In this mixing of narratives about the self and narratives about the world, we see the first indication of the methodological implications of the objectification of beliefs. This is

a clear example of an *ontological collapse*. In this case, the teacher's narratives about the world are collapsed with the narratives *about the narratives* that the teacher is endorsing. From a discursive point of view, an abundance of meaning is neglected with such a collapse. For example, at least in my own research experience with teachers, requesting a teacher to respond to a narrative that positions her as agreeing with (or 'believing') statements of the sort quoted above (about students' capacity) is very likely to raise a negative reaction. This still does not mean that teachers would not endorse (that is, express some level of agreement or even author themselves) many of the statements about students' fixed abilities. In other words, in discursive terms, narrating oneself as a 'believer' of something is very different than implicitly or even explicitly endorsing a certain narrative about the world.

The ontological collapse discussed above is carried on to many aspects of the method of analysis and reporting of the findings. For example, Beswick reports having examined interview transcripts for statements of beliefs, detailing "these were extracted, sometimes with re-wording for clarity, and used as the basis for inferring more general beliefs that seemed to underpin the specific belief statements" (p. 11). In other words, 'statements of beliefs' were, at least partially, the *researcher's* narratives about the participants' narratives, not their original stories.

In the findings chapter, Beswick reports that teachers' beliefs could be grouped into clusters that form 'aspects of a teacher's belief system'. Relating to one teacher whose belief system was exemplified in detail in the paper, Beswick states:

Her belief system seems highly integrated, coherent, and stable. In complexity terms, there is little diversity among her beliefs to balance the high level of redundancy. Any change would require a conscious desire on the part of Leanne to change. (p. 15)

This explanation, which uses the concept of 'motivation', is further linked to the concept of 'self' and beliefs that are central to the self:

Motivation to do so (change her beliefs) would appear to require her (the teacher's) beliefs interacting with others not currently part of her system and in a way that would undermine a large number of highly central beliefs while allowing her to maintain her sense of self. (p. 15)

In many ways, Beswick's writing echoes and takes into account the above mentioned critiques about beliefs research. She is very careful to hedge her statements about the results of her study (for example, using 'seems to be' rather than stating her findings as facts), uses qualitative methods rather than surveys, and bases her theorizing on 'systems theory' which eschews the simplistic cause-effect model of beliefs and behavior. Yet, as shown above, the discourse used and the methodological pitfalls the study encounters are still very much connected to the weaknesses of cognitive research as identified by previous discursive critiques. Next, I turn to discuss a study done within such a discursive framework. Not surprisingly, it has its own problems and shortcomings.

Shabtay & Heyd-Metzuyanim—Teachers' Identity

In our study of teachers' Pedagogical Discourses and their relation to teachers' identity, we are interested in several aspects that are quite similar to Beswick's interests. For example, we too wish to understand the ways in which teachers view the world, and in particular, the ways in which their views align or contradict current dominant Discourses [1] of 'reform' or explorative mathematics instruction. Like Beswick, we are interested in why teachers' views are often very difficult to change and how professional learning experiences may change their pedagogical beliefs/views/discourse. Notice that already in my attempts to describe the similarities in the phenomena of interest, the words needed to describe it become problematic. It would not be precise to say we are interested in teachers' 'views' (since this is a concept even less defined than 'beliefs' or 'discourse') yet we cannot say that both Beswick and we are interested in 'beliefs' (since our framework does not have an operational discursive definition for 'beliefs'). Given that, the best approximation of talking about Beswick's 'beliefs' in relation to our use of the word 'discourse' is by calling the beliefs 'endorsed narratives'. By that, I am using Beswick's own definition of beliefs as "anything that the individual regards as true" (p. 3). Thus, one could say that both studies are interested in teachers' endorsed narratives about teaching and learning mathematics, as well as narratives about themselves as teachers and doers of mathematics. At this point, it is time to explicate what I mean by 'narratives'. Narratives, in commognitive terms, are parts of discourses, where discourse is:

a special type of communication made distinct by its repertoire of admissible actions and the way there are paired with re-actions; every discourse defines its own community of discourse; discourses in language are distinguishable by their vocabularies, visual mediators, routines, and *endorsed narratives* (Sfard, 2008 p. 297, my italics).

With respect to endorsed narratives, Sfard (2008) is quite inclusive in what she considers as a narrative, including in it propositions such as ' $2 + 2 = 4$ ', as well as 'I am a mathematician' or 'She has a learning disability'. Narratives also figure prominently in Sfard and Prusak's (2005) well-known definition of identity as a collection of narratives about an individual that are reifying, endorsable and significant (p. 16). A somewhat less known conceptual tool that Sfard & Prusak offered in that same paper, is of the 'authoring tripartite' ABC —A being the author, B the protagonist of the story, and C the audience.

Anchoring our work in Sfard & Prusak's definition of identity, as well as Holland and colleagues' (Holland *et al.*, 1998) idea of identity as constructed in certain 'figured worlds', we conceptualized teachers' identity as drawing on distinct Discourses (Gee, 2011) existing in the public sphere. In particular, we differentiated between the discourse of reform (Explorations Pedagogical Discourse, or EPD) and the discourse of acquisition (Acquisition Pedagogical Discourse - APD). We based this distinction on previous studies (Boaler & Greeno, 2000; Ma & Singer-Gabella, 2011) that

have found these to be two distinct figured worlds. In addition, we operationalized the EPD based on a formal document of the Israeli ministry of education that exemplifies the dominant messages of the district where the participants of the study were teaching, and which was found to be quite similar to other ‘reform’ documents such as the NCTM (2000) *Principles and Standards for School Mathematics*.

Similar to Beswick’s study, our study included semi-structured interviews with teachers which were audio-recorded and fully transcribed. An additional similarity was that our interview protocol included questions eliciting teachers’ report of their practice, how they planned their lessons and their narratives about how students could best learn. Our interview protocol also included, like Beswick’s, paragraphs to which the teachers were asked to relate, yet we did not directly ask teachers to respond to certain ‘beliefs’ (or stories of themselves as holding certain beliefs). Instead, we presented them with vignettes of different teaching type ‘quadrants’ (based on Stein *et al.*, 2017 ‘quadrants survey’). We asked the teachers to relate to these vignettes (who they are like most, whom they dislike), through which we were able to analyze not only their self-identifications but also their interpretations of the vignettes. Interestingly, these differed considerably from one teacher to the other, although the texts themselves were identical. Such differing interpretations are parts of what Holland *et al.* (1998) call ‘realms of interpretations’ or ‘frames’ (Heyd-Metzuyanin, Munter & Greeno, 2018). In our study, it became clear that teachers’ interpretations were linked to other characteristics of their pedagogical discourse.

We coded the 12 teachers’ talk in the interviews on a line by line basis, characterizing EPD versus APD according to valued actions, outcomes, roles and responsibilities. This produced a ‘map’ of the teachers’ pedagogical discourse, placing it on a continuum between the APD and EPD. In addition, we analyzed the teachers’ mathematical talk when confronted with a problem that was slightly challenging for most of them. Again, links could be seen between teachers’ pedagogical discourse and their forms of engagement in the mathematical problem-solving episode. Whereas teachers whose talk was drawing more on the APD tended to participate ritually in the mathematics discourse (focusing on the procedure and relying on the interviewer’s guidance and authority) teachers whose pedagogical discourse was more aligned with the EPD also participated more exploratively in the problem solving episode. The most interesting group of teachers were those who self-identified as explorative teachers (claiming their teaching resembled that of the vignettes where students’ struggle was high) but implicitly marked as significant teaching actions valued in the APD (such as teacher explaining, students following pre-given procedures and producing correct answers). These teachers also tended to participate ritually in the mathematical discourse. It was within this group that $\text{Participant}^{\text{Participant}}\text{Researcher}$ narratives were most notably different than $\text{Researcher}^{\text{Participant}}$ narratives. In other words, while teachers described themselves as aligned with the EPD, our analysis of their discourse indicated alignment with the APD. However, we did not make claims about these teachers ‘true’ or ‘enacted beliefs’ which are ‘actually acquisitionist’. In other words,

we are not making claims about their pPp narratives, claiming that our rPr narratives are in some way a better indication of their pPp narratives than their pPr narratives. What we are showing, is that there is a wide variety of alignment with the two types of Pedagogical Discourses, and that certain narratives (mostly those that pertain to social aspects—how students are to talk and respond to tasks) seem to align more easily with the EPD than the mathematical aspects (for example, which mathematical objects and relations are foregrounded in the lesson).

What is foregrounded and what is backgrounded?

As shown above, some of the characteristics of the two studies are very similar, yet they arrive at different conclusions. While Beswick concludes that teachers need ‘motivation’ to change well established beliefs, and that certain beliefs are held more strongly because of their relation to teachers’ core selves, our study makes no such claims. In fact, we found it very difficult to explain why certain teachers align with the EPD and others align with the APD. Although relations to forms of participation in mathematical discourse were evident, they could certainly not be considered clear-cut. For example, we found one teacher who was relatively explorative in her mathematical discourse, yet self-narrated herself as a teacher who avoids explorations in the classroom.

On the other hand, our discursive lens was productive in showing the complexity and contradictions between teachers’ narratives-about-self and their narratives about the world. These contradictions, from a beliefs perspective, often seem like a gap between teachers beliefs and their ‘practice’ (Raymond, 1997) and are sometimes interpreted as indications that one cannot count on teachers’ self-reports because they are inconsistent with what they actually do in the classroom (see, for example, Stein, Correnti, Moore, Russell & Kelly, 2017). Notably, Beswick’s (2018) work attempts to distance itself from the simple cause-effect assumptions of traditional beliefs research, by conceptualizing beliefs as ‘emergent’ from complex systems. Yet it may be that a discursive lens is more effective for highlighting such complex and emergent characteristics. This, because the term ‘belief’ itself is a strong magnet for ontological collapses which obscure complexity and the situatedness of how narratives about self and teaching are authored.

As illustrated in Figure 1, ontological collapses, some which I have exemplified in Beswick’s study, others which are common in other studies of beliefs and affect, can be divided into three main types. One type (Type A in Figure 1) is the collapse of $\text{Participant}^{\text{World}}\text{Participant}$ narratives with $\text{Participant}^{\text{World}}\text{Researcher}$ narratives, that is narratives the person endorses privately about the world with those the participant authors to the researcher about the world. An example would be the statement ‘The teacher believes students have fixed abilities’, based on the teacher’s statement in an interview ‘I think students have fixed abilities’.

A second type of collapse is somewhat similar to the first one, only here the statements are about the Participant. Here $\text{Participant}^{\text{Participant}}\text{Participant}$ narratives are collapsed with $\text{Participant}^{\text{Participant}}\text{Researcher}$ narratives. For example a

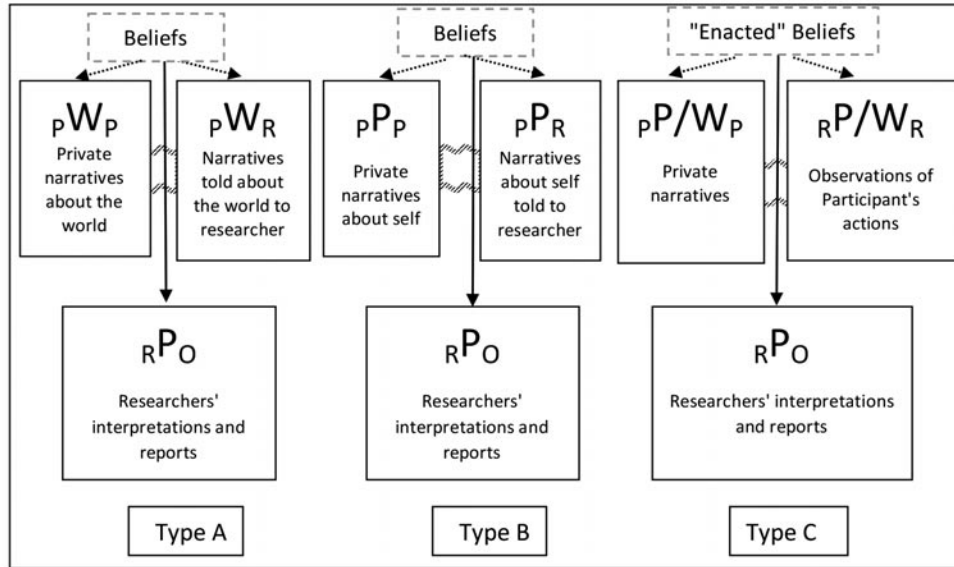


Figure 1. Types of ontological collapses common in research on 'beliefs' (P=Participant, R=Researcher, W=World, O=Others).

statement such as 'I am a fairly traditional teacher', authored to the researcher in the interview, is collapsed with 'The teacher believes in traditional teaching', or simply 'The teacher believes she is a traditional teacher'. Such collapses can often be found when reports of what the Participants say about their feelings/beliefs are collapsed with their 'actual' beliefs.

A third type of ontological collapse occurs when $\text{Participant}_{\text{Participant}}$ or $\text{Participant}_{\text{World}_{\text{Participant}}}$ narratives are collapsed with $\text{Researcher}_{\text{Participant}_{\text{Others}}}$ narratives, for example, when the Researcher forms narratives about the Participant's beliefs based on observing the Participant's actions (these are often called 'enacted beliefs'). Finally, as we saw in Beswick's methodology of asking teachers to relate to belief statements, even more complicated collapses are possible. For example, forming a statement about a teacher's beliefs based on how she responds to the statement 'You believe that...' is actually eliciting a narrative of the type $\text{Participant}_{\text{Participant}_{\text{World}_{\text{Participant}_{\text{Researcher}}}}$ and collapsing it with the $\text{Participant}_{\text{World}_{\text{Participant}}}$ to form a $\text{Researcher}_{\text{Participant}_{\text{Researcher}}}$ narrative ('The teacher believes that...')

The critical reader may object, asking "Well, what is this ontological collapse pointing to besides the fact that constructs that we use to describe a participant's thinking or affect are always the researcher's constructs?" To this, I answer in two parts. First, words are our main tool of research. The words 'ontological collapse' themselves, together with the routines for detecting different types of this collapse, can alert researchers more effectively to their pitfalls. Second, and more related to the difference between discursive and cognitive-psychological frameworks, my claim is that certain constructs (such as objectified mental concepts) invite ontological collapses more than others do. For example, the word 'narrative' is closely related to the verb 'to narrate', which refers to a (possibly) publicly accessible process of telling a story rather than to an inner mental state object. There is no

such verb that is associated with 'belief' ('to believe' is not a publicly accessible process). Thus, the basic conceptual tools of the cognitive psychological scientist may repeatedly trip her into ontological collapses, while discursive constructs such as narrating, subjectifying or identifying focus on the process rather than allude to objects [2].

Nevertheless, as pointed out by Skott (2015b), ontological collapses and objectifications of mental concepts also have advantages. In the case of Beswick's study, these can best be seen in the author's ability to use her framework to provide explanatory mechanisms of the teachers' psyche, something that is missing in the discursive framework.

When 'explanatory mechanisms of the psyche' are given up, the possibility of explanatory mechanisms is not lost. On the contrary, one could argue that discursive mechanisms of individualizing certain publicly available Discourses are explanatory just as theories of the psyche are. The more pressing problem for discursive theorists is that a whole century's tradition of psychological theories on human affect, including theories of motivation, the construction of self, personality traits and individual differences, is eschewed away and regarded as incommensurable or at least incompatible with the discursive lens.

It is by building on such theories of the psyche (whether implicitly or explicitly), that Beswick is able to make claims about beliefs and motivation being linked. This rich source of knowledge about human functioning is made inaccessible through using a discursive lens that is incommensurable with the language of mental concepts. This is probably the biggest challenge for discursive theories that attempt to treat affective phenomena.

What can be learned from this comparison?

I have attempted to compare and contrast not just different theories but rather different theoretical frameworks (or academic

discourses). The process has not been simple. As Radford (2008) aptly articulates:

A dialogue between theories is much more complex than it may appear at first sight. To talk to another theory means indeed to make an effort to be understood and to understand what the other theory says. In order to understand what is said in the language L of a theory τ , a theory τ has to translate it (at least at the beginning) into its own language L . (p. 318)

The case of teachers' beliefs versus teachers' identity shows just how such translation may be a complex matter. For example, I have translated 'beliefs' into 'endorsed narratives', yet this is just an approximation. The word 'beliefs' functions within a discourse where it is assumed that it has relations to other affective keywords (such as 'motivation', 'emotions' or 'self-concept'). 'Endorsed narratives', on the other hand, is a keyword that connects to 'discourse', 'voices', 'figured worlds' and other socially located phenomena. Moreover, using the 'beliefs' keyword usually implies that there is a dichotomy between internal mental constructs and externally observed actions (beliefs versus behavior), and that different statements are mere windows into the 'true' underlying belief. Some recent works, such as Sumpter (2013), have tried to circumvent the problem of inaccessibility of 'beliefs', by using the term 'belief indications' and operationalizing it as regularities in observed behavior. Although Sumpter (personal communication) insists that for her, the term 'belief indications' is concerned only with such regularities of behavior and does not imply any underlying mental constructs, the use of 'indications' implies there is something to be *indicated*. Yet what is indicated, is not specified in this conceptualization. This may be an example of how the field of affect is attempting to circumvent the problems identified by Wittgenstein (1953) more than half a century ago, yet is reluctant to let go of decades of productive research on these same 'problematic' concepts.

This reluctance brings me to the next point, which is the best ways by which the field can contend with what seem to be incommensurable discourses. Some may say that these incommensurabilities are not that critical. "Why not," would the eclectically inclined researcher ask, "simply use each framework for those questions that are best suitable for it?" According to such a conciliatory approach, 'narratives', located in the social sphere, become 'beliefs' when endorsed by individuals. If one wants to study the social aspects of teachers' world-views, one would study publicly available documents, media and teacher discussions. If one is interested in the private aspects of these world-views, including the psychological mechanisms underlying them, one would study them using frameworks on affect and beliefs.

Yet this conciliatory approach does not solve the problems of translation mentioned above, and thus fails to offer a way for research on identity to build on research on beliefs and vice versa. Moreover, as exemplified in the two studies reviewed above, studies of beliefs and identity tend to cross-over and deal with aspects that belong, according to the above suggestion, in the others' camp. For example, studies of identity are very often focused on participants' private experiences whereas studies such as Beswick (2018) are starting to address 'social beliefs' or beliefs of collectives.

Moreover, if thought about more deeply, the suggestion for a 'conciliatory approach' may actually hide the underlying assumptions of the cognitive framework, which assumes a division between the individual and the social, precisely the division which discursive approaches attempt to overcome.

Yet the rejection of a conciliatory approach is difficult for the researcher who wishes to achieve the basic hallmark of science: building on the findings of previous studies. To put it in practical terms—what would a theoretical review from a discursive-identity perspective include as evidence from studies of beliefs? Would it translate all the findings talking about beliefs to 'endorsed narratives' even though it is clear that this translation is not accurate? And even if translated as such, how would the discursive researcher make claims about endorsed narratives when ontological collapses often hide the fact that it was, in fact, the *researchers'* narratives about the participant's narratives, not first person authored narratives? The problems of the discursive researcher are met with similar problems on the side of the beliefs researcher. What is he/she to make of research claiming students' or teachers' endorsed narratives (or in his/her language—beliefs) to be situated, ever-changing, and contextually determined? How can these findings be connected to wider theories of the self and of human psychology, which attempt to predict consistencies in human behavior?

Often in the past, researchers confronted with these dilemmas have 'chosen sides'. This is an unfortunate occurrence, as has been pointed by many not only in the affect/identity field but also in the mathematics education field in general (Prediger *et al.*, 2008, Sfard, 2015). Yet the solution cannot come from suggestions such as avoiding 'isms' and adopting an eclectic approach to research in mathematics education (Goldin, 2003). In fact, such suggestions often come from the hegemonic discourse (namely the acquisitionist camp) which sees the claims for incommensurability as attempts for self-seclusion. In fact, incommensurability problems are indeed mostly claimed from the discursive side, yet this is not necessarily because of attempts to self-seclude. Rather, it may be the result of being able to see incommensurabilities only from the point of view of a discursive, non-Cartesian and Wittgensteinian perspective, which is mostly concerned with dis-objectifying, rather than objectifying human actions.

The field thus needs ways for the two frameworks in the field of affect and identity to take their differing and incommensurable descriptions of human social-affective phenomena into account. Even if straightforward 'building on each others' shoulders' is impossible, collective work on these problems may be very fruitful to the field.

Notes

[1] My use of Discourses with a capital D is inspired by Gee (2011) and the idea that these Discourses exist in the social sphere and pre-exist any individual adoption of these Discourses. I term the individualized adoption of these Discourses as a teacher's pedagogical discourse with a small d.

[2] Notably, the word 'identity' is just as susceptible to objectification as 'belief', especially if it is not defined operationally (for example, as a collection of narratives, as in Sfard & Prusak, 2005) and associated with the verb 'identifying'. Indeed, we have found in a recent review of the literature on identity (Graven & Heyd-Metzuyanin, 2019) multiple instances of such collapses in literature on mathematical identity. However, these were most often associated with studies that used identity as a 'catch all' (p. 369) term for affective constructs.

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