Why Teach Mathematics? Mathematics Education and Enactivist Theory

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"Why are we taking this? Asked yet again This time in the middle of an introductory fractions unit

I was only in my first year of teaching and I had already heard and responded to this question many times It was thus somewhat surprising to discover that, on this occasion, I didn't have a ready answer I don't recall my exact response, but I do remember that it amounted to, "You need to understand this concept so that you'll be able to understand the next one"—or, rephrased in more general terms, we study mathematics to prepare ourselves to study more mathematics Not surprisingly, neither my students nor I were particularly satisfied with that line of reasoning

Hoping that my colleagues would be able to offer some advice on dealing with such questions, I asked, *Why teach mathematics?*, at the end of that school day Unfortunately, their arguments for mathematics education were similarly uncritical, and the ensuing discussion would have been quickly forgotten were it not for the persistent and insistent voice of one teacher Her rationale was a concise, "You need it!"—and she used the statement as a point of exclamation for every suggestion that arose

"Most jobs involve some math "
"You need it!"
"Most other disciplines use math "
"You need it!"
"Universities require it "
"You need it!"
"We need scientists and engineers "
"You need it!"
"Math helps you to think clearly and logically "
"You need it!"

Although I was dissatisfied with these statements, at the time I disagreed with none of them They seemed so reasonable, so rational, so commonsensical that, while I found myself uncomfortable, I was unable to trace the source of that discomfort

Had I then been more aware of some of the critical discussion surrounding mathematics education, I would likely have been less troubled by this experience Numerous critiques of school mathematics have been offered, focusing on such issues as pedagogical practices, the central and isolated place of mathematics in the modern curriculum, and, perhaps most importantly, the privileged status of mathematics in Western cultures Mathematicians Davis and Hersh [1986], for example, point to the way that a mathematical rationality has permeated virtually all areas of scholarly endeavor They argue that this general movement has often led to inappropriate applications of mathematical procedures, particularly within the social sciences Others have offered more scathing critiques Postmodernist [e g , Borgmann, 1992], feminist, neo-Marxist [e.g., Walkerdine, 1988 See note 1], and ecological [e g , Bookchin, 1990] thinkers, for example, have elaborated on the personal, social, political, and environmental consequences of privileging a mathematical mode of thinking and the utilization of the resulting subject matter Characterizing mathematics as totalizing, masculine, colonizing, and divisive, these theorists link the discipline to social-, gender-, and racial-inequities, to our military culture, and to environmental decline

For the most part, the critiques of the central place of mathematics in the modern curriculum follow from these commentaries on the wider cultural situation Some of the criticisms also extend to the teaching practices and interaction patterns of the mathematics classroom Walkerdine, for example, characterizes mathematics teaching as "cruel" and mathematics learnings as "painful," in that it often requires a suppression of experience and intuitive understandings on the parts of learners She and other critics also perceive a narrowness to the field of mathematics education, accusing those involved in teaching and research as having an almost exclusive concern for issues of knowledge The ways in which that knowledge figures into the emergence of our collective and personal identities, for example, go largely unaddressed, as do such issues as the moral implications of mathematics teaching and the ecological impact of the technologies that are supported by our mathematics

In brief, then, theorists from a range of perspectives seem to be offering support for the claim that, when it comes to the study of formal mathematics, we in fact *don't need it*—at least in the form that tends to be enacted in the modern classroom Of course, few would actually state the case in such terms—not in the least because, if they are correct in their assessment of the centrality of mathematics in our society, then a knowledge of the subject matter is important for anyone concerned with evening out imbalances in capital and power To recommend that it not be studied is inconsistent with current discourses of empowerment and democratic citizenship

Few of the critics, however, would disagree with the suggestion that mathematics education needs to be rethought from the ground up Recommended sites for reconsideration include re-evaluations of the nature of the subject matter, its place in formal schooling, and its contribution to our collective identity A difficulty in this project, however, is that the recommendations from those who seek reform tend to be as diverse as the theoretical frameworks from which they work Demands for complete restructuring of curricula and schools often compete with calls for returning to the "basics "

There are, however, some convergences in various critical accounts—convergences that are common to an increasingly diverse range of disciplines and systems of thought In what follows, I draw from recent developments in philosophy, biology, ecological thought, phenomenology, and curriculum theory in an effort to re-formulate a response to the question, Why teach mathematics? Because there seems to be some compelling truth in the response, "You need it!", I structure the discussion around a re-interpretation of that phrase.

You need IT!

"What can you say about 2/6?"

This question was posed to a group of 12-year-olds In their small groups, most students began by laying two sixth pieces from their Fraction Kits side-by-side or end-to-end Several different sorts of activity then emerged: some students proceeded by comparing the two-sixths amount to other pieces from the kit; some started to search for combinations that covered the same area; others concentrated on a single sixth piece, discussing what they knew about that quantity and then extending their discussions to the two sixths amount

A few more groups seized upon the idea of listing combinations of pieces from their kits that covered the same area as two sixths, and very soon everyone in the class was busily listing possibilities. In an effort to facilitate this exploration, one student created a chart to record the combinations as they were identified.

Noting the advantages of using a table, the teacher requested that the student display her work on the chalkboard She did so, and soon almost everyone was making use of it. Some students quickly realized that the charts themselves could be used to generate answers, and some of them gradually put aside their fraction pieces A language of action began to emerge among those working strictly on their charts: a "one" in the twelfths column could be *traded* for a "two" in the twenty-fourths column; a fourth and a twelfth *covered* a third; three eighths *overlapped* two sixths.

What is the *it*—the mathematics—of this setting?

It is difficult to answer this question in conventional terms Is the mathematics the knowledge of fractions that these students seemed to be enacting—albeit on a largely unformulated level? Or is it the formal and abstract content of the program of studies? Is it the explorative activity? Or is it the formalized product of such activity?

As Ernest [1991] demonstrates in his overview of various philosophical orientations to the field, characterizations of mathematical knowledge are diverse. The varied descriptions can be grouped into two broad categories, however, since discussions of the nature of mathematics tend to be developed around attempts to locate mathematics either "in here" (as subjective and constructed) or "out there" (as objective and discovered, placed on either a Platonic or a social plane). That is, for the most part, the question, What is mathematics?, tends to be answered either in terms of knowing subjects or in terms of known objects

This subjective-objective tension seems to be almost inevitable—so much so that discussions of the nature of mathematics are often greeted with a sort of indifference, or even cynicism, by many involved in mathematics education. The situation is an unfortunate one. Whether or not we, as educators, choose to explicitly formulate our orientations to mathematics, we are inevitably enacting particular conceptions of the subject matter and/or the discipline In the school, for example, with the traditional emphasis on the development of technical proficiency, for the most part mathematical knowledge has been regarded as something "out there"—objectively true, pre-existent, independent of human agency School mathematics has thus been cast in the same terms as material commodities, and we have tended to be pre-occupied with such matters as the efficiency of production (i.e , acquisition of knowledge), the utility of the product, and the quality of the outcomes.

This orientation is currently the target of wide-spread critique, and important recent initiatives such as NCTM's Standards project are explicitly directed toward offering curricular and instructional alternatives that are more process-oriented and learner-centered—in effect, embracing a more subjective orientation to mathematics. As well, supported by the epistemological framework of constructivism, research in the area has overwhelmingly moved toward a greater awareness of the subjectivity of learners.

Arguably, however, neither subjective nor objective accounts of mathematical knowledge—alone or in tandem—seem to be powerful enough to interpret the events described in the above classroom example Characterizing the mathematics of that setting in terms of individual constructions is clearly inadequate; such a description would reduce the fluid and harmonious movement of the collective to mechanically coordinated action of fully autonomous agents. Similarly, characterizing the mathematics as a field of objectified truths toward which the learners are progressing is also unsatisfactory—as demonstrated by the simple fact that these students were dealing with an instance of partition theory that may never previously have been explored.

In other words, the interpretive power of the subjective-objective dichotomy is inadequate to address the question of knowledge, and so we are compelled to look elsewhere for other theoretical possibilities. One framework has been announced in the work of biologists Maturana and Varela [1980, 1987] and ecologist Bateson [1979]. Briefly, they begin by challenging two premises of the subjective-objective debate; first, the belief that mental operations and physical actions (i.e., mind and body) are in some way independent and separable, and second, the assumption that individual knowing agents are isolated from one another and from the known world.

Interestingly, in modern analytic terms, knowledge tends to be understood as a sort of bridge that links subject to object, knower to world, individual to collective, and mental to physical. Knowledge is popularly regarded as a "third thing" that links two opposites, and it is this conception that presents us with the problem of determining whether that third thing should be anchored in the subject/knower/individual/mental or in the object/world/collective/physical.

Much has recently been written challenging the mind-set that places the cognizing agent (i.e., the mind) and the world into two distinct categories. Merleau-Ponty [1962], for example, argues that this formulation is lacking because it has forgotten the body In his terms, the body is simultaneously a biological structure and a lived-phenomenological structure—it is, all-at-once, of the world and of one's self. Our bodies separate us from one another at the same time that they place us in relationship to one another Our bodies are shaped by the world that they participate in shaping; they render mind-and-world, subject-and-object, individual-and-collective, mental-and-physical inseparable. These phenomena are co-emergent: fluidly defined against one another.

The above classroom example might be used to support this point The students' use of terms such as "cover," "trade," and "overlap," as they worked with their charts, suggests that the terms are rooted in bodily movement, but have been abstracted from the paper-manipulating activity The students' (mental) understandings are thus tied to their previous (physical) actions Drawing distinctions between physical and mental becomes problematic here, particularly as students are observed to move fluidly between the two realms This blurring of the line between thought and behavior is succinctly stated in Maturana and Varela's aphorism, "knowing is doing." The students' language and action are not outward manifestations of their inner workings, they are visible aspects of these students' embodied (enacted) understandings.

An important and immediate implication of this conception of personal understanding is that each student's knowledge is entwined with every other student's. Knowledge and understanding in this frame, then, cannot be thought of in strictly subjective terms; collective knowledge and individual understanding are dynamically co-emergent phenomena. One might thus say that the mathematical knowledge is located in the activity—or, perhaps more descriptively, in the inter-activity—of learners. As the events of the lesson are re-traced, it becomes apparent that it was not so much the possibility for individual action as it was the opportunity for interaction that contributed to the flow of the mathematics.

The argument being mounted is that mathematics and mathematical understanding are collective phenomena, not individual ones Even when an individual is working independently and in apparent isolation on a mathematical task—as continues to be the case in many mathematics classrooms—the action is social, for it is framed in language and procedures that have arisen in social activity. As such, while studying the actions of a particular learner can be informative, the focus of our inquiries into the natures of knowledge and knowing must not be so narrow as to block out the collective.

An assumption being made here is that, contrary to popular belief (particularly as maintained by those working from more subjective versions of constructivism), the phenomena of *joint* action and *shared* understanding are quite possible To state the point differently, Descartes' assertions that each of us is isolated from one another and insulated from the world—assertions which serve as the foundation for modern analytic philosophy—are in error. Challenging these notions, Maturana and Varela [1987] use the term "structural coupling," following Merleau-Ponty's [1962] use of "coupling," to describe interpersonal relationships, suggesting that we are capable of (and indeed already are) acting together as a unity.

Recent studies in the field of complexity theory [see Waldrop, 1992; Casti, 1994] also support the notion that we are capable of joint action—that is, of acting collectively in ways that give rise to structures that transcend each of us. An analogy that is often drawn to illustrate complex (joint) action is that of the subsystems that comprise our bodies Just as cells come together to form organs and those organs, in turn, come together to form bodies, higher order unities can emerge in joint action. Important in these instances of coupling is that the properties and behaviors of higher order unities tend to be highly complex—in general, far more complex than might be predicted from a knowledge of the unity's subsystems. Moreover, the higher order unities are capable of patterns of acting that could likely never be attained separately by any of the individual subsystems.

This notion of coupled action might be used to interpret the events of the Fraction Kits setting. One thought sparks another, an idea spreads through the room, knowledge in this setting seems to exist and consists in the participants' patterns of interaction.

In light of this sort of event, an analogy might be drawn between one's body and a collective's body of knowledge. Just as mind and world are made inseparable by our physical-phenomenological body, so knowers and known world are brought together in a body of knowledge (This latter body is a fluid structure consisting entirely in the complex choreography of knowers) Similarly, just as my changing body is the locus of my personal identity—simultaneously setting me apart from while situating me in the world—so our dynamic knowledge is the locus of our collective identity—providing an integrity that distinguishes us from a background while placing us in communion with that background. Our body of knowledge—that is, our established and mutable patterns of acting—can thus be thought of as our collective self. As part of this body, we constantly participate in its shaping, just as it serves to shape our own perceptions and identities.

In a sense, then, mathematical knowledge is like the subject matter of a conversation It exists only in conversing, and its nature, its structure, and its results can never be anticipated, let alone fixed Such was certainly the case with the Fraction Kits activity. The teacher could hardly have predicted the curriculum path that eventually unfolded—much less have caused it to happen

The question of the nature of mathematical knowledge can thus never be determined once-and-for-all. Further to this point, if our knowledge and our action are co-implicated, then we are also compelled to acknowledge that the character of our mathematics changes as a result of engaging in mathematical activity Instead of thinking of our body of mathematical knowledge as essentially unchanging (and of mathematics learning as conforming to that fixed body), this enactivist view of knowledge suggests that individual conceptions and collective knowledge take shape simultaneously The question of whether mathematics is a process or a product, then, is replaced by the assertion that the two are inseparable.

So, what is "it"?

The critical task seems to be not so much determining the nature of mathematics, for in posing the question in those terms, there is an implication that we can somehow consider the body of knowledge as determinable, fixable, and separable from ourselves—as though we could somehow step outside our mathematics An enactivist turn on the question of knowledge would be to ask how we are knitted together in this particular body How does the discipline contribute to our perceptions and define our actions? How does the subject matter help to shape the responsive word that we perceive and within which we act?

You NEED it!

Noting that most of the groups had moved on to determining coverings for other fractional amounts, the teacher chose to call for their attention in order to discuss other avenues for investigation

"If you put away the kits and the charts," he asked, "what are some of the other things that you can say about 2/6?"

"It's equal to lots of different fractions," Lori offered.

"Can you list some fractions that are equal to 2/6?"

"Sure . 1/3. 3/9 . 4/12.. "

"Three-ninths?" Jiema interrupted

Lori's response was a straightforward, "Yeah Three-ninths is equal to two-sixths "

"Well," Jiema countered, "It's not as equal as the others. "

"Not as equal as "

In commonsense terms—and certainly according to the truths of school mathematics—Jiema's suggestion that equality occurs on a relative scale is utter nonsense. quantities are either equal or they are not There are no shades of gray.

Yet, in this classroom, "not as equal as" is treated as a reasonable notion. In this instance, the teacher went on to ask Jiema for further explanation, and based on her roundabout way of saying that the connection between two-sixths and one-third is much more obvious than that between two-sixths and three-ninths, it was agreed that some fractions really did seem to be more closely related than others. It was further agreed that a fraction like 63/189 "is not very equal at all to 2/6," whereas 4/12 and 8/24 "are very equal "

What was happening here? Was it appropriate for the teacher to encourage Jiema, and then not to correct her obvious misconception?

To understand this event and the questions it raises, I believe it is necessary to examine the logical assumptions underlying both the traditional conception of equality and the conception that Jiema has announced. Regarding the former, for the most part, Western systems of logic are founded on a belief in the possibility of clean definitions, crisp edges, and unambiguous categories. Our numeration systems as well are developed around the assumption that those things that are to be counted, sorted, added, or compared can be unproblematically defined and grouped. The same 1-0, all-or-nothing mentality is evident in much of our formal interaction, and likely has been at least since Aristotle formulated his axioms of logic. [2] Those phenomena that belie clear-cut classification or quantification tend to be dismissed as mis-defined or not-yet understood But is this the way we really think?

It is clearly not the way Jiema was thinking in the reported classroom situation. And, according to fuzzy logicians, it is not the way we think most of the time. Cups that we consider full are usually less than full; a robin is a better example of a bird than a penguin; some blues are more blue than others. And, it seems, some equivalent fractions are more equal than others. The contention of fuzzy logicians is that the crispness suggested by whole numbers and our 1-0 logic is an illusion—founded on an assumption that, through un-interrogated habit, has been elevated to the level of a fact too obvious to question.

Fuzzy logic—along with non-Euclidean geometries, complexity theory, and other areas of mathematical study—has helped us to uncover some of what we have assumed to be universally, and not just mathematically, true Most phenomena cannot be clearly defined or rigidly bounded, parallel lines might meet or they might diverge, the whole really can be greater than the sum of the parts It is interesting to note that, both in the mathematics community and in society at large, these ideas have not always been readily embraced. Very often, in fact, extensive effort has been made to explain them away as special cases, matters of definition, or not really mathematical. It seems that we are not completely comfortable with the challenges they offer to what we believe to be true [3]

Why are we so reluctant to welcome these sorts of ideas? Might it be that the traditions that they challenge are so much a part of the way we think that we are unable to acknowledge their relevance? Or might it be that these sorts of ideas challenge not only what we know and believe, but who we are?

To address these sorts of questions, we need to have a strong sense of how our mathematics helps to determine the way we perceive and act. Gadamer's [1990] notion of "prejudice" is helpful in this regard He uses the term to refer to the knowledge and experiences that give shape to our understandings and perceptual capacities (individual and collective). Divesting the term of its negative connotations, he suggests that prejudices do not simply limit our perceptions and constrain our actions, they are in fact what make perception and action possible. We *need* our prejudices—without them we would be unable to draw meaningful images out of the noise of sensorial possibilities that surround us.

Western mathematics, in its privileged position within our culture, plays a profound role in shaping our prejudices—an assertion that is supported by numerous anthropological studies of other cultures. [4] Quite literally, how we see the world is shaped by our mathematics, and in helping to shape our pre-judgements and perceptions, our mathematics has become an integral part of our individual and collective identities. It might, in fact, be more appropriate to say that our mathematics *has* us, rather than insisting that we have control of our mathematics. It so permeates our culture and so frames our thinking that, whether we have a deep understanding of its subject matter or not, we situate ourselves in mathematical ways. Davis and Hersh [1986] demonstrate this point in their tour through some of the places and ways that mathematics has infused our thinking/acting

It is interesting to note that, in many of the critiques mentioned at the start of this paper, mathematics and mathematics rationality are often singled out as *the* main contributors to the isolating and reductive efforts of Western systems of thought. [5] While these criticisms are justifiable on some levels (e g , our mathematics enables—or perhaps compels—us to ignore contextual factors, thus risking the "reduction" of events, persons, and situations to mere quantities), we must bear in mind Gadamer's assertion that prejudices also enable perception While it narrows our vision in some ways, our mathematics also powerfully links us to our world Rhythms and patterns that would otherwise go unnoticed are revealed through mathematics to be repeated in all forms of life and at all levels of observation. And so, far from simply reducing or separating us from the universe, mathematics reveals that we are in conversation with it—hinting at complex orders and tangled relationships which inevitably exceed our attempts to understand surpass our efforts to control Conversely, our mathematics presents us (i e., makes us present) in these harmonies, attuning us to the movements of the planet, subtle and great. Mathematics provides us with, in Bateson's [1979] terms, the "pattern which connects" the singular to the cyclical and the fragment to the whole.

At the same time, we must bear in mind that, as is inevitably the case, a solitary mode of reasoning—whether mathematical, literary, historical, or other—compels us to focus on particular details while ignoring others. To this end, mathematics is a fairly self-aware body of knowledge—in that an important part of any mathematical inquiry involves explicitly laying out what is assumed And, as enacted in non-Euclidian geometries and non-traditional logics, these assumptions are available for renegotiation. Mathematics thus, perhaps more than any other field, offers us a route to interrogating the way we think.

It seems that this is precisely what was occurring in Jiema's interaction with the teacher, although it was not made explicit in that interaction One way of interpreting this classroom event is to suggest that the participants were engaged in contrasting the crisp conclusions of one mathematical system with the fuzzy way we sometimes think While not formulated in the established language of fuzzy logic, Jiema and the teacher had touched on a topic of current widespread discussion within the mathematics community.

That this occurred in a unit on fractions is important, for it demonstrates that even with the most *basic* of topics, occasions for mathematical anthropology [6] do arise. What is needed to investigate how our mathematics shapes us is not so much a revised syllabus as a change in mindset. Mere facility with fraction concepts was not the goal of learning in the reported classroom; rather—following the enactive orientation to knowledge announced in the preceding section—fraction concepts served as a site for participating in the shaping of that mathematics. An enactivist interpretive framework, then, does not imply that a revised program of studies is needed. Rather, it calls for a loosening of the prescriptive framework we have applied to curriculum [7] and a re-defining of what it means to behave mathematically

On this latter point, each of us is already behaving mathematically, by virtue of the fact that we live in a culture permeated by a mathematical rationality. It is thus that, I would argue, the topics that are present in a typical grade school curriculum already offer a rich ground for exploring what tends to be taken-for-granted—not just in mathematics, but in virtually every area of inquiry typical to our society The laws governing our systems of logic and numbering, as noted above, have been identified as a possible site. We might also, for example, explore the relationship between rational numbers, ratios, and "rationality"— noting that this last term is derived from the same logical processes that underlie our work with fractions. Similarly, the foundational truths of geometry, having long ago been demonstrated to be anything but universally applicable, offer a site for investigating how we think These are a few of the possible locations for interrogating our mathematics—or, more accurately, for studying ourselves and the prejudices that shape our world

And so, we *need* to study mathematics. But that need should not be understood in the utilitarian terms of equipping children with the skills necessary for adult life, nor in the political terms of providing the understandings needed for democratic citizenship—in part because we have repeatedly demonstrated ourselves to be poor predictors of the sorts of competencies that will be needed even a few years hence. Rather, bearing in mind the plethora of current "crises"—of identity, of environment, of social equity, etc —it behooves us to be attentive to our assumptions [8] We *need* to study mathematics to begin to understand our prejudices and to explore other possibilities for acting With the emergence of such areas of inquiry as chaos dynamics, complexity theory, and fuzzy logic—each of which challenges, on some level, conceptions and assumptions of traditional mathematics—there seems to be good reason to be optimistic about the possibility of mathematical study that makes a difference.

An implication of this formulation is that our understanding of what it means to "do mathematics" may need to be broadened. The conventional focus of mathematics educators on epistemological issues (i e., dealing with questions of knowledge, programs of study, learning, and instruction) eclipses the fact that education is an ontological issue (involving us in questions of existence, being, and identity).

YOU need it!

After verifying their lists against one another and assuring themselves that all the possible combinations for 2/6 had been identified, groups selected other amounts for the same sort of analysis.

Lori chose 3/4 for her group and she was well into listing various combinations when the teacher noticed that Jake, one of her partners, was hastily copying answers from her chart into his own rather than participating in the search for possibilities.

Listening in for a moment, the teacher noted that Lori had a much deeper sense of the generative potential of the tables, relative to the demonstrated understandings of her classmates. She had clearly devised a systematic means of using the chart to generate not just some, but *all* the combinations for any fraction amount.

Realizing this, the teacher asked Lori to describe to her partner what she was doing. She complied, but only reluctantly, and with a explanation that was too brief and too rapid for Jake to follow. But the intervention did have a positive effect: Jake stopped copying and returned to his manipulations of the pieces

A short time later, the teacher felt a small poke in the middle of his back. He turned to see Jake, chart in hand On it, he had recorded that 24/24, 12/12, 8/8, 6/6, 4/4, 3/3, 2/2, and 1/1, all made 1, and this series of combinations formed a diagonal pattern across his page. "I know everything about one," Jake announced.

Clearly, Jake did not know "everything" about one—in fact, for a seventh grade student, his insight hardly seems remarkable. To appreciate its significance, however, the event has to be placed in the larger contexts of the classroom situation and Jake's schooling experience. First, Lori and Jake appeared to be the only two students to realize that the charts could be used to generate the complete solution to some questions—albeit that Lori's understanding was more sophisticated than her partner's Jake's insight, then, was likely tightly linked to Lori's action, and it seems reasonable to assume that the teacher's intervention (i.e , having Lori explain her reasoning), while not at all effective in helping Jake to understand Lori's actions, did help him to regard the charts differently

It is helpful to note that Jake and Lori were assigned to work together in accordance with the co-operative learning recommendation that lower achievers be given opportunities to work with their more advanced classmates. Jake was politely described by his teacher as "not really mathematically oriented. He tries, but he just doesn't see things." His consistently failure-level scores on tests and assignments bore up this assessment. Lori, on the other hand, was noted for catching on quickly and for always achieving high grades That the insight into the power of patterns in the charts was shared by two learners who represented the extremes of school mathematics achievement—and apparently by no others in the setting—again points to the importance of active and communicative classrooms

The real importance of this event was only to come to light some weeks later As the unit progressed, Jake's understanding of fraction concepts became more and more sophisticated By its end, he was achieving passing grades and, perhaps more importantly, was a regular contributor to class discussions and group activities In effect, some time during the unit, Jake ceased to be a person who "tries, but just doesn't see things" and he became a mathematizer—a movement that was acknowledged when he received the school-wide honor of "most improved student." In effect, Jake's identity within the mathematics classroom had been completely re-configured. He was not the same person

At first hearing, this sort of claim sounds preposterous. Jake was still Jake, and everyone recognized that fact. This challenge to the commonsense belief that the self is essentially static, however, is currently being argued by such philosophers as Gadamer [1990] and Taylor [1989] who place identity in the same fluid and negotiated event-space as knowledge and action. For them, what one knows and what one does cannot be dissociated from who one is. As Maturana and Varela concisely phrase it: Knowing is doing is being [9]

In brief, the commonsense conception of self—i e., that of a stable, coherent, autonomous, and isolated unity—is argued to be lacking Rather than regarding self as some *thing* (which might undergo change), the self is re-cast as the instigator and the result of change—in effect, the producer, the product, and the on-going process, all-at-once. This interpretation of identity is more often framed in terms of a text that is continuously re-interpreted [Taylor, 1989], as an ever-evolving conversation [Gadamer, 1990], or as a constantly revised narrative [Kerby, 1991]. Moreover, conventional identity theorists call attention to the complex interplay of persons and events, arguing that, while we tend to think of our selves as independently constituted, our identities are tied closely to the co-evolving identities of those around us To use an autobiography metaphor, our stories are told together and against one another Or, to use a mathematical comparison, the relationship between our identities and the character of the collective is not one of fragment to whole, but of fractal to completed image. The whole unfolds from the part and is enfolded in the part

This blurring of the distinctions between Self and Other and between individual and collective leads to the assertion that *you* are not distinct from *me*, nor can *us* be unproblematically distinguished from what is generally considered to be *not-us* [10] The constructions acquire their shapes by casting themselves against the background of the other—and so their shapes (i e , our identities and the character of our surroundings) co-emerge.

As far as mathematics teaching goes, an important implication of these ideas is that knowledge is not something that is merely incorporated into one's knowledge structure; each and every learning helps to re-shape what was learned earlier and, simultaneously, to determine what will next be noticed. Moreover, the effects of such learning can never be predetermined—and certainly cannot be thought of in the singular terms of one life. By teaching, then—as the account of Jake illustrates—we are affecting, however subtly, the individual and collective identities of the learners and of ourselves. Jake became a mathematizer This claim is an important one because it is not merely a statement that his knowledge-base was broadened or that he became able to deal with more sophisticated ideas. Nor is it merely about his private identity It is a statement of how he stood in relation to others; it is a statement about not just Jake's understandings, but about the conflation of his and our knowledge, his and our actions, his and our identity.

An implication is that discussions of education and curriculum are not simple matters of determining what might be important to know or of ways and means of ensuring that appropriate understandings are developed. The school, as an agent of society, does not merely transmit the knowledge of one generation to the next; it participates in the transformation of that knowledge In focusing on *this* idea and not *that* one, it is assigning a value to both; in teaching it *this* way and not *that* way, it is privileging particular ways of acting over others The institution of formal education is part of our culture's process of re-creating, re-interpreting, and re-negotiating itself The conventional understanding that the purpose of formal education is to prepare the young for their role as adults is thus problematized: merely by intervening in the lives of learners, we are affecting the shape of our collective world The world we strive to prepare them for will be different, partly as a result of such efforts

The point being made here is not that the school or the curriculum has to be fundamentally re-structured It is, rather, that learning mathematics affects who we are, what we do, how we stand in relationship to others, and how we situate ourselves in our world It is thus a call for a more mindful attendance to the consequences of our actions/knowledge, a call to recognize that we, as teachers, are thoroughly implicated in what is taught and learned. Bruner [1986] urges us to regard education as an agent of "culture-making" and as a "forum for negotiating and re-negotiating meaning"—functions that, according to an enactivist framework, are already and inevitably being fulfilled What is needed is a complexified [11] awareness of the way we participate in the re-negotiation With this deepened understanding, I believe, come increased awarenesses of our ecological situatedness, the moral implications of our attempts to educate, and the role of mathematics in our world

You-need-it

Why teach mathematics?

A quick response to this question is provided by Davis and Hersh [1981]: mathematics is one of the humanities In studying mathematics, we are never merely learning about what's "out there" Nor can we be approaching mathematical study as a means of fostering whatever is "in here" Mathematical knowledge is simultaneously about the dynamic co-emergence of knowing agent-and-known world, of self-and-collective

It is important to note that, in terms of current discussions, the enactivist framework does not deny the insights of constructivism, although it does offer a challenge to, for example, the narrowness of its scope In focusing on the individual cognizing agent, both the participation of that agent in the larger community and the fluidity of the context are obscured Similarly, enactivism does not dismiss the varied critiques of mathematics education Rather, the framework offers a means of incorporating cultural commentary with discussions of individual cognition it does so by arguing there is a certain self-similarity between processes of individual cognition and of collective action—an argument that is supported by recent developments in mathematics and related fields (some of which I have referred to in this writing)

It thus makes little sense to study the emergence of an individual's understandings without considering the social and political contexts in which those understandings arise Conversely, the broader context cannot be understood as fixed, oppressive, and all-pervasive, but as subject to the movements of individuals' conceptions In terms of current tension between the epistemological framework of constructivism and the many-faceted critiques of mathematics education, then, it must be understood that each *needs* the other—just as the enactivist framework from which the current discussion arises requires the tension between constructivism and its critiques in order to exist

You-need-it In this rationale for mathematics education, subject-and-object, self-and-collective, knower-and-knowledge, critic-and-criticized are tightly linked, suggesting that the rigidly enacted dichotomies that pervade much of educational discourse are mere heuristic conveniences The point is not that such dichotomies should be discarded, but that they cannot be considered static and unambiguous—much less, truths that can be taken-for-granted

In particular, the distinction that tends to be made between teacher and learner must be problematized In the classroom examples given, the teacher learned at least as much as the students who, in turn, played an important role in teaching Not all of these teaching/learning acts were deliberate or conscious Much remained implicit—unnoticed and unannounced (enacted)—in their (inter)actions This, again, is the place that enactivism urges us to look: the fluid, constantly re-negotiated space of our interrelationships

In terms of immediate and practical recommendations for teaching and for teacher preparation, several suggestions emerge from the preceding discussion Teacher education programs, for example, with their traditional emphases on subject matter and teaching methods, do little to prepare teachers to teach differently than they were taught Founded on a fairly static conception of the subject matter, and unlikely to be engaging prospective teachers in critical analyses of either that subject matter or the conditions around which schools are structures, such programs can do little more than prepare teachers to replicate existing patterns of acting In such settings, the answer to the question, Why teach mathematics? is generally assumed to be so self-evident that there is no need to ask it

But such is not the case if prospective teachers are engaged in studies of such topics as (mathematical) anthropology, history, philosophy, and recent developments in mathematics Within such study, it may be possible to uncover some of the inadequacies of our common sense Perhaps then the rationale, "You need it!", while no less true, will not seem so self-evident

Notes

[1] Walkerdine [1988] offers an introduction to the critiques of both some feminists and some of those working from a critical pedagogy perspective

[2] Specifically, his Law of Contradiction (*A* cannot be both *B* and *not-B*) and his Law of the Excluded Middle (*A* must be either *B* or *not-B*)

[3] Numerous examples of the sorts of hostility that have greeted some of these areas of inquiry are presented in various popular accounts of their emergence. See, for example, Gleick's [1987] comments on non-linear dynamics, Waldrop's [1992] on complexity theory, and McNeill & Freiberger's [1993] on fuzzy logic

It is interesting to note that Fuzzy Logic has not been well received in the western world, either within academia or by business. Noting the rapid acceptance and widespread application of fuzzy logic in Japanese, Chinese, and other Eastern cultures, McNeill and Freiberger argue that the difference in attitudes toward the field has to do with a general reluctance of westerners to loose their grip on Aristotle's axioms of logic. Eastern cultures, which do not share the tradition of drawing rigid distinctions, have thus had comparatively little difficulty recognizing and exploiting the power of fuzzy logic.

[4] Ong [1982] elaborates on one such study. He compares the perceptions of persons from an oral culture (who had little formal knowledge of western mathematics) to what we would consider "normal," noting, among other points, that our taken-for-granted use of abstract categorization, geometrical figures, formal logic, and definitions were not shared by members of the oral culture

[5] This criticism is particularly prominent in those approaching the issue from feminist [Walkerdine, 1988] and ecological [Bookchin, 1990] perspectives. In particular, these critics focus on the ways that personal experience and unforeseeable consequences of mathematically-reasoned actions tend to be marginalized (or ignored) in our abstract and reductive approaches to mathematics education.

[6] Here I am borrowing from and adapting the notion of "literary anthropology" as developed by Iser [1993]

[7] To this end, Pinar & Grumet [1976] have reminded us of the verb root of curriculum, *currere*. They use the term to help re-direct our attention away from the impersonal goals of conventional curriculum projects and onto the process of moving through the melée of present events. In brief, "curriculum" is recast as the continuous interpretation of lived experience and is thus valued for its transformative rather than its transmissive potential.

[8] To elaborate, as suggested at the start of the paper, many commentators maintain that it is the uninterrogated assumptions underlying our actions—and not so much the actions in and of themselves—that present the real dangers. The assumption that mathematics is "out there" (and, hence, value-neutral), for example, has been linked to various crises by cultural critics

[9] Adapted from Maturana & Varrela [1987]

[10] I am borrowing here from an idea developed by Sumara (forthcoming). Following Geertz [1988], Sumara discusses the *us/not-us*—by which he means to call attention to the inextricable intertwining of identity and the background out of which that identity is carved

To tie this notion to an idea alluded to earlier, it seems that one of the reasons we have difficulty accepting the possibility that our selves are not self-contained has to do with our predisposition toward defining objects and phenomena according to Aristotle's axioms of logic: *A* cannot be both *B* and *not-B* (the Law of Contradiction) and *A* must be either *B* or *not-B* (the Law of the Excluded Middle). Fuzzy logic blurs the boundaries of *B* and *not-B*, just as identity theorists have blurred the distinction between *self* and *not-self*. Again, this sort of analysis points, I believe, to the potential of mathematical study for interrogating the assumptions that underlie our perceptions and conceptions.

[11] This term is borrowed from Casti [1994]. Consistent with an important conclusion in studies of complexity, part of my intention here is to point to the importance of considering both the specific event and the broad circumstances—i e , the way each affects the other and their self-similar structures—in our discussions of mathematics teaching

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