Reading to Learn Mathematics: New Connections, New Questions, New Challenges*

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Mathematics is one area of the curriculum where, traditionally, little reading and writing occur — the few exceptions being some rare consultations of the textbook, the very specialized reading necessary for the solution of the word problems, and the schematic writing required in responding to assigned technical exercises. In the past few years, however, mathematics and writing educators, alike, have begun to weave a variety of writing experiences into mathematics instruction [Rose, 1989] Part of the rationale for introducing writing into the content areas is the belief that improving students' writing ability is the responsibility of all teachers, not just English teachers. But more importantly, proponents of the "writing across the curriculum" movement argue that writing experiences can make a unique contribution to students' subject matter learning if the instructional focus shifts from the *written product* to *writing as a process* When thought of as a mode of learning which stimulates and shapes thinking, writing becomes integral to all content areas.

Could a similar reconception of the nature and role of reading in mathematics instruction contribute to our students' learning of mathematics? Interestingly, we have not yet seen a movement in this direction [Siegel, Borasi and Smith, 1989]. Teachers and educators often voice complaints about students' reluctance to read the mathematics textbook, or their difficulties solving word problems due to poor reading skills Researchers have responded to these concerns by examining the topic of "reading mathematics" [see reviews by Nolan, 1984; Pinne, 1983; O'Mara, 1981], with an emphasis on strategies for comprehending word problems and for teaching the "language" of mathematics Yet the way the problem has been formulated — "How can we overcome students' reading deficiencies so that they do not interfere with their learning of mathematics?" — as well as the solutions subsequently proposed, seem very limited and do not take advantage of the radical reconception of reading comprehension which has recently taken place within the reading education community (see Harste [1985] for a summary of this shift). At the same time, a few interesting collections of readings on mathematical topics have been published to supplement the traditional textbook (see, for example, classics such as Newman [1956] and more recent initiatives such as Campbell and Higgins [1984]). Unfortunately, very few teachers actually use these readings in their mathematics classes, and no effort has been made to help interested teachers take advantage of such texts through the use of appropriate reading strategies. We suggest, on the contrary, that looking at reading as a mode of learning along the lines suggested by the most recent work in reading research could provide mathematics education with a powerful new instructional strategy

To appreciate how differently "reading mathematics" could be conceptualized, it may help first to look back at the more familiar paradigm shift that occurred with respect to writing in mathematics. In that case, we witnessed a *radical reconception of the nature and role of writing* itself from a process of transcribing existing information and fully formed ideas onto paper to a process of thinking and learning [Britton, 1970; Emig, 1977] This reconceptualization of writing, in turn, called for *new kinds of writing experiences* in the context of mathematics instruction — for example, "expressive" writing tasks such as keeping a journal or personal notebook throughout a course, where each student may report thoughts, feelings, and difficulties encountered in learning the material, without being concerned about the quality of the writing product *per se* [Borasi and Rose, 1989]. The introduction of these new ways of writing in the mathematics classroom also raised questions about the *mathematical content worth writing about*. In addition to traditional technical expositions, students are encouraged to write about the process of doing mathematics, the nature of the discipline and its applications, affective as well as cognitive components of the learning of mathematics, and so on [Stempien and Borasi, 1985]. It seems clear that a similar rethinking of the *nature and role of reading*, as well as the kind of *reading activities* and *mathematical texts* which could be used in the context of mathematics instruction, will be necessary if mathematics educators wish to take full advantage of the potential of reading in the context of mathematics instruction.

Three assumptions

The strategy of "reading to learn mathematics" proposed in this article makes the following assumptions with respect to each of the three fundamental elements identified above:

- **On the nature and role of reading:** Traditionally, reading has been interpreted as the *act of extracting the message encoded in a written text by the author.* This view of reading has recently been challenged by the theoretical models of the reading process which portray reading as a *transaction* involving reader, text and context [Carey and Harste, 1985; Eco, 1979; Rosenblatt, 1978]. Unlike reading models which view reading as the transfer of information from text to reader, transactional models suggest that reading always involves each reader actively in the transformation of the text

* This study was partially supported by a Grant from the National Science Foundation (award #MDR-8850548) The opinions and conclusions reported in this article, however, are solely the authors'.

read, mediated by his/her past experiences, knowledge and beliefs as well as the context and purpose of the reading experience itself. Thus, the written text is seen as a springboard for generating new meanings, rather than as a template against which a reader's understanding is measured. Doubts, anomalies and ambiguities play an essential role in this process and are thus greatly valued as starting points for engaging in knowledge production and refinement [Dewey, 1933; Siegel and Carey, 1989]. Transactional models of reading are grounded in a more general conception of thinking and learning as constructive and generative activities, involving each person in the continuous effort to make sense of new experiences. The "reading to learn mathematics" strategy is based on a transactional model of reading and thus views reading as an active, personalized and contextualized process of constructing new meanings.

- **On appropriate reading strategies:** Reconceptualizing reading as a transactional process logically calls for different instructional strategies to help students comprehend a given text and take advantage of its potential for learning. While a traditional view of reading would emphasize instruction in developing vocabulary, identifying text structures, and making correct inferences so as to better "grasp" the author's message, a transactional perspective would promote strategies which call on the reader to invent and transform meaning. For example, some transactional reading strategies engage the readers in the transformation of a given text into a different medium (such as drama [Grumet, 1985] or drawing [Siegel, 1984]), so as to help them explore it from a different perspective. Others build upon the heuristic power of analogies and metaphors [Hayes and Tierney, 1982] to raise new questions and to gain new insights into the material, while at the same time bringing the reader's background and interests to bear on the experience. Later in the article we will illustrate how some of the transactional strategies proposed in the reading literature can be adapted to help students create and share alternative interpretations, make connections, approach the ambiguity inherent in language as an opportunity for learning rather than an obstacle, and above all become critical and reflective readers of mathematical texts.
- **On relevant mathematical text:** A transactional approach to reading and specific transactional reading strategies may be applied to any mathematical texts. In fact, one may observe that technical mathematical expositions always need a generative kind of reading, with "paper and pencil," as the reader works out for him/herself the individual steps and overall logic of the author's argument, usually presented in a very synthetic and unintuitive form. At the same time, we suggest that the educational potential of "reading to learn mathematics" can be greatly enhanced if we consider a greater variety of texts as relevant to mathematics instruction. These texts move beyond technical expositions and include essays addressing the history and philosophy of mathematics, reports on mathematical applications to real-life problems, accounts of strategies employed to generate, frame and/or solve mathematical problems, biographies and anecdotes about great mathematicians, and so on. This range of content could help mathematics students become aware of and appreciate the more humanistic aspects of mathematics, which are too often neglected in the school mathematics curriculum. In addition to new *content* to be explored, new *formats* for the reading material employed should also be considered, since both elements can influence the way students read, and, consequently, what they may learn from the experience [Borasi and Brown, 1985]. For example, the concern for clarity and organization which characterizes mathematics textbooks and technical essays allows their authors to present only the end product of mathematical activity, ignoring the process that produced such results as well as the possibility of alternative solutions. In contrast, formats such as historical and philosophical essays, dialogues, narratives, and even poems and cartoons, may be more effective in communicating the process of producing and learning mathematical knowledge. Such texts highlight the possibility of debate and controversy within mathematics — which may, in turn, stimulate reflection and exploration on the part of the reader.

"Reading to learn mathematics" in practice

The nature of the new integration of reading and mathematics grounded in the principles outlined above can best be explored by looking at one illustration of "reading to learn mathematics" in practice. To this end we will present in what follows a unit which promoted inquiry into "Alternative Geometries" through a variety of "reading to learn mathematics" experiences.

This unit of study took place in the context of a semester-long graduate course, developed by the authors for secondary mathematics teachers interested in exploring the potential of integrating reading activities into their own mathematics classes [1]. The course had two main goals: (a) engaging the participants in activities that would allow them to genuinely *experience as learners* the potential of using reading to support the learning of mathematics; and (b) building up a collaborative research team in preparation for the design and implementation of "reading to learn mathematics" units within secondary school settings the following year. The unit on "Alternative Geometries", developed during our first six meetings, played a crucial role in achieving the first goal of the course.

To capture the richness of "reading to learn mathematics" within the space constraints of this article, we will report on the experience in three complementary ways:

- first, we will provide a brief overview of how the whole unit developed, so as to illustrate the variety of texts and strategies employed, and how they were integrated with other learning activities in order to achieve the instructional goals of the unit;
- second, we will focus on a specific "reading to learn mathematics" event, so as to report, in detail, at least one case of what it means to read mathematics in a transactional way;
- third, we will reflect on reading mathematics by using the data from this event to explore some questions about the nature, value and implications of integrating reading and mathematics in the way we are suggesting.

An overview of the unit

The choice of the topic of "Alternative Geometries" was moti-

vated by the desire to engage the participants in learning some mathematical content new to them, yet neither too technical nor complex. Though most of the teachers in our course were aware of the existence of geometries other than Euclidean geometry, none had explicitly studied one of these geometries nor had put much thought into the implications of accepting alternative geometries for mathematics.

The unit began by engaging the participants in the exploration of *taxi-geometry*, a simple non-Euclidean geometry implicitly introduced in the *mathematical story* "Moving around the city," written previously by one member of the group, John Sheedy [2]. This story reports the adventures of a protagonist as he searches for an ideal apartment in Manhattan and finds his way around this city. These adventures provide a context for a non-technical introduction to a non-Euclidean geometry, present mathematical problems whose solutions are not always fully provided and might even depend on the way the problem itself is defined by the reader, stimulate the generation of new problems and questions, and deal explicitly with feelings. The choice of a text generated by a course participant also served the purpose of indicating the value we placed on authoring, both literally and metaphorically [Harste, Woodward, and Burke, 1984; Rowe and Harste, 1986; Solsken, 1985].

In order to support a generative reading of this story, a strategy called "say something" [Harste, Pierce, and Cairney, 1985] was introduced. This strategy invites readers to pair up and work their way through the material, stopping at self-selected points to raise questions, make predictions, share an image or feeling, summarize, suggest alternatives, connect to other texts and contexts, and so on. "Say something" demonstrates a transactional approach to reading in several ways. First of all, it encourages students to take ownership of their reading experience by allowing them to decide what points in the text merit further study and discussion. It employs social interaction to help the readers consider perspectives other than their own and recognize and exploit anomalies present in the text. Finally, the mere fact that class time is devoted to the reading event demonstrates to the students that the teacher values reading as a legitimate learning activity. We will take a closer look at this activity in the next section, as it well illustrates the dynamics and power of "reading to learn mathematics," and had a significant impact on the way many of the participants thought about reading and its relation to the learning of mathematics.

The exploration of *taxi-geometry* initiated by reading "Moving around the city" was further pursued by each participant with the support of a technical text on this topic: Krause's *Taxicab geometry* [Krause, 1986]. This book develops the mathematical properties of plane taxi-geometry in an explicit and technical way. Yet the style and organization of this textbook are unusual and educationally interesting inasmuch as the author does not assume that the reader will attend lectures in advance about the topics covered in the book; rather, the text is written in a manner that encourages the reader to pursue the study of *taxi-geometry* on his/her own, with the help of the few explanations and a well-sequenced series of exercises.

In this case we decided to assign the reading of the text for homework without specifying what strategy to use, but rather emphasizing the purpose that the reading should serve: that is, to support the participants' own exploration of *taxi-geometry*. Our intention in assigning such an open-ended task was to provide an opportunity for the participants to spontaneously use the reading strategies they had developed throughout years of learning mathematics, and become better aware of those strategies as well as of possible alternatives by sharing their reading approaches with the class. The experience indeed revealed some interesting differences in expectations and approaches to the reading of a technical mathematical text among the participants.

To further the participants' understanding and appreciation of *taxi-geometry*, a "transmediation" activity involving Sheedy's story and Krause's textbook was then proposed by the instructors. The class was asked to try to use some of the problems and results found in the technical text as the basis for writing an extension of "Moving around the city." The assumption behind this activity was that the act of transforming the same content from one format (technical exposition) to another (narrative) allows students to view such content from a different perspective, and thus generates new connections and implications. At first, the students seemed hesitant about the feasibility and value of the activity — perhaps partly due to a lack of confidence in their ability to actually write a decent mathematical story. In order to support the engagement of all participants, the task became a group activity, which began with a sharing session in which ideas and results about *taxi-geometry* that could be used to continue the story were generated. After a slow start suggestions began flowing and, to the participants' surprise, this process yielded new mathematical questions and results as well as an outline for a genuine continuation of Sheedy's story (which was later written by one of the participants).

The exploration of *taxi-geometry* conducted through the previous three activities not only produced some surprising mathematical results but stimulated questions about the wider implications of accepting geometries other than Euclidean geometry in mathematics and mathematics instruction. To pursue these issues the instructors suggested two additional readings: an historical essay on the development of non-Euclidean geometries [from Kline, 1980] and a mathematical novel, *Flatland* [Abbott, 1952].

A new transactional reading strategy — "cloning an author" [Harste, Pierce and Cairney, 1985] — was introduced for the reading of these texts. In the spirit of this strategy, the participants were asked first of all to read each text at home with a set of blank 3×5 index cards in hand. When they came to an important concept or something they did not understand, they were to record it on a card; another suggested use for the cards was to write down personal observations and thoughts generated as they read. In other words, the cards were to be used to create a record of personal questions and reflections generated in the course of reading, on a flexible and versatile medium which could later facilitate the sharing of interpretations and ideas with other readers.

A number of different activities were then organized in class so as to make use of the cards each of the participants had prepared. In the case of Kline's historical essay we asked the participants to break into pairs. Each person in the pair was asked

to arrange his/her own cards so as to create a "map" emphasizing conceptual relationships among the ideas recorded on individual cards and then ask his/her partner to interpret the map thus created. The author of the map would then have the opportunity to explain what he/she had actually tried to express through the cards and their organization. In the class discussion that followed this activity most of the participants expressed surprise at the number of new insights and ideas that resulted from both "mapping" their own cards and looking at other people's maps. At the same time this activity also led some of the participants to suggest alternative ways of using the cards — such as categorizing the cards and creating labels to describe each category; selecting only a very small number of most significant cards (according to some previously established criterion) from all those initially prepared; focusing only on those cards that highlight confusion or problems, and trying to find some resolution with the help of the partner or even the whole class; and so on. Given the wealth of interesting possibilities that were generated, we suggested that each pair choose one way to make use of the cards prepared in reading *Flatland* and report their impressions on this variation of the "cloning" strategy during the next class meeting.

As a wrap-up for the whole unit the participants were encouraged to come up with their own metaphor for "alternative geometries." This task evolved from one of the participants' spontaneous association of "alternative geometries" with "alternative teaching methods." He noticed the connection between the way several members of the group had first resisted the value of studying alternative geometries and the resistence often encountered when a new instructional method or approach is introduced to teachers. Several people in the group were intrigued by this metaphor and chose to develop it further, whereas others proposed new metaphors — for example, the idea that an alternative geometry is a different "culture," which one of the participants suggested after reading *Flatland*.

This brief overview illustrates how "rich" mathematical texts, read transactionally, could be integrated into a unit of study. Note that the texts and reading strategies were not "enrichment" activities but set the learning process in motion and propelled the inquiry forward. The reading strategies played a central role in achieving this integration; they drew learners into the texts and encouraged them to raise questions, make connections, take new perspectives, and, in general, actively work out meanings with the support of peers.

Each of the reading events that occurred during our exploration of "Alternative Geometries" is rich with insights into the learning potential of "reading to learn mathematics." In the section that follows, however, we have chosen to go back and focus in on the very first "reading to learn mathematics" activity developed in the unit — reading the story "Moving around the city" with the "say something" strategy. We have selected this event for two reasons: first, because of the uniqueness of using a mathematical story to learn mathematical content, and, second, because the activity made explicit the participants' beliefs and practices vis-à-vis reading and mathematics, and thus set the tone and the direction for the rest of the unit.

A close-up on the "say something" experience

To get things started, the "say something" strategy was first briefly described and then the whole class was asked to read the first paragraph silently and share initial comments. After this brief demonstration the class broke into two and threes and continued reading using the strategy.

One pair, Raffaella and Dave, decided at first to proceed paragraph by paragraph, stopping to interact at the end of each. After finishing the second paragraph (see Figure 1) they shared their comments.

> The prospect of going out on foot and making my way around filled me with excitement and a spirit of adventure. I decided that I would not ask directions to my destination, partly to avoid exposing myself as an outlander, and partly to answer the challenge that finding my own way posed. I started walking toward the East (toward the morning sun, still obscured by tall buildings). Every door on the street was designated by the letter "S" followed by a number, but halfway down the block the even numbers that had been escalating by two's reverted back to "2" and began the progression again, now with a different letter designation. "A". Engaged with this first "clue", I did not pause to note the street sign as I crossed the street straightaway. Now, the designations on the doorway reverted back to "S", with even numbers descending. Instinctively, I looked over my shoulder at the street sign on the corner. It read A38 09- S on both faces that I could see. I approached it again to gain a perspective on its other markings. Ninety degrees-clockwise, it read A38 09- T on both faces. I continued around to find A38 09- C; then A38 09- A
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> 
>

Figure 1

Dave mentioned that he had been more taken by the feelings expressed by the author than by the problem of understanding how the city addresses were formed. Raffaella also liked the weaving of feelings and emotions into the story, yet could not help be puzzled by the problem and was interested in resolving it somehow before proceeding further in the story. At the same time, she wasn't sure that would be an appropriate response to the text since they were using the "say something" strategy and solving the problem, for her, went beyond reading as it would also require some drawing, thinking, and so on. She suggested that perhaps they could continue the reading, leaving the problem unsolved. Dave, however, observed that if he were reading the story on his own, he would, in fact, not proceed but try to figure out how to decode the addresses. They decided to follow their instincts and solve the problem.

Immediately they took paper and pencil and tried to use a drawing to interpret what the author was describing with words. This was neither immediate nor easy. Raffaella started by drawing a street corner, putting some numbers and letters down based on what was said in the story. She moved her pencil around as if it were the protagonist, making him go from right to left. When she showed her attempt to Dave (who had drawn his own design on his copy of the story) he noticed that in her drawing the protagonist did not move East as the story indicated. Surprised, she went back to the story and realized

that indeed that information had been clearly stated at the beginning of the paragraph, though she had missed it in her reading Dave suggested that she simply relabel her drawing (putting East on the left and West on the right), but Raffaella didn't feel comfortable with the idea and preferred their drawings to be compatible so that they could be compared later on. Thus she proceeded to re-do her drawing using the usual East-West convention while Dave continued working on his.

The process of using a drawing to construct the protagonist's movements was difficult and confusing They went back and forth between their drawings and the second paragraph, not necessarily reading it linearly, but trying to catch elements which could confirm or contradict their interpretations and hypotheses At one point Raffaella stopped and discussed with Dave the strange fact that, after crossing the road, the new block would start with descending even numbers; it was not consistent with what had happened in the previous block, and thus broke the pattern that was starting to take form. Dave agreed that it was odd, but neither felt like pursuing the issue further at that point

A little later they spent more time trying to interpret the sentence "Ninety degrees-clockwise, it read A38 09-T." Neither Raffaella nor Dave could figure out what "ninety degrees-clockwise" actually meant. They tried interpreting it as "the protagonist, who at the time was facing West, turned to the right 90 degrees, and then looked at the sign." But if that were the case, then the sign should have read A38 09-A and not 09-T, as written in the story. At this point, Raffaella said she thought the author had made a mistake whereas Dave said he'd rather assume that what was written in the story was correct, and perhaps they had not interpreted it correctly. Raffaella admitted that she probably would've felt the same way if she had not known the author personally but, still, she was willing to give the author the benefit of the doubt. Intrigued by this possible conflict they spent more time trying out different interpretations of "ninety degrees-clockwise," as well as trying to visualize the street sign and what was written on all its faces

Though they had not reached a satisfactory explanation of how addresses were created in the city, they decided to proceed with the reading. Both started reading the third paragraph of the story (see figure 2).

> What sort of crazy system is this? To be inconspicuous, I walked away from the intersection before I scribbled what I had seen on a piece of paper. My head, dancing with confusion, I wandered into what appeared to be a coffee shop and sat down at the counter A neatly-dressed young man appeared and informed me that things were a bit hectic and that it would be at least 30 minutes before I could get to see a counselor. Meanwhile, he continued, if I chose to wait, I should feel free to help myself to the beverages and snacks that were at the end of the counter. Then, he presented me with a numbered token and disappeared through the doorway beyond "the refreshment" stand. What sort of place I had happened into concerned me only briefly as I poured myself a cup of coffee and sat back down to try to orient myself Where am I?, I thought Where have I been?

Figure 2

At the end of this Dave commented that while he enjoyed the story line developed in this paragraph he did not want to stop and "say something" at that point and rather preferred to read on so as to find the solution to the problem. Raffaella agreed, and they read also the fourth paragraph of the story (see figure 3), implicitly considering the map at the bottom of the page an integral part of that paragraph

> I reconstructed my observations on a napkin. Lighting a cigarette, I noticed the address of the hotel I was staying at, on the back of my matchbook: A37 09 -S22 From that starting point I moved in a straight line to the intersection . . . it's a simple grid; it must have a center at (0,0), ASTC are quadrant designations; and, the blocks, too, are divided into quadrants, for consistency. I'm in quadrant "A", on the 38th vertical block, 9th horizontal block (This is what I generated)

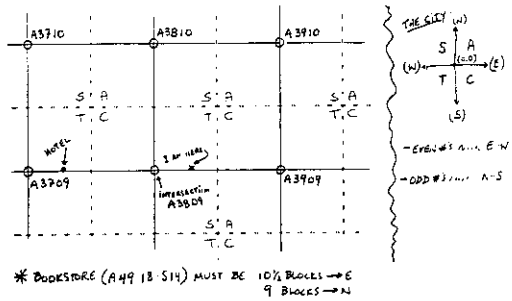

Figure 3

Now, without even thinking about it, they began to talk about the fourth paragraph, using the map provided by the author. Arriving at an interpretation for what it stated meant a lot of going back and forth, bit by bit, between the story and the map. They often referred back to their own drawings as well, comparing them to the map provided in the text, and began to feel as though they were getting a handle on the situation They returned briefly to the confusion they had left unresolved in paragraph two (was there an error in the sentence "Ninety degrees-clockwise, it read A 38 09- T"? but were still not able to find a satisfactory explanation. Nor were they able to decide whether the sentence didn't make sense because of their interpretation or because the author had made a mistake [3].

At this point the class was called back together to briefly share their experiences with the strategy. Raffaella and Dave had to stop, both quite disappointed not to have been able to read the rest of the story.

Reflections on reading mathematics

The reading event reported in the previous section shows how a generative reading of a mathematical story may promote the kind of mathematical thinking called for in the new N.C.T.M. Standards [NCTM, 1989] as Dave and Raffaella were continually engaged in posing as well as solving mathematical problems, exploring and conjecturing, and discussing both their mathematical results and processes. But it also raises important questions about what it means to read mathematics and how to support it in classroom settings. In an effort to better understand the role that reading might play in learning

mathematics, we focus here on these questions, drawing on the reflections of other class participants in addition to those of Raffaella and Dave.

What role can anomalies play in reading mathematics?

For Raffaella and Dave the anomalous address (A49 18-S14) and the unusual layout of the city drew them into the meaning-making process. The confusion they felt led them to define the problem that guided their engagement; it set up a puzzle they felt compelled to solve. The contradictions they discovered as they constructed a map of the city and the difficulty of interpreting the sentence, "Ninety degrees-clockwise, it read A38 09-S," encouraged them to delve further into the problem, trying out different interpretations and challenging both their own system and that of the author.

Not everyone saw these anomalies as problems, however. During the second class period, the whole class read and discussed the report of Raffaella and Dave's experience. Lisa and Tracy's comments suggest that whether confusion engages the reader in problem definition also depends on his/her expectations for the reading event.

Lisa: Well, after reading yours [the report], I felt like when Tracy and I discussed it . . .

Tracy: We skipped through that whole thing . . .

Lisa: I think we skipped over all the mathematics and just read it for comprehension.

Tracy: I just assumed that the mathematics would fall into place. (Lisa agrees.) I mean, I figured John wrote it [so] it's going to fall into place.

Lisa: And if there had been a question at the end, then maybe I would have gone back in [to the reading].

Notice that Tracy and Lisa were guided not only by their expectation for reading for "comprehension of the story" but by their belief that the author, a friend, wouldn't purposefully set out to confuse them. As this discussion continued, the effects of instructional history and connections to other texts became clear.

Lisa: I recognized that there was math in it, but I didn't find that I had to solve a problem . . .

Dave: What's the "have to?" What compels one to solve the problem?

Lisa: A question.

Dave: I perceived a problem and I was compelled to solve it.

Lisa: What were you solving though?

Dave: Discerning what the pattern was and how the city was laid out. (Lisa nods.) What the progression of the pattern would be.

Lisa: (to Tracy more than to Dave) So I guess we assumed that we were going to be told what it was and they wanted to figure it out on their own?

Tracy: Yea, because remember we said, "Do you think we should stop and figure this out?" And we said, "No." We just didn't see it that way.

Donna mentioned that reading "Moving around the city" brought to mind other texts she had read.

Donna: I had an entirely different idea, because I'm a person who reads *Games Magazine*. And if any of you read *Games Magazine*, this is a typical introduction to a *Games Magazine* puzzle. It's exactly how I pictured it. I had to look carefully at some of the clues and had to figure out what's going to be the key question at the end. Am I reading it carefully enough so that I don't have to go back.

Later in the class, Denise returned to the issue of whether anomalies necessarily require the reader to stop and reflect.

Denise: . . . I still see that second paragraph as — that was a sense of confusion. And not that we had to immediately clarify that. That he was confused and you either accepted his confusion and went on to see what he did and — I didn't see that it was my problem to — at that point in the story — to solve the problem.

Dave: The reader has control over that. You can decide it's not a problem and I can decide it is a problem.

Clearly, anomalies and their role in meaning-making are not determined by the text alone. The readers' expectations, which grow out of reading experiences with other texts and in other contexts, together with the context of the situation, influence the significance the reader assigns to an anomaly. Yet the understanding that confusion, contradictions and doubt are not necessarily a fault of the reader, but rather play a key role in making sense of the text, is crucial to the notion of reflective thinking. With respect to mathematics in particular, it may challenge the common perception that mathematics is the discipline of certainty, and encourage students to realize the importance of problem setting, values, and personal judgement in the learning of mathematics.

What role can social interaction play in reading mathematics?

The "say something" strategy offers readers the opportunity to interact in the course of reading. Raffaella and Dave found that after some initial adjustments to each other's reading speed, they fell into a smooth rhythm of reading, talking, and problem solving. They tended to use each other as a sounding board for working out ideas and as a support system for handling points of confusion. Other members of the group also commented on how the social interaction promoted the generation of ideas. Lisa enjoyed the opportunity the strategy provided for seeing the piece from another perspective.

Lisa: You get to voice your opinions with somebody. You get a feel for if you're on the right track. You're broadened by new ideas, somebody else's viewpoint. You're able to see more, I think, and go more places when you get another person's opinion — someone else's input and it just makes it more enjoyable to share, I think.

The sense that interaction provided a support system when readers were confused was a theme in many of the participants' comments.

Ken: It's less frustrating to be confused when you have somebody confused with you. (The class laughs in agreement.) . . .

Denise: There was a sense of confusion that this portrayed. And I got to thinking, well, you know it was OK to say, "I'm confused. Are you confused?" And then I thought if somebody tries to read something in math they are often confused. But they don't have somebody to say, "Are you confused? Yes, I'm confused. So let's go on." They won't. They say, "I'm confused — I quit!" And that's what they do.

The participants also noted that the strategy affected their reading strategies.

Laurie: It forced me to slow down. I don't have enough time to read and so I read fast and read for key ideas and I had to stop and wait . . . So I found it made me slow down and think. And I had to process it because I had all that time.

In short, approaching reading as a social process provided a context that supported risk-taking and reflection, both of which are critical if learners are to move beyond algorithmic thinking in mathematics.

What does it mean to comprehend when reading mathematics?

From a transactional perspective, to comprehend a text means to transform it in such a way as to produce understanding, that is, to duplicate the author's creative role and not simply the author's message [Rosenblatt, 1978]. This is what Raffaella and Dave did when, naturally and almost immediately, they picked up pencils and began to draw their own maps in order to understand the geometry of that strange city. They seem to have taken the idea of comprehension as transformation literally! As Ken noted in his journal, "say something" in mathematics may often become "say and do." Reading with pencil and paper allowed Dave and Raffaella to read reflectively; rather than take the author's portrayal for granted, they took on the role of the author, consequently raising questions, seeing patterns, and exploring new possibilities as they moved through the material.

Some members of the group also felt that translating the text in this way was essential to comprehension, though they may not have chosen to do so on this occasion.

Anna: . . . I think that we tried to solve the problem . . . and we didn't. For a moment we tried to but we didn't and then we read. And our mistake was that we didn't take a pencil and a piece of paper and draw something. . . . Donna said that there is a drawing [in the story] but the point is that you don't have to look at [it]. You're not supposed to look at the drawing — you have to make the drawing.

Ken pointed out that the reader may decide not to construct a drawing in the case where there's an expectation that the author will clear up the confusion.

Ken: There's that notion of waiting for the author to give you the answer. . . . If you do the drawing yourself, you are looking for the answer on your own. If you just keep reading and wait to see what the author's explanation is, then it's a whole different kind of thing.

. . .

Denise: Yes, but there's accepting the drawing and verifying the drawing. And . . . even though the drawing was there, they [Raffaella and Dave] were checking it. They seemed to think there was a discrepancy in the drawing. So more than accepting what was there, they were verifying or trying to reproduce it.

Though the participants did not agree on whether actually drawing their interpretations was important, they did see comprehension as an active process of sense-making. This perspective stands in sharp contrast to the idea that reading mathematics is the quintessential case of denotative reading, that is, reading to extract the message from the text. The "say something" strategy drew readers into the search for meaning and may serve as a demonstration that the process of reasoning mathematically is as important as the outcome of that search.

Concluding thoughts

The experiences described in the previous sections suggest that "reading to learn mathematics" activities could be a valuable addition to school mathematics. At the very least they offer mathematics students a variety of effective strategies for reading mathematical texts and, thus, help address the problem of students' reading difficulties as identified by most mathematics teachers. We believe, however, that integrating "reading to learn mathematics" into mathematics instruction could have a more radical impact, as it could contribute to a redefinition of what it means to understand and learn mathematics, as well as what it means to read.

First of all, "rich" mathematical texts can help challenge the commonly held view of mathematics as a rigid body of predetermined rules by providing information on the construction of mathematical knowledge, the limitations inherent in mathematics results and their applications, and the affective as well as cognitive dimensions of doing mathematics. In turn, the transactional strategies employed in reading these texts can reinforce this view in that they may help students understand that any real learning requires an active and generative involvement and that their attempts to master mathematics by simply memorizing information transmitted by an authority is doomed to failure. Our hope is that "reading to learn mathematics" activities, by inviting students to be active readers, will also encourage them to become active problem solvers and critical thinkers in the context of mathematics instruction and beyond.

The questions we have raised about the learning potential of "reading to learn mathematics" are intended to open up a conversation on new ways in which language can support the learning of mathematics. The challenge before us is to draw mathematics educators, teachers, and students into the discussion about reading mathematics so as to begin collaborative research. Only in the world of classrooms, in collaboration with teachers and students, will we learn how far it is from the potential to the actual benefits of the framework we have proposed, and what it will take to close the gap.

Notes

[1] This course, entitled "Reading to Learn Mathematics," was offered at the University of Rochester in Spring 1989, as an integral part of the research project on "Reading to Learn Mathematics for Critical Thinking," supported by the National Science Foundation (grant #MDR-8850548). The course met weekly for about three hours, for a total of 16 meetings.

The course was co-designed and taught by the authors, Raffaella Borasi and Marjorie Siegel, and built on their respective expertise and background in mathematics and reading education. Ten mathematics teachers from a variety of schools in the area and three doctoral students in mathematics education participated in the course: George Isgrigg — teaching in an urban high school in the Rochester City School District; Dave Baker and Judi Fonzi — teaching in an alternative high school in the Rochester City School District; Lisa Grasso and Tracy Markham — teaching in suburban-rural middle schools; Laurie Platt and Donna Rose — teaching in suburban-rural high schools; John Sheedy — teaching emotionally disturbed adolescent at the Convalescent Hospital for Children in Rochester; Denise Anthony — a certified secondary school mathematics teacher who is currently working with teachers, parents and students as a "mathematics consultant" for a suburban elementary school in the Rochester area; Ken Steffen — a certified mathematics teacher currently

involved in social work and tutoring; Richard Fasse, Constance Smith and Anna Sotiriadou — full time doctoral students in mathematics education

Careful documentation of the experience was collected. All our meetings were audio-taped, and a very detailed report of what happened in each session, including the verbatim transcription of most of the dialogue, was prepared. In addition, the participants kept a weekly "class journal" where each member of the group shared ideas, reflections, experiences, etc. related to the theme of the course. Each entry was duplicated and distributed to all the members of the class, thus providing an additional forum for discussing "reading to learn mathematics" as well as valuable data about each individual's development throughout the course. The quotes reported in this article have been taken verbatim from this material, with the permission of the participants.

[2] A revised text of this story (which was unpublished at the time the experience took place) can now be found in Borasi, Sheedy and Siegel [1990].

[3] Confronted later with this argument, the author of the story felt challenged to go back to the text and settle the matter. In his next journal entry he wrote: "Rest assured, faithful readers, there are mistakes," and explained that inadvertantly the text distributed to the group (and reported in figures 1, 2 and 3 in this article) was an earlier incorrect version of the story; he clarified all doubts by providing everyone with a copy of the latest "correct" version of the story (the one now reported in Borasi, Sheedy and Siegel [1990]).

References

Abbott, E. A. [1952] *Flatland.* New York: Dover

Borasi, R., & Brown, S. I. [1985] A novel approach to texts. *For the Learning of Mathematics*, 5 (1), 21-23

Borasi, R., & Rose, B. [1989] Journal writing and mathematics instruction. *Educational Studies in Mathematics* 20 (4), 347-365

Borasi, R., Sheedy, J. R., & Siegel, M. [1990] The power of stories in learning mathematics. *Language Arts*, 67 (2), 174-189

Britton, J. [1970] *Language and learning.* London: Penguin Press

Campbell, D. M., & Higgins, J. C., (Eds.) [1984] *Mathematics: People, problems, results.* Belmore, CA: Wadsworth

Carey, R., & Harste, J. [1985] Comprehension as context: toward reconsideration of a transactional theory of reading. In R. Tierney, P. Anders, & J. Mitchell (Eds.) *Understanding readers' understanding.* Hillsdale, NJ: Lawrence Erlbaum Associates

Dewey, J. [1933] *How we think.* Boston: D. C. Heath

Eco, U. [1979] *The role of the reader.* Bloomington, IN: Indiana University Press

Emig, J. [1977] Writing as a mode of learning. *College Composition and Communication*, 28, 122-127

Grumet, M. [1985] Bodyreading. *Teachers College Record*, 87(2), 175-193

Harste, J. [1985] Portrait of a new paradigm: reading comprehension research. In A. Crismore (Ed.) *Landscapes: a state-of-the-art assessment of reading comprehension research 1974-1984.* (USDE-C-300-83-0130) Washington, D.C.: U.S. Department of Education

Harste, J., Pierce, K., & Cairney, T. (Eds.) [1985] *The authoring cycle: a viewing guide.* Portsmouth, NH: Heinemann

Hayes, D., & Tierney, R. [1982] Developing readers' knowledge through analogy. *Reading Research Quarterly*, 17, 256-280

Kline, M. [1980] *Mathematics: the loss of certainty.* Oxford University Press

Krause, E.F. [1986] *Taxicab geometry.* New York, NY: Dover Publication

Newman, J. R. (Ed.) [1956] *The world of mathematics.* New York, NY: Simon and Schuster

Nolan, J. [1984] Reading in the content area of mathematics. In M. Dupuis (Ed.) *Reading in the content areas: research for teachers* (pp. 28-41). Newark, DE: International Reading Association

Oaks, A. [1987] The effect of the interaction of mathematics and affective constructs on college students in remedial mathematics. Unpublished doctoral dissertation. University of Rochester, Rochester, NY

O'Mara, D. [1981] The process of reading mathematics. *Journal of Reading*, 25, 22-30

Pinne, S. [1983] Teaching reading in the mathematics class (ERIC Document reproduction no. ED 228 919)

Rose, B. [1989] Writing and mathematics: Theory and practice. In P. Connolly & T. Vilardi (Eds.) *Writing to learn mathematics and science* (pp. 15-30). New York, NY: Teachers College Press

Rosenblatt, L. [1978] *The reader, the text, the poem.* Carbondale, IL: Southern Illinois University Press

Siegel, M. [1984] Reading as signification. Unpublished doctoral dissertation, Indiana University, Bloomington, IN

Siegel, M., & Carey, R. [1989] *Critical thinking: a semiotic perspective.* ERIC/Reading and Communication Skills Monograph Series. Bloomington, IN: Indiana University

Siegel, M., Borasi, R. & Smith, C. [1989] A critical review of reading in mathematics instruction: The need for a new synthesis. In S. McCormick & J. Zutell (Eds.) *Cognitive and social perspectives for literacy research and instruction* (pp. 269-277). Thirty-eight Yearbook of the National Reading Conference. Chicago, IL: The National Reading Conference, Inc.

Stempien, M. & Borasi, R. [1985] Students' writing in mathematics: Some ideas and experiences. *For the Learning of Mathematics*, 5 (3), 14-17

Several nights when Wittgenstein was in a bad way — raw and restless, distracted — Russell saw it: nothingness made palpable. Russell had long felt that there was something fearful and disfiguring in solipsism, a willful blinding, like Oedipus dashing out his eyes. And it pained him to see Wittgenstein clawing to get a foothold on something, a crevasse of life or logic — himself. (...) What the solipsist *says* — that the world is *his* world — this seemed to Russell disagreeable and somewhat boring; above all, it was *lonely*. But still it contained more than a grain of truth about the nature of our lives, of how we view life from the inside, peering out through crabstalk eyes. Wittgenstein talked and talked about this but that he spoke at all, Russell thought, gave the lie to his contentions. If he spoke at all, it seemed to Russell, then he must believe there are ears to listen. For Russell it was an article of faith: despite what the solipsist says, people's hopes and sensations *must* somehow be connected, however haphazardly, like beads on the slenderest of threads.

Bruce Duffy