Steering between Skills and Creativity: a Role for the Computer? [1]

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Background

Children's and adults' mathematical knowledge frequently appears to be in a state of crisis - a crisis of skills or a crisis of creativity. In the U K and the U S A , there are now waves of enthusiasm for basic skills, mental arithmetic and target setting. Studies comparing England's performance in mathematics with other countries have shown England to be performing relatively poorly in comparison with others For example, evidence from the Third International Mathematics and Science Survey (TIMSS) indicated that our Year 5 pupils (aged 9 and 10) were among the lowest performers in key areas of number out of nine countries with similar social and cultural backgrounds (see Harris, Keys and Fernandes, 1997) A huge, multi-million pound National Numeracy Strategy is now underway in the U K and in its first report (DfEE, 1998), the TIMSS studies were cited as one reason for the new focus on numeracy.

At the same time, the news from the Pacific rim reports rather different pressures for change. For example, Lew (1999) describes Korea, a country which scores very highly on most international comparisons of mathematics attainment, as being in 'total crisis' in mathematics. He illustrates graphically how most students seem quite unable to relate their well-developed manipulative skills to the real world. Lew argues that:

> the direction of the mathematics curriculum in Korea should change from emphasis on computational skills and the 'snapshot' application of fragmentary knowledge to emphasis on problem-solving and thinking abilities. (p. 221)

Similarly, Lin and Tsao (1999) present a picture of test obsession in Taiwan, where college entrance examinations dominate students' (and parents') lives Both of these countries are encouraging more 'open' curricula to include opportunities for mathematical creativity: that is, adapting their curricula to be more like those currently being vilified in the U.K. and the U.S.A.

Other data from TIMSS suggest that English children are comparatively successful at applying mathematical procedures to solve practical problems and are generally positive about mathematics. Is it possible to retain these strengths while at the same time consolidating arithmetic skills and developing the ability to construct deductive arguments? (The latter area is one in which we have shown our students to be surprisingly weak: see Healy and Hoyles, 2000.)

The challenge for the international mathematics education community perhaps appears at first sight to be the design of a globally-effective balanced curriculum. From a U.K. perspective, this would build on the wealth of informal mathematical knowledge students bring to school, while at the same time drawing attention to mathematical structures and properties and introducing them more systematically to mathematical vocabulary The mathematical curriculum of the next millennium should harness children's motivation without losing their mathematics - and I envisage that the computer might offer just the context to help do this

A role for the computer

I was inspired in the early 1980s by Seymour Papert's (1980) radical vision of a mathematics that was playful and accessible, but at the same time rigorous and serious We [2] dreamed (and still do) of children actively expressing mathematics in different ways We wanted children to learn by conjecture, reflection on feedback and debugging, as part of their own meaningful projects that required planning, sustained engagement with mathematical ideas represented in diverse ways and the bringing together of a range of skills and competencies Logo was the vehicle or the catalyst for many of us to try to achieve those dreams. In doing this work and studying the work of others, our eyes were opened to students' strategies and potential - computer interaction was a window onto possibilities, an environment to illuminate pupil meanings and interpretations (Hoyles, 1985; Hoyles and Noss, 1992)

Since that time, we have designed a range of microworlds around different 'open' software and have further developed the notion of technology as a means by which knowledge can become concretised and connected. We have also undertaken more systematic investigation of the nature of the child's activity and how it can be better understood. Inevitably, the boundary of what is and is not mathematics has been explored (see Papert, 1996): some say that working experimentally with the computer counts as mathematics, some that it is not, and many are not sure. The software may have changed but the issues have not and the location of this boundary is still a matter of hot dispute.

If we want to design investigative environments with computers that will challenge and motivate children *mathematically*, we need software where children have some freedom to express their own ideas, but constrained in ways so as to focus their attention on the mathematics. Are there lessons to be learned from all the work that has been done with these sorts of environments over several decades? What do we actually know about how children can better learn mathematics with technology?

Mathematics comprises a web of interconnected concepts and representations which must be mastered to achieve proficiency in calculation and comprehension of structures (for elaboration of this theoretical framework, see Noss and Hoyles, 1996) Mathematical meanings derive from connections - *intra-mathematical connections* which link new mathematical knowledge with old, shaping it into a part of the mathematical system and *extra-mathematical meaning* derived from contexts and settings which may include the experiential world. Yet how are these meanings to be constructed? How is the learner to make these connections? To what extent can the software tools encourage this process of meaning-making and connection-making?

A critical weakness of many mathematical learning situations has been the gap between action and expression and the lack of connection between different modes of expression Over many years, our central research priority has been to find ways to help students build links between seeing, doing and expressing (see, for example, Noss, Healy and Hoyles, 1997) We have shown that technology can change pupils' experience of mathematics, but with several provisos:

- the users of the technology (teachers and students) must appreciate what they wish to accomplish and how the technology might help them;
- the technology itself must be carefully integrated into the curriculum and not simply added on to it (see Healy and Hoyles, 1999);
- most crucially of all, the focus of all the activity must be kept unswervingly on mathematical knowledge and *not* on the hardware or software.

Computers and the curriculum

To date, work with computers in mathematics education has largely been concerned with construction and the potential of software to aid the transition from particular to general cases - specific instances can be easily varied by direct manipulation or text-based commands and the results 'seen' on the computer screen (see, for example, Laborde and Laborde, 1995). Yet, even if students develop a sense of how certain 'inputs' lead to certain results, there remains the question of how to develop a need to explain, a need to prove, as part of (rather than added on to) this constructive process

In countries like the U.K., where proof has all but disappeared from the school curriculum, this issue must be addressed urgently if we are to avoid limiting the mathematical work for most children by the introduction of computers If we fail, the majority of our students will simply be subjected to even more convincing empirical argument - for example, using powerful dynamic geometry tools simply to measure, spot patterns and generate data.

There is an alternative which we are in the process of investigating We have designed tasks where, through computer construction, students have to attend to mathematical relationships and in so doing are provided with a rationale for their necessity. Thus, the scenario we envisage is one where students construct mathematical objects for themselves on the computer, make conjectures about the relationships among them and check the validity of their conjectures using the tools available.

This forms part of a teaching sequence which also includes reflection guided by the teacher away from the computer, and the introduction of mathematical proof as a particular way of expressing one's convictions and communicating them to others It is in this way, we suggest, that constructing and proving can be brought together in ways simply not possible without an appropriate technology: formal proof is simply one facet of a proving culture, revitalised by the 'experimental realism' of the computer work (Balacheff and Kaput, 1996).

Lulu Healy [3] and I have devised algebra and geometry teaching sequences which follow these criteria Our activities were developed after analysing students' responses to a nationwide paper-and-pencil survey to assess students' conceptions of proving and proof (see, for example, Healy and Hoyles, 2000). This survey was completed by 2,459 fifteen-year-old students of above average mathematical attainment from across England and Wales. Each teaching sequence was designed 'to fit into the curriculum' and to fill at least some of the gaps the survey had revealed in the understandings of students. Overall, eighteen students from three schools have worked through the sequences, each of which took nearly five hours of classroom contact supplemented by homework.

I now present snapshots from case studies of two students who engaged in these sequences The first illustrates the gains that can be made by connecting skills to creative exploration through computer interaction, while the second points to potential pitfalls in planning 'the best' mathematics curriculum incorporating technology

Tim: making the step to explaining in algebra

Tim was a quiet and diligent student who knew about proof as something that involved verification and explanation, though only recognised it in the context of algebra - a natural consequence of our curriculum with its emphasis on generalising and explaining number patterns (see Figure 1)

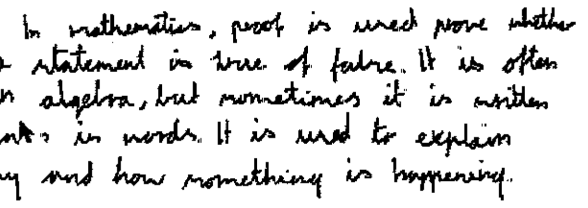
In mathematics, proof is used prove whether a statement is true of false. It is often in algebra, but sometimes it is written down in words. It is used to explain why and how something is happening.

Figure 1 Tim's initial view of proof

It was also clear from Tim's choices in the paper-and-pencil survey that he had a preference for visual argumentation: he evaluated the visual 'proof' by Yvonne in exactly the same way as a 'correct' formal algebra proof; when asked about this, it was clear he 'saw' the general structure *through* this particular visual example (see Figure 2).

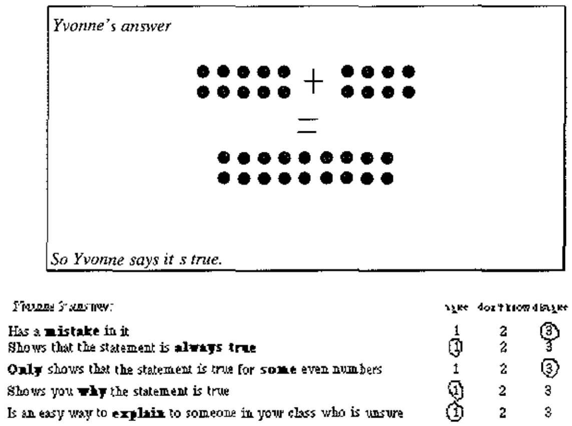

Figure 2 Tim's evaluation of a visual proof

In the first algebra session of our teaching sequence, students are introduced to our microworld, *Expressor*, in which they build 'matchstick' patterns of number sequences by constructing simple programs. They are encouraged to connect their computer constructions to corresponding mathematical properties and hence find a general formula for the number sequence explaining why any conjecture is true or false, by reference to computer feedback and to the mathematical structures they have constructed. Similar work with more complex number sequences is undertaken in the third session.

Tim found this work of generalising through programming both engaging and challenging - in fact, he described it as the most enjoyable part of our teaching. He also saw a strong connection between proving and his computer work.

> I liked the programming stuff - that helped [to write proofs] because it sort of showed how it was constructed so [ ] It helped prove because it showed you how they were made [...] how that construction was made step by step

In the second session, students are introduced to writing formal algebraic proofs and helped to 'translate' their Logo descriptions of the mathematical structures into algebra. They are also taught how to construct deductive chains of argument - systematically starting from properties they had used in their constructions in order to deduce further properties. Both of these tasks are unfamiliar to U K students.

Here is an example. Students are asked to investigate the properties of the sums of different sets of consecutive numbers. They construct by programming a visual representation of numbers as columns of dots (shown in Figure 3 below). [4] Students can, for instance, move the bottom right dot to the bottom left, see that it would 'even up' the three columns, and convince themselves that the conjecture that the sum of three consecutive numbers is divisible by three is always true.

Although these moves can be achieved by, for example, using counters, in *Expressor*, the visual arrangement has a simultaneous 'algebraic' description which is constructed by the children. In Figure 3, a program *col* has been written to generate 6 (n), 7 and 8 columns. The dots can be dragged into columns as with real counters; but as this is done, a recorded 'history' of the actions is stored (see the history box in Figure 3) in the form of fragments of computer program

This code is executable: that is, it can be 'run' to produce the output (or part of the output) which produced it. There is, therefore, a duality between the code and the graphical output of the dots; the *action* (on the dots) to produce a new visual arrangement and the *expression* (in the form of pieces of program) are essentially interchangeable. The code is a rigorous description of the student's action to construct a particular image, and her actions are executable as computer programs. A box n is used to store the smallest of the three numbers and our student might see that what is in the box n hardly matters, and therefore come to realise that the theorem is independent of the first number

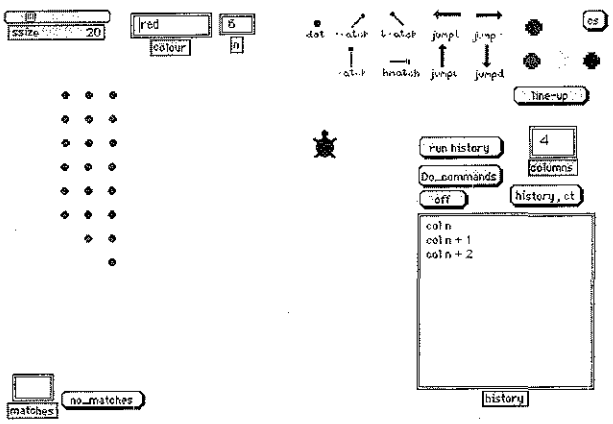

Figure 3 A typical Expressor screen to explore the sum of three consecutive numbers

How did Tim cope with this activity? In his first session, he had been seeking explanations for a general rule in the general symbolic expressions he had constructed (in the form of programs). He built his three columns of dots in *Expressor* and was faced with a screen rather like Figure 3. Then he wrote: $n + (n + 1) + (n + 2) = 3n + 3$.

But, he obtained this equivalence not as a result of a manipulating algebra but by reference to our microworld: he noted that the three original columns could be changed to three columns of length n and a 'tail' of three

Tim generalised this method to find factors of sums of different numbers of consecutive numbers - always considering columns of dots and a tail, but flexibly using visual manipulation and argumentation. For example, to show that it was impossible for the sum of four consecutive numbers to have a factor of four, and so could never add up to forty-four, he visually moved dots, as he describes in Figure 4.

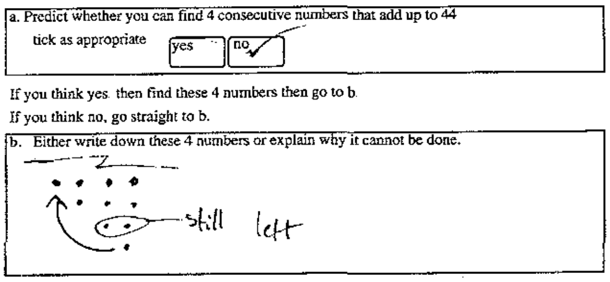

Figure 4 Tim's proof that the sum four consecutive numbers is not divisible by four

Finally, together with his partner, Tim also came up with a new, visual 'proof' that the sum of five consecutive numbers must have a factor of five. He again focused on the tail of dots, but combined their total using inductive reasoning starting from the case when n was 0

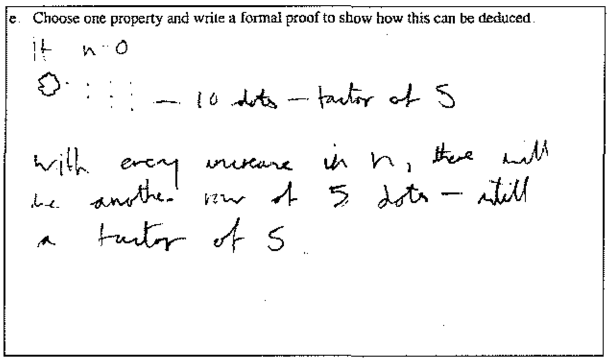

Figure 5 Tim's inductive proof that the sum of five consecutive numbers is divisible by five

By this time, it was clear that Tim had found two well-connected ways to explain it: constructing symbolic code and manipulating visual expressions. His explanations came from linking logical and general arguments with visual representations (columns of dots) – and not from algebra, even though he clearly recognised its importance. This gap in his repertoire of skills is well illustrated in his final homework (Figure 6). Tim creatively generalised 'the dots' microworld into thinking of multiplication as a rectangular array of dots, whose rows could be paired off leaving 'one left over'. But, he was still unable to multiply out brackets correctly.

In contrast to Tim, Susie could say nothing about what proof was and appeared clearly confused about the generality of a mathematical argument. She selected empirical arguments as her own approach in all the multiple-choice 'proofs' in our survey, both in geometry and algebra, and described these as both general and explanatory. She thought mathematics was quite complicated and, in fact, admitted to hating it.

Although Susie offered no description of proof or its purposes, it emerged in interview and by watching her computer

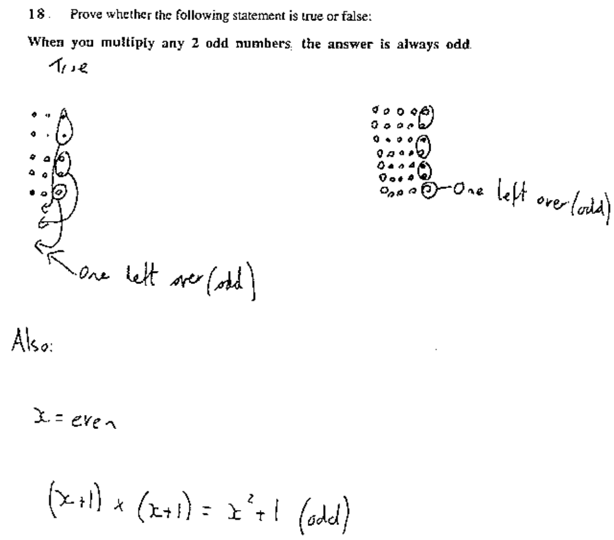

Figure 6 Tim's two explanations

work that she did have a view about proof – it was about examples (many examples) It was enough to have shown a statement was true many times Additionally, for Susie, there was another important aspect of proof, which was a rule or formula But, its role was to obtain more marks from the teacher rather than to confer generality – the examples were enough for this

Although Susie could write formal proofs, she did not see them as general and found them no more convincing than empirical evidence: her two 'modes of proving' – examples and formal proofs – were apparently completely disconnected. She believed, for example, that, even after producing a valid proof that the sum of two even numbers is always even, more examples would be needed to check that the statement holds for particular instances

In our teaching experiments, both in algebra and geometry, we noticed that Susie followed all the instructions carefully, but rarely if ever experimented with the computer. She found it hard to see the computer as a means to try things out when unsure, hard to learn from feedback

I will illustrate Susie's work in algebra by reference to the same tasks described earlier. Susie was considering the sum of four consecutive numbers She constructed the columns of dots and came up with the formula $4n + 6$, ostensibly by making the connection of the '+6' with the 'tail of dots'. For five consecutive numbers, she apparently used the same method to come up with the correct sum of $5n + 10$.

Then she changed her mind. She crossed out the +10, explaining this by writing that she had checked and 'it was 6' From this point on, her written work and explanations were disconnected from any generality suggested by the visual display she had constructed in the *microworld*, except she persisted in showing pictures of columns of dots with a six-dot tail, as illustrated in her homework following this session.

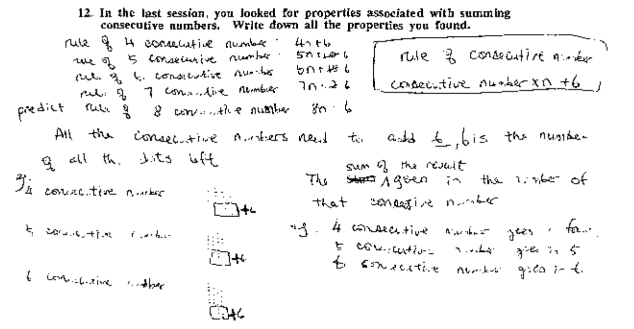

Figure 7 Susie's rule for consecutive numbers

The rupture between particular examples and generalisations can be explained by reflecting on what we had discovered as Susie's goal in mathematics – to find examples and then a *rule*. She had achieved this: she had found a rule in which numbers could be substituted and even had pictures to illustrate it, but the pictures did not represent the structure of consecutive numbers.

Susie's story is not completely negative. She did make progress after engaging in these teaching sequences. By constructing matchstick patterns, Susie was beginning to appreciate how an algebraic expression could express generality (and not serve merely as something to be manipulated) and, although proving for Susie remained solidly 'a rule plus examples', she did seem to be beginning *to want to explain* as well.

Some snapshots from the geometry sequence

Briefly, here are some insights gained from the teaching sequence in geometry, simply to illustrate further some points raised in the previous sections. This sequence followed a similar pattern to that in algebra. In the first session, students are encouraged to construct simple geometrical objects on the computer with dynamic geometry software, to describe their constructions, connect each with a corresponding mathematical property and use the computer to explore or reject conjectures.

In the second session, students are encouraged to construct familiar geometrical objects (parallelograms, rectangles) on the computer, identify properties and relations of a figure that had been used in their constructions and distinguish some properties that might be deduced from those given by exploring with the computer. In much the same way as in algebra, students are also taught at this point to construct logical deductive chains of argument and write formal proofs based on their computer constructions. In the third session, students are faced with more unfamiliar constructions and proofs, which again they can tackle experimentally on the computer.

So how did Tim fare in geometry? Geometry for Tim, as for most of our students, was far more problematic than algebra. He did make some progress in that he learnt to write clear descriptions of his constructions, translate them into given properties and 'see' deduced properties. The computer work helped Tim 'see' relationships and convinced him of their necessity, but the links he could make between constructions and proofs or even explanations were much more tenuous than in algebra.

> T: Well, you could actually see like if they were congruent – you could take however much you were allowed to take and actually make a triangle. If it was congruent then you could [..] tell it was
>
> CH: Tell it how?
>
> T: Just by seeing
>
> CH: And did that help you write your formal proofs?
>
> T: Not really – not the formal stuff. But, well it made it more enjoyable.

Tim found it hard to appreciate and reproduce 'the game' of proving: that is, systematically separate givens from deduced properties and produce reasons for all his steps. He found the language of formal geometry proofs inhibiting – it stopped him 'seeing it all'.

The construction task in the third session was important in his progress. He had to construct a quadrilateral where adjacent angle bisectors were perpendicular and describe and justify its properties. Tim found this hard, but, after much experimentation and 'measuring' lots of angles, he eventually 'saw' the key relationship – two parallel lines – not by 'just seeing them' but by noticing two numerically equal angles and dragging. The important point is that the measurements for Tim were not simply collecting empirical evidence: they were not only part of the conjecture but also and crucially part of his proof. When he talked about two angles of 44°, it was clear to us that he was seeing *through* the numbers to the general case – just as he had done in *Expressor*. As in algebra, Tim was using his interaction with the computer to help him to find explanations.

Susie again presents us with a different picture. When it came to constructing proofs, Susie's responses were quite unlike the majority of students in our survey. Her proofs in geometry were far better than in algebra and the proof she constructed for the more complex geometry question (a standard Euclidean geometry proof) was much better than almost all the survey students. [5]

Yet despite being able to write these perfectly formal proofs, we found on interview that they were, for Susie, rituals, disconnected from any appreciation of the generality of the mathematical properties and relationships she used. As in algebra, she believed that even after proving a statement, its validity had to be verified in any specific set of cases (see also Chazan, 1993). In her response to the survey, for example, Susie was certain that she needed examples to check that the statement (the sum of the angles of a triangle is always 180°) held for right-angled triangles. These findings point to the complexity of proving in geometry.

So how did Susie manage in our teaching experiment in geometry? In fact, we found rather little evidence that Susie made any progress in geometry as a result of engaging in these tasks. Computer interaction did not seem to help Susie come to appreciate the generality of a proof and the proving process. Also, before she started the sequence, Susie could already construct formal geometry proofs in the context of familiar and fairly routine problems Faced with more unfamiliar situations such as those described above, she was lost and, unlike Tim, was unable to use the computer to help her

We can throw light on this lack of progress, by reference to two factors: her interactions with the computer and her interpretation of feedback First, as I have mentioned, Susie did not exploit the computer to test hypotheses or try things out. But success in our tasks *relied* on experimental interaction – we did not *expect* our students to know immediately what to do. Second, Susie interpreted the feedback from the dynamic geometry software in a way which certainly was unexpected. For her, dragging a Cabri construction was *not* testing a relationship, exploring a property – but merely a way of generating many examples.

Once we had noticed this, we could see it was completely consistent with Susie's view of proof Susie's reflections on the use of the computer in mathematics are also relevant. When interviewed, it was clear that she thought the computer had given her ideas about 'what it was all about' and had done so quickly: but, rather crucially, it makes examples and checks them. [6]

There were positive outcomes for Susie in relation to her response to mathematics. All through the teaching experiments, Susie picked the most enjoyable aspect of her mathematical work as 'finishing it', 'getting it right', 'writing down the results' Yet, in algebra, we were beginning to catch glimmers of enjoyment and engagement: Susie began to mention her *activity* rather than simply its end point. In her final interview after the teaching sequences too, she spontaneously offered how much she had enjoyed the work with the computer, although it must be admitted this was only as a contrast with 'normal maths'. Even so, this more positive attitude might be a key to Susie's further development.

Discussion and conclusion

To begin an explanation of the two very different student responses to our teaching sequences and work with computers, we have to consider cultures and curricula – huge issues well beyond the scope of the article, but which simply cannot be ignored. Susie's profile is somewhat less 'odd', if it is known that she had only studied mathematics in an English school for one year – she was, in fact, from Hong Kong and had been educated there, although the language of instruction had been English. Unlike most other students in our survey, Susie had been taught formal geometry proofs as well as algebraic formulae and manipulation and had little experience of 'doing investigations'.

As I have tried to show, Susie's lack of progress might at least partly be explained by the disjuncture between the assumed starting points of our tasks, particularly those with computers – in terms of sense of proving and student-computer interaction – and Susie's world. We had students like Tim in mind when we designed our sequences; students reared in an investigative culture – who wanted to explain but who lacked the tools to do it

Susie was at odds with this culture in terms of her beliefs about mathematics and about proof Our activities did not build on *her* existing framework for proof, did not help *her* to connect her informal mathematics to our agenda Our story of Susie provides compelling evidence that we must take seriously prevailing beliefs about mathematics and computers in our curriculum planning and resist the temptation simply to import 'exemplary tasks' from other cultures.

The comparison of Tim and Susie's work cautions against any assumption that the computer will lead to a single set of learning outcomes or bring about particular changes We can only design optimal activities within very limited parameters, given that how children interact with and learn from software depends on their expectations and beliefs Curricula must seek to build on student strengths – in the case of U.K. on a confidence in conjecturing and arguing – and connect these strengths to new representations Students like Tim respond positively to the challenge of attempting more rigorous proof alongside their informal argumentation Susie was less successful as the culture which had shaped our teaching and task design was not shared by her

Clearly, not all U.K. students are like Tim or students from Hong Kong like Susie. But the purpose of elaborating their stories is to guard against the stupidity of 'transferring' curricula simplistically across cultures, replacing a curriculum which over-emphasises an empirical approach with one in which students are simply 'trained' to write formal proofs. It is all too easy for countries simply to flip between two states of the 'skill and creativity' crisis while attempting to model curriculum innovations which look so alluring to the distant observer. (For an interesting discussion of the cultural implications of traditional Chinese views, see Leung, 1999.)

So, returning to my initial question about the desirability of a globally-effective mathematics curriculum, I can only conclude that this goal is fundamentally misguided We should not set our sights on the same curricular sequences and targets, because these are not the same in any reality Incorporating what look like comparable tasks into our curricula will not mean that the meaning derived from them will be comparable. [7] Cultural effects might even be magnified when activities involve technology, which carries its own sets of beliefs and agendas. I have tried to illustrate how the power of microworlds to engage our students with mathematics rests first and foremost on what they believe about curriculum goals and intentions and about what they can learn from computer interactions.

Our aim in mathematics education may be to reach a common goal – a mathematical literacy comprising a better balance between skills and competencies on the one hand and engagement with mathematical thinking on the other. We might even agree that the computer might have a useful role to play. Although it is deeply illuminating and exciting to move beyond the surface features and slogans of international comparisons and focus on what *mathematics* and what *education* we are striving to achieve in our countries, ultimately we have to tease out different routes toward this common goal for ourselves

Notes

[1] This paper is a modified and updated version of a keynote address presented to the ICMI-East Asia Regional Conference on Mathematics Education

[2] 'We', in this article, refers to my close colleague in Mathematical Sciences at the Institute of Education, Richard Noss

[3] See the U.K. ESRC project, *Justifying and Proving in School Mathematics* (Ref R000236178). I wish to acknowledge the central work of Lulu Healy in all aspects of this project.

[4] Noss (1998) borrowed this task in *Expressor*, along with some student work from our pilot study, to illustrate how alternative representations can be used to offer 'a channel of access to the world of formal systems' (p 10).

[5] She produced an almost perfect formal proof – something only achieved by 4 8% of the students in the survey and one which 62% of students did not even start

[6] She also thought that the computer helped her to remember, but there was a disadvantage: 'you can't use the computer in exams'

[7] Similar points have been made in relation to the meaning of test items in TIMSS which are not the same simply because it is the same test (Keitel and Kilpatrick, 1999)

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