The Clinical Interview in Psychological Research on Mathematical Thinking: Aims, Rationales, Techniques

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This paper presents an analysis of the clinical interview procedure as employed in psychological research on mathematical thinking My thesis is that the clinical interview has legitimate uses for three different research purposes and in each case involves somewhat different techniques A subsequent paper will explore the criteria by which one can evaluate the clinical interview and discuss its applications to educational practice.

Historical background

As a prelude to the discussion of the clinical interview's research roles, it is useful to consider the historical roots of the method. Piaget originally developed the clinical interview method as an instrument for psychological research In 1920, working with Simon in the Binet laboratory in Paris, Piaget was engaged in the development of a standardized French version of an English reasoning test. A standard test presents the same items to all children in the same way. No flexibility or variation in questioning is permitted Such standardization of procedure, it is argued, eliminates tester bias and subjectivity. If standardized methods are employed, differences in subjects' performance can be attributed not to variations in the questions but to real differences in the subjects' psychological characteristics. When constructed properly, standard tests permit one to rank children against their peers on some psychological trait in an objective and reliable manner

While the rankings provided by a standard test may be useful for some purposes, Piaget became intrigued with other kinds of issues. For him, what the test failed to measure was far more interesting than what it purported to deal with He observed that children gave unanticipated responses that were quite fascinating. Children's mistakes in particular gave important clues concerning the nature of their thought and seemed to show that it is qualitatively different from adult cognition. For Piaget, the important goal was not to rank children on standard tests but to gain insight into the fundamental nature of their thought Since existing psychological research methods seemed inadequate for research into cognitive development, Piaget felt it necessary to create a new technique. Consequently, he drew on his experiences with psychiatric procedures to develop the "clinical interview", a flexible method of questioning intended to explore the richness of children's thought, to capture its fundamental activities, and to establish the child's cognitive competence.

For some 50 years, Piaget applied the method, in one form or another, to the study of children's thinking. The results are well known: Piaget's work has revolutionized the psychology of cognitive development, so that today, Piaget's is the dominant theory within developmental psychology

Current impact

Piaget's work has also had an enormous impact on research into mathematical thinking. First of all, we have accepted Piaget's basic research goal, namely to explicate the nature of thought. Following Piaget, we wish to understand the intellectual processes underlying mathematical knowledge Whether or not we accept the details of Piaget's theory, we attempt to follow in his footsteps by investigating cognitive structure, specifically the structure of mathematical knowledge As a corollary, we are uninterested in the investigation of mere behavior or in the results of achievement tests. Everyone — almost everyone — agrees that our goal is the study of the mathematical mind in action.*

Another Piagetian influence on our field has been methodological. In recent years, several researchers have championed the use of the clinical method in investigation of children's mathematical thinking [Easley, 1977; Erlwanger, 1975; Ginsburg, 1977]. Indeed, one major journal, *The Journal of Children's Mathematical Behavior*, specializes

*We are not referring here to an "orthodox" Piagetian influence which is reflected in some investigators' interest in classical Piagetian problems, particularly the conservation of number, classification, seriation, formal operations, and the like These investigators employ traditional Piagetian concepts, like the concrete operations, in an effort to understand and explain children's mathematical knowledge. Such investigations are Piagetian in a narrow sense. Other researchers believe that traditional Piagetian concepts suffer from severe limitations with respect to explaining academic knowledge [Ginsburg, 1981] and that other theoretical systems [e.g. information processing theory: Davis, Jockusch and McKnight, 1978] are potentially more useful than Piaget's. In order to achieve Piaget's general goal of understanding the structure of thought, it is therefore necessary to "go beyond" Piaget's admittedly vital contributions by abandoning the specific content of his theory.

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in publishing clinical interview studies

And yet, while achieving a measure of popularity, the clinical interview method remains controversial. We may all accept Piaget's aim of investigating the structure of thought, but we are not unanimous in adopting the method Piaget employed to conduct his research. The clinical interview is an unorthodox method, apparently contradicting what many of us learned about scientific procedure. "Science" dictates that we use standardized, reliable, replicable procedure; the clinical method demands that we treat each child flexibly and therefore often differently from every other. Some investigators concerned with mathematical thinking, and with cognitive research generally, feel that while the method may be productive in informal pilot work it should not be employed for the serious task of gathering data in controlled investigations. In this view, the clinical method is "soft": it is no substitute for the rigorous methods of science. Therefore, the proper approach to the clinical interview is to standardize it, to eliminate its informality, its sloppiness [e.g. Tuddenham, 1970]. Other investigators believe that the clinical method is superior to standard testing and makes possible a degree of insight that is virtually impossible to obtain through standard tests. Clearly, researchers in mathematics education differ in their appraisal of the role and utility of the clinical interview procedure

Perhaps the controversy derives, at least in part, from the fact that the clinical method has received little critical examination. It is quite astounding that although Piaget used the method for almost sixty years, even he did not devote much theoretical attention to the procedure. Piaget himself seems to have written only one major piece on the clinical method, and that in 1926 (the introductory chapter to *The child's conception of the world*). Since that time, other writers seem to have produced few analyses of the method. The contrast with standardized testing is striking: tomes have been published on its methodology. In any event, as a consequence of the lack of critical examination, we understand little of the clinical interview. We are not certain what it should be used for, nor consequently how to evaluate it. So long as this is true, we run the risk of using it inappropriately, thereby subverting our attempts to understand children's mathematical cognition.

The purpose of this paper, then, is to achieve a better understanding of the clinical method as it can be used in research into mathematical thinking. I argue that the commonly accepted goal of investigating the activities of the mathematical mind in fact involves three aims, suggested by Piaget's work: the *discovery* of cognitive activities (structures, processes, thought patterns, etc.) the *identification* of cognitive activities, and the evaluation of levels of *competence*. In any given study, the distinctions among these aims may be blurred, and more than one aim may be involved. But the aims are separable, and they are as valid for us as they were for Piaget. Reviewing the aims separately, I attempt to show how each requires a somewhat distinctive type of clinical method, characterized by particular techniques. Consider then in turn the three aims of cognitive research, their methodological rationales, and the techniques they require

Discovery

Aim In Piaget's view, one basic task of cognitive developmental psychology is to discover the cognitive processes actually used by children in a variety of contexts. At the present stage of our knowledge it is unproductive to begin research, Piaget felt, with an *a priori* "definition" of intelligence. If, at the outset, one "defines" intelligence as "abstract thinking" or the "manipulation of verbal symbols" or "whatever the IQ test measures," the main outcome is to set artificial and unnecessary limits on one's research. To be sure, the definitions focus research on a relatively specific topic; but they also exclude from consideration other, perhaps more interesting topics. In the absence of extensive theoretical knowledge concerning human cognition, such definitions are premature and overly exclusive. In Piaget's view, our main interest should be in discovering what intelligence is. To do this, we must carefully observe, explore, and attempt to discover; "definitions" must wait until later

In the area of mathematics education, the situation is virtually identical. At the beginning stages of research it makes no sense to define "mathematical thinking" in some arbitrary fashion; instead a process of discovery must be employed to determine the main developmental features of children's mathematical thought. As these are discovered — and many findings will surprise us — our conceptualization and definitions of mathematical thinking must necessarily evolve.

Rationale If we grant that one important goal is discovery, what research methods should be used to achieve it? How should we go about discovering the unknown phenomena of intelligence or mathematical thinking? One possibility is naturalistic observation — the careful examination of children's thought in various natural settings. It would be extremely valuable to observe, without interfering, how children add in the natural setting or make spontaneous use of concepts of infinity. But such observations, however desirable, are exceedingly difficult to make, partly because much of children's thought is private and partly because the occasions on which the thought is public are few and far between. One can wait a very long time indeed before observing the child's spontaneous use of arithmetic ideas at the dinner table. Consequently, whatever its merits *in principle*, naturalistic observation is usually not practical as a technique. Also, naturalistic observations are sometimes difficult to interpret: spontaneous behavior may indicate competence (the best a child can do at his current level) or it may not. Often one needs non-naturalistic data to decide the issue

A second possibility, the standard test, is of limited value for purposes of exploration. By necessity, the standard test focuses on a predetermined range of subject matter. The tester must ask certain questions and does not enjoy the freedom to vary the form and nature of the questions nor to follow up on interesting leads by asking new questions. By its very nature, standardization precludes exploration. The standard test may, on the other hand, permit some forms of discovery, as when, for example, a particular group displays an unusual pattern of response to a given set of items.

This result may be of interest, perhaps telling us something new about children's cognition. At the same time, the test does not offer the kind of flexibility required for extensive exploration, for the immediate pursuit of interesting phenomena, and for the checking of intriguing hypotheses.

To overcome the limitations of these two methods, Piaget designed the clinical interview procedure, an unstructured and open-ended method intended to give the child the opportunity to display his "natural inclination." Piaget describes the essence of the method as follows: "The clinical examination is dependent on direct observation. The practitioner lets himself be led... and takes account of the whole of the mental context" [Piaget, 1929, p. 8]. While beginning with a set problem, the clinical interview is designed to permit a kind of naturalistic observation of unanticipated results. This is intended to satisfy the first research aim, discovery, and should be as valuable for research into mathematical thinking as for research into cognitive development generally.

Technique. When the discovery function is stressed, the clinical interview procedure begins with (a) a *task*, which is (b) *open ended*. The examiner then asks further questions in (c) a *contingent* manner, and requests a good deal of *reflection* on the part of the subject. Consider the following example in which the interviewer is Barbara Allardice and the child a third grader:

Interviewer (I): First, I'll ask you what you are doing in arithmetic or math. Which do you call it?
Butch (B): Math.
I: Can you show me some kind of problem that you are working on? Write something down.
B writes: 7 + 15 = 22
I: Okay. What does that say?
B: Seven plus fifteen.
I: And then what?
B: Twenty-two.
I: How did you get that answer?
B: Added it up... on my fingers.
I: On your fingers? Can you sort of show me what you did? You had seven...
B: I started with the biggest number first. Fifteen, sixteen, seventeen, eighteen, nineteen, twenty, twenty-one, twenty-two.
B. counted on his fingers as he said this.

Note that in the example the experimenter began with a *task* and did not simply wait for the child to exhibit relevant mathematical behaviors, as one might do in the case of naturalistic observation. While the clinical interview sets a task, it is typically *open-ended*. The aim is to avoid putting words in the subject's mouth, and to allow him to structure the task in any way he sees fit. The danger, of course, is that the examiner may suggest answers to the child so that there is only the illusion of discovery. Every attempt must be made to avoid biassing the child's response. In the present instance the child is asked questions like "What kinds of things are you doing in math?" rather than "Show me how you can carry." Furthermore, the follow-up on the subject's answers is *contingent* on them. The subject's response determines the nature and sequencing of the interviewer's questions. The contingency of questioning appears to be the essence of clinical interviewing and sets it apart from the predetermined standardization of the method of tests. Thus, in the present example, when Butch maintained that he was engaged in doing problems like 7 + 15 = 22, the interviewer pursued her method for solving problems of that type; when Butch said that he was counting on his fingers, the interviewer attempted to discover his method. In both cases, the interviewer's specific questions were not predetermined but were rather contingent on the previous line of questioning and answers. Another feature of the clinical interview procedure is that the examiner constantly requests *reflection* on the part of the subject. The interviewer asks the child to verbalize his thoughts, to give reasons for his actions, and generally to reflect on what he has done. Note that two types of reflection are often involved. One involves process, as when the interviewer asks the child *how* he got a particular answer and a second involves the rationale for a solution, as when the interviewer asks the child *why* he used some process. In the present example, the interviewer asked Butch to reflect on his method for adding on the fingers. Butch was able to give a very explicit account of the process: "I started with the biggest number first." The interviewer could also have asked for a rationale: "Why did you count on your fingers?", which would have led to a very different sort of answer, like, "I don't know any other way." All these techniques are intended to draw from the child, in an unbiassed manner, rich material allowing the discovery of unsuspected cognitive processes.

Identification

Aim. In Piaget's view, once interesting intellectual phenomena are discovered, the cognitive processes underlying them need to be identified and described. If for example, we know that young children maintain that the sun follows them, we need to explicate the cognitive processes in terms of "structures", which are general mental capacities presumably underlying performance on a wide range of tasks, and which may be described in terms of logico-mathematical models. Thus, to explain success at the conservation task, Piaget postulates the "concrete operations", a complex model of cognition assuming logico-mathematical form. Other theorists do not speak of "structures" and instead postulate "information processing", "symbolic activities", "strategies", etc.

In the study of mathematical thinking, researchers have a similar aim: to describe precisely the thought processes ("structures", "cognitive activities", etc.) involved in mathematical work. Of course, in doing this many theorists do not employ the Piagetian concepts or terminology, which do not appear to be very useful for understanding children's school mathematics. Thus, Davis, Jockusch and McKnight [1978] offer an information processing model. Yet while the terminology and concepts differ, the aim is essentially the same: the clear description of mind.

It is important to note that the portrayal of cognitive structure is usually a complex inferential process. Often, it begins with the observation of a behavior which may have been produced by any one of several different underlying structures or processes. To take a simple example, the answer "10" to the problem 5 + 5 = ? may be obtained by immediate memory (which itself needs to be explained), by counting on the fingers, or by an "invented strategy" like, "4 + 4 = 8 and then 9, 10, because there 2 left over." the identification of structure requires that the researcher imagine the various alternative possibilities and then decide among them This is often impossible to do if the researcher has available for analysis only the initial response In the present example, if the sole datum is the answer "ten", then *all* of the proposed processes are equally plausible Choosing among them thus requires additional data — data which, we shall see, may be most efficiently gathered by clinical interview techniques

Rationale. Suppose the researcher wishes to determine which of the particular strategies is employed. She may attempt to accomplish this in several different ways. One is to use *naturalistic observation*. She may wait until the child spontaneously verbalizes the method of solution or otherwise displays behavior useful for deciding among the various alternatives As noted above, in most instances naturalistic observation is simply not practical.

Another possibility is the standard test. The researcher may design problems intended to reveal various cognitive processes. For example, if the hypothesis is that the child subtracts by always taking away the smaller from the larger, the researcher might present true-false items of the following types,

$$\begin{array}{r} 23 \\ -\ 17 \\ \hline 14 \end{array} \quad \text{and} \quad \begin{array}{r} 81 \\ -\ 18 \\ \hline 77 \end{array}$$

If the child believes they are correct, the researcher has evidence consistent with the "smaller from larger" hypothesis Indeed, much non-Genevan research on Piagetian problems involves standardized methods. For example, a conservation problem is presented to all children, questions are asked in the same manner ("Which row has more or do they both have the same?"), and responses, including requested explanations are recorded ("This one has more because it's longer") Flexible questioning is not employed.

Sometimes such testing, whether in the form of a standard test or a standardized laboratory procedure, is effective in identifying and describing underlying strategies. This is apt to be true particularly when a given cognitive phenomenon has been well explored and when the possible underlying cognitive processes are few and relatively easily described

But there are circumstances in which standardized testing may not be the method of choice When the underlying cognitive processes are numerous and complex, standard tests may be ineffective or at least inefficient. Suppose one wishes to probe a Master's understanding of chess or a coach's understanding of football strategy. A small set of standard questions, or even a large one, will not suffice to capture the richness and complexity of the relevant cognitive structure Similarly, genuine mathematical understanding is extremely complex [see, for example, Michener, 1978], so that standardized items are likely to be overly limited in exploring it. Is there a professor of mathematics or any other academic discipline (including psychometrics) who would dream of employing standardized tests to assess the knowledge of an advanced doctoral student?

We see then that at least for the identification and description of complex cognitive structure, it is desirable and usually necessary to employ a method other than naturalistic observation or standard tests. For Piaget, and for researchers concerned with mathematical thinking, the method of choice is the clinical interview

Technique. For purposes of identifying and describing structure, the clinical interview involves three especially relevant sub-goals. First, the clinical interview is intended to facilitate rich verbalization which may shed light on underlying process. The complexity of knowledge may not be revealed by simple response; extensive verbal reflection may be more informative. Put simply, if you want to know what someone is thinking, ask him Yet, at the same time, verbalizations can be misleading since the child may not have direct access to his cognitive processes or may have poor command over language Hence, the user of the clinical method must beware of misinterpreting language while simultaneously encouraging it This then is the second important sub-goal: the clinical method attempts to check verbal reports and clafiry ambiguous statements. Third, the method uses procedures aimed at testing alternative hypotheses concerning underlying processes. If the subject's response seems to indicate that he is subtracting the smaller from the larger, one question will be asked; if the response suggests diagonal subtraction

$$\text{e g} \quad \begin{array}{r} 23 \\ -\ 1 \\ \hline 12 \end{array}$$

then another probe must be undertaken.

Consider now the details of how the clinical interview achieves these subgoals when the aim is to identify structure by eliciting verbalizations, evaluating them, and checking alternative hypotheses, the clinical interview procedure (a) employs tasks which *channel* the subject's activity into particular areas; (b) it demands *reflection;* (c) the interviewer's questions are *contingent* on the child's response; (d) the interviewer employs basic features of the *experimental method*; and (e) some degree of *standardization* may be possible. These are illustrated in the following example in which two interviewers are involved, Barbara Allardice and the present writer, and the child is a third grader:

Given the problem 29 + 4, Patty (P) wrote

$$\begin{array}{r} 29 \\ +\ 4 \\ \hline \end{array}$$

and as she did so, designated it as "plus"

Before placing the 4 under the 2, her hand hesitated under the 9: apparently she could not decide where to place the 4. Patty then said, "You put the 4 over here... that would be... that's 9... (she whispered) 2, 3, 4, 5, 6,.. 69" She had counted to get the sum of 2 and 4 She wrote:

$$\begin{array}{r} 29 \\ +\ 4 \\ \hline 69 \end{array}$$

Up to this point, the interviewer's intervention has been minimal: she simply presented a problem which elicited a rich stream of behavior, talk and writing, all of which was recorded. Next, however, the interviewer challenged Patty:

I: Are you sure that 29 and 4 are 69?
P: Altogether?
I: Uh, huh (Yes)
P: No
I: How much are 29 and 4?
Patty then used tallies to calculate the correct result, 33.
I: How come this says 69 (the written problem)?
P: 'Cause you're not doing it like that (pointing at the tallies)

After making the observations described above, the interviewer conceived of several hypotheses to explain the child's behavior. One hypothesis was that the child experienced difficulty with written addition involving addends having different numbers of digits, and yet could solve essentially the same problems when they were presented in concrete form. A second hypothesis, which at the time seemed less likely (because in previous interviews the interviewer had seen many cases confirming the first hypothesis), was that the source of Patty's difficulty involved the word "plus"; when the problem was presented in terms of "altogether" she had no difficulty. To decide between the two hypotheses, the interviewer proceeded as follows:

I: Let's do this. There are 10 of these (chips) and here's 1 more. How many do you think, altogether?
P: Altogether, it would be 11.
I: OK. What about 10 plus 1, not altogether, but plus?
P: Then you'd have to put 20.
I: What if we write down on paper, here's 20, now I write down another 1, and you want to find out how much the 20 and 1 are altogether?
P: It's 21.
I: Now what would 20 plus 1 be?
P: 20 plus 1?
Patty wrote

$$\begin{array}{r} 20 \\ +\ 1 \\ \hline 30 \end{array}$$

We may characterize this example in several ways.

(a) After formulating the hypotheses, the interviewer presented several specific tasks intended to *channel* the child's behavior into specific areas. Thus the examiner did *not* ask, "What are you doing in school now?" or some similar open-ended question, but instead directed the examination into the areas of addition with chips and numerals.

(b) On some occasions, the examiner asked for *reflection*. This was not particularly successful with Patty, who responded to the request to explain her answer, 69, by saying: "'Cause you're not doing it like that", which did not give much information to the interviewer.

(c) The interviewer's questions were *contingent* on Patty's responses. If Patty had not responded to the question "Are you sure that 29 and 4 are 69?" with another question, "Altogether?", the examiner would not have been interested in the issue of "altogether" versus "plus".

(d) In the focused section of the interview, the examiner employed the *experimental method*, in the sense of holding some variables constant while deliberately varying others. Thus, the interviewer deliberately varied the problem's language ("plus" vs. "altogether") and its concreteness (chips vs. numerals). This is essentially a factorial arrangement, and if undertaken with a large enough number of subjects (rather than one) and an appropriate counterbalancing procedure (rather than none), it would be an example of traditional experimental design. Such a design was necessary to decide between alternative hypotheses.

(e) In the present example, the interviewer developed the questions on the spot, in response to the subject's behavior. Nothing was "standardized", although the interviewer did have a clear plan in mind. By contrast, when an area has been thoroughly investigated (like the conservation of number) the interviewer *may employ virtually the same set of questions* to identify cognitive structure in all subjects. She may ask everyone, "Do these rows have the same number?" and later, "Why do you think this row has more?" In the genuine clinical interview, this standardized set of questions may not result in an identical sequence for all subjects, nor may all questions be used. Instead, the various "standard" questions will be used contingent on the subjects' responses, and may be omitted or substituted for as necessary. If all the questions are given to all subjects in the same manner and order, without omissions or additions, then the result is no longer a genuine clinical interview but a standard test. *Contingency defines the clinical method.*

We see then that the identification of structure is a complex process requiring a good deal of flexibility and subtlety of the method of investigation.

Competence

Aim. Piaget's theory is primarily concerned with intellectual "competence", not "mere performance". It aims at establishing underlying competence — the child's highest ability at his current stage of development — and does not often focus on typical performance — the child's ordinary behavior on a particular occasion. Piaget is more interested in determining that Pierre is *capable* of the concrete operations than that on a particular day he does not exhibit evidence of them, perhaps because of fatigue, boredom, laziness, misunderstanding of the problems, dislike of the examiner, etc.

The study of mathematical thinking also involves a concern with competence. Often researchers wish to gain insight into what the child "really knows" about division even though his performance in class or an achievement test may be poor. Or we wish to determine the preschool child's highest ability with respect to informal addition, even though he seldom if ever displays it. Of course, we do not *limit* our interest — as perhaps Piaget does — to competence. It often makes sense to focus on performance, as when we need to know how the child ordinarily approaches column addition under the typical conditions of classroom instruction. Nevertheless, one of the important aims of research on mathematical thinking is the determination of competence.

The investigation of competence seems to involve three components, the assessment of motivation, subjective equivalence, and strength of belief. The examiner needs to determine (a) whether the child is motivated to perform the task and takes it seriously; (b) whether the child understands the question in the way described; and (c) whether the child's response is deeply held. All of these activities require the flexibility of the clinical interview, as we shall now show

Rationale. Consider first the problem of motivation. A prerequisite for the expression of competence often seems to be a motivated state. Usually, the child must be properly motivated — "trying hard" — to exhibit his highest possible level of performance. Of course it is possible for some children to exhibit their competence, or lack of it, while half asleeep, especially if the problem is so easy that virtually no effort is required to solve it or so hard that no amount of trying will suffice. Yet if the problem is challenging, a high degree of motivation is usually required for its competent solution.

How then to establish a high degree of motivation? Sometimes, naturalistic study may focus on situations in which individuals are highly motivated and therefore display competence An example is Labov's [1970] study demonstrating the complexity of Black English in the natural setting of gangs. But more often naturalistic observation is an inefficient method for observing highly motivated mathematical thinking Ordinary classroom behavior does not always reveal competence, especially in those students who have had a poor record of academic achievement To determine competence, some kind of examiner intervention is usually necessary.

The standard test attempts to motivate children through instructions designed to establish rapport. These often stress that the test is a "game" which children will "enjoy" playing. Sometimes such instructions are effective, especially in the case of middle class children who have been effectively socialized in test taking But often attempts at rapport are ineffective. Many children, particularly lower class youngsters who have done poorly in school, are not motivated to take standard tests In the view of these children, the tests are not games

Indeed, from a Piagetian point of view, the use of standardized procedures to establish rapport is a contradiction in terms. If one wishes to motivate a child, one needs to deal with him on an individual basis, to determine whether he is interested in a particular task and understands the instructions, and to tailor the task to his individual needs Flexible methods are required to establish motivation in many children; standardized procedures, especially when group tests are used, are apt to be counter-productive.

Consider next the determination of "subjective equivalence". To measure competence in a given area, we must guarantee that all subjects perceive a problem's basic features and requirements in a particular way We may wish, for example, to investigate competence in the cognitive processes underlying mental addition. For our purposes, it is therefore necessary for all subjects to understand the problem as requiring addition How to do this? Naturalistic observation is again unsuitable, if only because subjects seldom provide spontaneous behavioral evidence of their understanding of problems. Some kind of intervention on the part of the examiner is required. The usual procedure, exemplified by standard tests, is to present all subjects with the identical problem in a standardized fashion, perhaps with some pre-tests or other kinds of preparatory activities. But this "objective equivalence" of the problem does not necessarily guarantee "subjective equivalence" (although often it does). Presented with the objectively identical problem, different children may understand it in very different terms. If so, the test fails to tap the competence of some children Similarly, members of different cultures may interpret the "same" problem in different ways [Scribner and Cole, 1976], with the result that the problem fails to measure competence in some cultures. If standardization does not lead to subjective equivalence of meaning, the examiner may find it necessary to use the non-standard approach of the clinical method If one set of instructions does not induce the child to understand a problem in the intended manner, then a clarification of the instructions or even a new set of instructions may be required Clinical interviewing permits this; standard testing does not Similarly, in cross cultural research ". equivalence is best approximated by use of culture-specific materials that are equivalent along cultural dimensions but are not carbon copies of each other." [Scribner and Cole, 1976] In brief, the establishment of the common understanding necessary for the measurement of competence — subjective equivalence — may require the use of non-standard methods.

Finally, consider the determination of "strength of belief" Assume that the child understands the question as we would like and is properly motivated. He may nevertheless respond in a way that does not accurately reflect his cognitive status For example, suppose a young child is given the conservation problem and asserts that the two rows are characterized by equal numbers The examiner notes that the child appears hesitant and uncertain When challenged, the child reverts to a pre-operational response, asserting that one row has more than the other The examiner concludes that the child is in a transitional stage His belief in conservation is uncertain, not firmly held; his competence is not that of the concrete operational child

It may be difficult for naturalistic observation to provide insight into the strength of belief since the observer may view only the child's initial response Some kind of active intervention by the examiner seems required. Standard tests also may find it difficult to deal with strength of belief, although one possible solution is to examine consistency of response to similar test items. The more inconsistent the pattern of response, the less firmly held is the belief likely to be. While this approach might help, the flexible nature of the clinical method seems more appropriate to the task of establishing firmness of belief.

Technique. To establish *proper motivation*, the clinical interview (a) exploits the *one-to-one relationship* between interviewer and child; (b) involves a flexible construction and presentation of *tasks*; and (c) utilizes *persistence and repetition*. Let us consider each in turn.

(a) The clinical interview attempts to capitalize on the *one-to-one relationship* between the interviewer and child. A sensitive, non-threatening interviewer can put an apprehensive child at ease, especially when the interviewer takes time to familiarize herself with the child before the actual tasks are administered. Of course, the personal relationship between interviewer and child may have adverse effects. Some children may feel threatened by an inept or overbearing interviewer and others may be more comfortable in the anonymity of a group-administered standard test. Despite these very real dangers, the skilled clinical interviewer can use a relationship with the child to set him at ease. It goes without saying that this relationship cannot be standardized.

Thus, as the interview with Butch progressed, the interviewer refrained from "pushing" him when it became evident that he felt badly about his ability to do arithmetic; the interviewer attempted to ask questions which Butch had a reasonable chance of answering and did not stress his failure; the interviewer even offered explicit praise for a correct answer.

(b) A second procedure for establishing motivation is the *flexible construction and presentation of tasks*. For whatever reasons, sometimes obscure, a child may find one form of a task boring or overly difficult, but not another. One set of instructions, but not another, may captivate a child. Indeed, Piaget himself pointed out [1972] that the tasks which he and Inhelder used to measure the formal operations of adolescence may be inadequate to stimulate the interest of lower class adolescents. Different tasks should be employed before concluding a lack of competence in that group. The same point holds for all children: the measurement of competence requires flexible and innovative construction and presentation of tasks. Moreover, this must be done, often on the spot, in response to the needs of individual children.

Thus, a child may be unable to deal with written addition when it is presented in standard vertical form but may be able to when the problem is written horizontally. Or, as the present writer observed in one case, a child may be unable to do informal arithmetic by finger counting, but may be able to solve the same problems by using tallies.

(c) Third, motivation may sometimes be achieved through *persistence and repetition*. The child may not feel sufficiently competent to attempt a problem when first presented. If the interviewer does not give up and repeats the problem later, the child may then be prepared to do it. Of course, there are obvious dangers in the interviewer's persistence: repeated failure may serve only to humiliate the child. Nevertheless, the technique sometimes works. Thus, the first two times he was asked, Butch claimed that he could not do "times." It was not until the third occasion that he solved a multiplication problem.

To establish *subjective equivalence* of tasks, the primary technique is *flexible verbal presentation and interrogation*. Instead of presenting task instructions in a standard fashion, the interviewer both feels free to modify them if the child seems to misunderstand and to question the child in order to determine if the new instructions have been successful. For example, one child given the Stanford-Binet IQ test failed to understand the instruction, "Tell me what is *foolish* about this." When the word "silly" was substituted for "foolish" she indicated comprehension and indeed succeeded at several of the relevant items. According to the rules of standardized testing, such a modification of procedure is unacceptable; in clinical interviewing, the change is desirable if it enhances the child's understanding of the basic problem and thereby ensures subjective equivalence of the problem across children.

Finally, to assess the child's *strength of belief* the interviewer employs two basic techniques. One is *countersuggestion*, in which the interviewer directly challenges the child's response. If the child vacillates or immediately changes it, the interviewer concludes that the initial response was not based on firm belief. A second technique designed to assess firmness of belief is simple *repetition* of the problem or *introduction of highly similar problems*. If the child's responses to the repetitions or variations are inconsistent, then the interviewer may conclude that the child's beliefs or strategies are not deeply based.

Conclusions

Research into mathematical thinking has three basic aims: the discovery of cognitive processes; the identification of cognitive processes; and the evaluation of competence. Theoretical analysis shows that the clinical interview is the most appropriate method for accomplishing these aims, although it is far from foolproof and although other procedures, chiefly naturalistic observation and standard tests, may also have appropriate uses. The clinical interview is a heterogenous collection of techniques which may be blended in different ways to pursue each of the research goals. More than one research goal may be involved in a given investigation, and the boundaries among goals are often indistinct. When exploration is involved, the method is open-ended and employs a kind of naturalistic observation. When the aim is identification of structure, the method is relatively focused and may employ elements of experimental procedure. When the evaluation of competence is involved, the method focuses on the variation of instructions and presentation of tasks. There is, in effect, no one clinical method; there are three, each designed for a different research purpose.

The clinical method makes theoretical sense. For some research purposes, it makes more sense than traditional methods. Despite this, the clinical interview's effectiveness needs to be evaluated. We must deal with such issues as the extent to which verbalizations (reflections) yield insight into thinking, the "reliability" of the method, individual differences in interviewer skill and style, and the uses of the clinical interview in educational practice. A future paper will consider some of these topics.

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