Twenty Years of French *Didactique* Viewed from the United States [1]

JEREMY KILPATRICK

> L'auditeur et le professeur, le lecteur et l'auteur doivent s'entr'aider mutuellement Il y a dans chaque science des choses qui ne peuvent s'enseigner, et que l'élève doit acquérir par lui-même; c'est l'habitude des procédés de la science, ou autrement le mécanisme des opérations qu'elle prescrit: en arithmétique et en algèbre ce sont des calculs, en géométrie, des constructions. (Lacroix, 1816, p. 184) [2]

One cannot begin considering the topic of this colloquium without asking, why twenty years? Why not two hundred? Two hundred years ago, Silvestre François Lacroix was about to be named chief officer of the Commission Exécutive de l'Instruction Publique. Out of that experience, together with his long career in instruction, especially as professor of mathematics at the École Centrale des Quatres Nations, he drew an analysis of teaching that has seldom been surpassed in its deep insights and broad understanding of education in general and mathematics education in particular (Lacroix, 1816).

Or, why not one hundred years? One hundred years ago, Alfred Binet had established the first French psychological laboratory at the Sorbonne and was trying to make pedagogy a scientific, experimental subject In words that anticipate more recent views on research, he contrasted scientific pedagogy with what had come before:

> La pédagogie nouvelle se distingue surtout de l'ancienne par la très large place qu'elle entend faire à l'observation et l'expérience; elle cherche à remplacer les affirmations *a priori* par des résultats précis et par des chiffres. (Binet, 1899, p. 29) [3]

It was clear to Binet that experimental pedagogy could not be done in one's study or laboratory:

> elle exige une étude directe des enfants Par conséquent, elle devrait être faite surtout par les maîtres, qui sont en contact continu avec les élèves (Binet, 1899, p 30) [4]

Certainly that emphasis on the direct study of learners has been a hallmark of the didactics of mathematics in France over the past decades.

Finally, why are we not celebrating a twenty-fifth anniversary? Most of us remember quite well the months of May and June 1968. The student demonstrations and the elections of 1968 launched a re-examination of French education that continues today and a movement toward the decentralization of higher education that has had profound consequences for the development of mathematics education in centers around France. I was a new faculty member at Columbia University in New York twenty-five years ago and I remember those times well Although the wave of educational reform in the United States after the student demonstrations at Columbia, Berkeley and elsewhere never became as powerful as they did in France, we all work today in educational institutions shaped by those unforgettable days of 1968

Although we could be celebrating other milestones in our progress as mathematics educators, we are at this colloquium celebrating twenty years of didactics of mathematics in France. That number suits me fine. Many countries around the world (including France) can trace activity in mathematics education back several centuries or more, but as communities of researchers we have a short history. Two decades covers the time not only when French didactics of mathematics began to flourish through the establishment of research groups in universities but also when the international community of researchers in mathematics education began to develop its own identity (Kilpatrick, 1992) French didacticians - Guy Brousseau and Gérard Vergnaud, in particular - have played a major role in the development of that community.

In this article, I will attempt to examine the contributions of French didactics of mathematics from an American perspective. (I restrict the term *American* in this article to the United States of America) I am aware of the risks of this proposal. As Alexis de Tocqueville (1835) wisely observed:

> N'amenez pas l'Américain à parler de l'Europe, il montrera d'ordinaire une assez grande présomption et un assez sot orgueil. (p. 352) [5]

I will take my chances; you can decide

Cultural and linguistic barriers

I have made mistakes before. At the Seventh International Congress on Mathematical Education held in Québec in 1992, in a working group on research methodologies, I innocently showed an overhead transparency on which I had used *"recherches en didactique des mathématiques"* as a translation for "research in mathematics education". That touched off a long discussion whose only clear message to me was that the two phrases are not equivalent

> [Didactique des mathématiques] recouvre l'étude des rapports entre enseignement et apprentissage dans leurs aspects qui sont spécifique des mathématiques. (Laborde, 1989, p. 47) [6]

In French, the term *« didactique »* does not mean *the*

art or science of teaching. Its purpose is far more comprehensive: it includes teaching AND learning AND school as a System, and so on. (Douady and Mercier, 1992, p. 5)

Research in this field requires a dialectic between our research questions and our theories. It requires a *problématique*. French *didactique des mathématiques* has a framework grounded in the hypotheses that pupils construct their own meaning for mathematics and that intellectual productions become meaningful knowledge only if they have been identified in solving important practical or theoretical problems (Balacheff, 1990b).

Why are the two terms "research in mathematics education" and *"recherches en didactique des mathématiques"* not equivalent? Both claim a broad domain: the teaching and learning of mathematics in a social context. Both draw upon other disciplines such as mathematics, epistemology, psychology, sociology, linguistics and anthropology as sources not only for constructs to be used in the research but also for research methodologies. Both argue for the centrality of theory *and* practice (or "action"). I can only conclude that the terms are not equivalent because they reflect strong, culture-bound conceptions of our field.

In American English, two terms strongly associated with European educational thought - *didactics* and *pedagogy* - have taken on negative connotations. To call someone a *didact* is to imply not only that the person is someone who teaches but also that he or she is likely to be moralistic about it. A *pedagogue* is not just a teacher; the word implies that the teacher is long-winded and boring, rather like a *pedant*. Americans use *education* and *educator* to avoid these unpleasant connotations. They view the field of education as one that, though low in status and prestige, has begun to establish its place in academe.

The connotations in France of *énseignement, éducation, éducateur/éducatrice* are unknown to me. It appears, however, that *pédagogie* as a tertiary subject may have become too strongly associated with courses in teaching methodology that draw upon practical experience but not a body of scholarly knowledge. Consequently, *didactique* seems to have been needed to express a particular scientific approach to our field. In Germany, there is a similar usage: the term *Mathematikdidaktik* refers to mathematics education considered as an academic field. Americans, however, resist the term *didactics*. They use *mathematics education* to refer both to the activity and to the field. I am not the person to make a linguistic analysis of these terms. I wish only to draw attention to the way in which our language is embedded in our cultural experiences in ways that make translation difficult across those experiences.

Over the past twenty years, French didacticians of mathematics have engaged in many serious efforts to cross the cultural divide, not only by becoming familiar with what others are writing in English but also by putting their own work into English. The early volumes of the journal *Recherches en Didactique des Mathématiques* contained articles in English by authors from the United Kingdom, Canada and the United States (and it still accepts papers in English and Spanish, as well as French). French didacticians have been active from the beginning in the activities of various international groups concerned with research in mathematics education and much of their work there is available in English (see, for example, Balacheff, 1984, 1990a; Brousseau, 1984, 1988; Brousseau, Davis and Werner, 1986; Chevallard, 1988; Nesher and Kilpatrick, 1990; Vergnaud, 1978). A *Bulletin Inter-IREM* (Braemer, 1984) was produced in English for the Fifth International Congress on Mathematical Education in Adelaide to report on activities of the IREMs, accompanied by special sessions at the congress, also held in English. Similarly, at the 1992 ICME congress in Québec, some articles in *Mathématiques Chez les 11-16 Ans en France* (Commission Inter-IREM premier cycle/Commission Inter-IREM probabilites-statistiques, 1992) were translated into English, a special issue of *Recherches en Didactique des Mathématiques* (Douady and Mercier, 1992) contained selected articles in English and, as at previous congresses, several French didacticians gave talks in English.

What have American mathematics educators done in return? Shamefully little, I am afraid. Handicapped by their disinclination to learn other languages, they cannot make translations of their scholarly productions into French (or any other language, for that matter). Unless English-language versions of French research appear in American journals or books, they tend not to know about that research. For example, the book on the work of Guy Brousseau [7] being prepared by Martin Cooper and Nicolas Balacheff, if it becomes easily available in the United States, will be the first opportunity for most American mathematics educators to encounter Brousseau's ideas directly. (Exceptions include Balacheff, 1990a, 1990b, and Warfield, 1992, but they interpret rather than quote Brousseau. Some important but little-known references are Balacheff, 1984, and Brousseau, 1984, 1988.) American mathematics educators rely on their relatively few colleagues who may have visited France, know some of the French work and can explain it to them in American terms.

The barriers are gradually coming down, however, and each culture of mathematics education/*didactique* is getting a clearer picture of how it looks to the other.

Characteristics of French research

An American who encounters French *didactique des mathématiques* is struck first by the density and elaboration of the theoretical argument. The references to Piaget and to constructivism seem familiar, but the language of *didactical situation, didactical contract, didactical transposition, didactical variable* and *didactical engineering* has only recently begun to lose its strangeness and seeming artificiality. The kind of intensive epistemological analysis given to mathematical concepts in French *didactique* is not found in U.S. research. In the early 1970s, under the influence of Piaget's work, Mike Rosskopf (1975) and some of his students at Teachers College attempted something similar, but it was not nearly so elaborate as the analyses in French *didactique*.

Mathematics itself seems to be used more extensively as a source of metaphors for thinking about *didactique*. An American reader is struck by the extensive use of algebraic

symbolism in the French literature on *didactique*. For example, the notation R(X, O) might be used to describe the relation of student X to an object O, and then R(X, R(X, O)) as the student's relation to that relation. The modification of student B's knowledge over time might be expressed as: (B(t), B($t + \Delta t$)). A concept C might be conceived of as a triple of three sets, C = (S, I, s), in which S is the reference [*la référence*] the set of situations which give meaning to the concept, I the signified [*le signifié*], the set of invariants on which the operativity of the schemes rests and s the signifier [*le signifiant*], the various forms used to represent the concept symbolically. Such notation appears to reflect the desire to be as precise as possible in confronting complex phenomena. Although the use of 'variable' in terms such as 'task variable' or 'teacher variable' seems to be fading from American usage (and was never common British usage), French didacticians happily use *variables didactiques* to characterize changes in the didactical situation that provoke qualitative changes in pupils' procedures (Balacheff, 1984; Gras, 1992).

French didacticians make frequent use of mathematics, but are clearly not limited by it. In pointing out that the emergence of competence can be described in mathematical terms by means of 'theorems-in-action' [*théorèmes-en-acte*], for example, Vergnaud (1988) notes that:

> the analysis of these competences requires more than the stabilized formal description that is usually offered by mathematics. [...] the analysis of additive structures requires change over time to be taken into account, also the use of a unary-operation model, also of children's own untaught procedures and errors. Mathematics does not usually take change over time into consideration, and sees addition as an internal binary law of combination. Actually students have to do with both a unary and a binary conception of addition, depending on the situations they have to master. There lies the conceptual problem. (pp. 43-44)

Vergnaud's (1981, 1990) introduction of a relational calculus and his theory of conceptual fields, with its study of the continuities and discontinuities between the different steps of knowledge acquisition from the point of view of their content, its analysis of invariant operators and its use of homomorphisms, illustrate how abstract mathematics fuels *didactique des mathématiques* in France.

French didacticians proceed from elaborate, *a priori* analyses to experimentation in the classroom. This movement from thought to action has been characteristic of their work for two decades. With the development of didactical engineering [*l'ingénierie didactique*] (Artigue, 1992; Artigue and Perrin-Glorian, 1991; Douady and Comiti, 1987) has come a greater emphasis on case study methodology and the use of an *a posteriori* analysis in confrontation with an *a priori* analysis as a means of validating the research hypothesis.

Also characteristic of French *didactique* has been its sustained attention to classroom teaching and what appears to be a growing interest in the social context in which teaching and learning occur. The study of classroom situations – linking concept and activity – virtually defines *didactique des mathématiques*, as Americans see it (e.g. Warfield, 1992). The teacher is an actor in that situation, an actor with didactic intentions:

> Mais un milieu sans intentions didactiques est manifestement insuffisant à induire chez l'élève toutes les connaissances culturelles que l'on souhaite qu'il acquière. (Brousseau, 1986, p. 49) [8]

To American eyes, French *didactique des mathématiques* has a remarkable unity. Obviously, there are different emphases in the work done by different didacticians, but there appears to be a common epistemology and a shared methodology. The community is not split by factionalism or dispute. The past 20 years have seen the emergence of a school of French *didactique des mathématiques*.

Influences of French research

> En Amérique, la partie purement pratique des sciences est admirablement cultivée, et l'on s'y occupe avec soin de la portion théoretique immédiatement necessaire à l'application; les Américains font voir de ce côté un esprit toujours net, libre, original et fécond; mais il n'y a presque personne, aux États-Unis, que se livre à la portion essentiellement théoretique et abstraite des conaissances humaines. Les Américains montrent en ceci l'excès d'une tendance qui se retrouvera, je pense, quoique à un degré moindre, chez tous les peuples démocratiques. (de Tocqueville, 1840, p. 552) [9]

Viewed from the United States, French *didactique des mathématiques* can be seen as having had a significant, but largely indirect, influence on American research and scholarship in mathematics education. The influence has come primarily through the international community of mathematics educators and especially through some people on each side of the Atlantic who have worked to build a bridge of common understanding (with most of that effort, as I have said, on the part of the French). The field of research in mathematics education as a whole has moved away from behaviorist research paradigms, with much of that movement having been influenced by French didacticians.

More than a decade ago, I noted that American research on mathematical learning and thinking suffered from a lack of attention to theory (Kilpatrick, 1981, p. 369). That was not an entirely original observation of the American character:

> L'esprit américain s'écarte des idées générales, il ne se dirige pas vers les découvertes théoriques.
> (de Tocqueville, 1835, p. 349) [10]

Since that time, however, American researchers have begun to pay more attention to some of the theoretical constructs we are using. We have begun to subject those constructs – from *representation* to *algebra*, from *metacognition* to *ethnomathematics* – to more serious analysis. French *didactique* has helped us to take our constructs more seriously and to analyze them more carefully.

French *didactique* has also helped us move from an almost exclusive preoccupation with learning under highly controlled, laboratory-like conditions to the study of teaching and learning in the mathematics classroom. In the 1970s, E. G. Begle attempted to place American research more directly in the mathematics classroom by developing highly structured teaching units that would allow variations in

teaching methodology to be controlled (see Begle, 1979, p xi). In some ways, Begle's work is reminiscent of French *didactique* American researchers did not, however, follow Begle's lead They went into the classroom and began to study teaching, but they also largely abandoned the analysis and control of didactical variables. French *didactique* has supported the movement in the United States from learning to teaching as the focus of research and from controlled experimentation to case studies as the preferred research methodology It has not been successful in focusing American attention on didactical variables. Our increased attention to the analysis of theoretical constructs has not included many didactical constructs

Many American researchers in mathematics education today are so deeply embedded in a peculiarly radical constructivism that they find the French concern with such issues as didactic transmission and reproducibility of situations (Arsac, Balacheff and Mante, 1992; Artigue, 1992) almost impossible to understand. *Transmission* is now a bad word, a hangover from the old days when we presumably thought children were empty vessels to be filled with information (although I cannot find evidence that any American ever actually believed that). *Reproducibility* suggests the teacher might want to guide instruction in a specified direction, when any up-to-date mathematics educator in the United States will tell you that the teacher's role is simply to facilitate the child's learning of the child's own mathematics We are surprised by Vergnaud's (1982) claim that:

> when sequencing and observing the same series of lessons in different classrooms, with the same teacher or with different teachers, at different levels or at the same level, one can observe that some events happen and happen again and the same coherently and hierarchically organized behaviors appear and appear again (p. 41)

Too many American researchers seem to believe that such regularities are impossible to find

Our researchers shun the metaphor of research as engineering; they consider *variable* the residue of a discredited positivism; they have abandoned not just hypothesis testing and hypothesis generation but hypotheses themselves. They are wandering in a kind of wilderness where *radical constructivism, the child's mathematics* and *qualitative methods* have an incantatory power and questions of validity are *passé* Constructs such as 'conceptual field', 'didactical situation' and 'didactical contract' may appeal to them, but they cannot see clearly how to use such constructs Some interesting work has been done in the United States by researchers wanting to follow up Balacheff's (1990a) ideas about counter-examples (Sekiguchi, 1991) and Chevallard's (1985) development of the didactical transposition (Kang, 1990; Kang and Kilpatrick, 1992), but it is perhaps no accident that neither of the studies cited was done by an American

The true contribution of French *didactique des mathématiques* to American thinking is still to come. Americans need to consider how we might use such notions as 'epistemological obstacle' (Brousseau, 1983; Sierpinska, 1988) in our research. We need to undertake a more serious examination of constructs such as representation (Vergnaud, 1978, 1981). Vergnaud has shown us a pragmatic constructivism; Brousseau has shown us how to confront and analyze the paradoxes of our field. These are tools that can help American mathematics educators get beyond the anti-scientific attitudes that have begun to infect much of their work.

Chevallard (1990) claims that:

> where mathematics education is concerned, the scientific debate that no research community can dispense with has gradually given way to mutual bowing and scraping, which leaves little or no room for healthy intellectual squabbling. (p. 26)

I agree, except that it is not clear that we have ever had scientific debate in our research community. Certainly we need more debate and the confrontation between American and French views of research is healthy.

The French have audaciously attempted:

> une rationalité des phénomènes aussi complexes que recouvrent les rapports entre enseignement et apprentissage (Laborde, 1989, p. 60) [11]

They have built an imposing scientific edifice. For Americans who mourn our apparent loss of a scientific spirit to sustain our work, French *didactique des mathématiques* offers a reassuring prospect.

Notes

[1] This article was first presented as a talk at the colloquium *Vingt Ans de Didactique des Mathématiques en France* held at the Institut National de Recherches Pédagogique (INRP), Paris, 15-17 June 1993. I am grateful to Colette Laborde for translating it into French It was first published (in French) as Kilpatrick (1994)

[2] "The listener and the lecturer, the reader and the author have to aid one another In each science, there are things that cannot be taught and that the student must acquire for himself; it is the habitual way science behaves, that is, the operational mechanism it prescribes: in arithmetic and algebra, it is calculations; in geometry constructions."

[3] "The new pedagogy can be distinguished from the old above all by the very great emphasis it proposes to give to observation and experience It seeks to replace *a priori* assertions by precise results and statistics "

[4] "It requires the direct study of children Consequently, it especially needs to be done by teachers, who are in continuous contact with pupils "

[5] "An American should never be led to speak of Europe, for he will then probably display much presumption and very foolish pride " (de Tocqueville, 1835/1945, p. 329)

[6] "[*Didactique des mathématiques*] covers the study of the relationships between teaching and learning in those aspects that are specific to mathematics "

[7] Now available as *Theory of Didactical Situations in Mathematics* (Brousseau, 1997)

[8] "But a milieu without didactical intentions is manifestly insufficient to induce in the pupil all the cultural knowledge we wish him [or her] to acquire "

[9] "In America, the purely practical part of science is admirably understood, and careful attention is paid to the theoretical portion which is immediately requisite to application. On this head the Americans always display a clear, free, original, and inventive power of mind But hardly anyone in the United States devotes himself to the essentially theoretical and abstract portion of human knowledge In this respect the Americans carry to excess a tendency that is, I think, discernible, though in a less degree, among all democratic nations " (de Tocqueville, 1840/1945, p. 43)

[10] "The spirit of the Americans is averse to general ideas; it does not seek theoretical discoveries." (de Tocqueville, 1835/1945, p 326)

[11] "a rationality of sufficiently complex phenomena that cover the relationship between teaching and learning."

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