An Exchange about Word Problems

ROBERT THOMAS, SUSAN GEROFSKY

Robert Thomas sent a message by e-mail to Susan Gerofsky after reading her article, "A linguistic and narrative view of word problems in mathematics education", in FLM 16(2). When Susan Gerofsky replied, also by e-mail, she suggested to Robert Thomas and to me that the two messages might be printed in a subsequent issue of FLM as a contribution to public discussion We agreed Both authors were given the chance to revise their messages for publication and they made a small number of changes of a minor nature —David Wheeler

Dear Dr Gerofsky,

I have just finished reading your paper in the recent FLM, and am moved to write to you. While there were aspects of your paper that I found interesting and even wish to follow up with further reading, I am moved chiefly by my exasperation that in the situation you describe at the beginning of your second-last section ("I have no answer ... the purposes of word problems") you did not ask someone. I have visited the Mathematics and Statistics Department of Simon Fraser University, and I am confident that almost anyone in that Department could have told you what they are for. Since, however, you did not do so, and since your puzzlement illustrates things I wrote of in my paper just a few pages earlier in the same issue of FLM, I ought to mention to you why I construct word problems for tests and examinations—-and why they are needed at earlier stages of the learning process (appear in textbooks, for example).

As I say in my paper, a mathematical process that needs to be taught involves the finding of mathematics in non-mathematical situations, the solving of the mathematical problems that one reads into those situations, and the carrying back of the solution to the situation, where sometimes it is useful The stuff by Pinder that you seem to quote approvingly concerns the solution of practical problems by non-mathematical means ("turn off the taps"). If we need to be told to turn off the taps in Alcuin's situation, we are in trouble The point is mathematical It would be lovely, had we but world enough and time, to get our non-mathematical situations in real life, but the exigencies of classroom instruction—not to mention examination—make us get along with stories that serve to convey somewhat artificially the aspects of real life in which we can find the mathematics to solve. The "story" part of the problem is the link with real life It is sometimes tenuous, but it is there as a link not because you have to have a story in a problem you are going to call a "story problem"; lots of people don't call them "story problems". I refer to them usually as "the hated word problems".

Component 2, as you call it, expresses the proto-mathematical relations in which the student is to find the mathematical problem to solve. The conversion of the proto-mathematical relations into the mathematical equations, inequalities, or whatever, that will be solved is one of the main points of the problem. You have expressed component 2 on page 37 already in mathematical terms, but in the standard word problem, they are usually times of day (Alexander's ten o'clock) or other coded real-world matters that the student needs to work on to translate into mathematics. The student is working entirely within his or her own constructs, both of mathematics and of the world in which the story takes place. Numbers of nickels and dimes have to be multiplied by five and ten to convert them into cash equivalents This is work, thoughtful work, and children need not only practice at doing it if they are going to use mathematics outside the classroom, but also to be taught to do it The most valuable mathematics lesson of my life was given me by a mathematics teacher (a family friend) visiting my home when I was beginning algebra; he taught me what was involved in doing such problems.

When the conversion has been done and the mathematics is sitting there "undressed" as you put it, the student has to solve the naked mathematical problem. This is customarily the execution of an algorithm, which you seem to think is the point of the problem. It is emphatically not the point of such a problem when I set it, and I think that this is the standard situation Any fool can execute simple mathematical algorithms—even computers can do that. I am not teaching undergraduates to be computers, and I wish that teachers did not teach high-school students to be computers Many don't, but some do. Those that do are doing their students a disservice for a variety of reasons, among which are that they are teaching that mathematics is boring, that it is mechanical, that there is only one right way to do problems, all of which are false. The interesting, non-mechanical part of mathematics, in which the variety of possible approaches exercises the student's creativity, is the setting up of the problem for the execution of the boring algorithm that will produce an answer The point is the development by practice of the exercise of mathematical insight. The important test of whether one has learned the operations of algebra is whether one can see them in proto-mathematical situations; if one can only execute algorithms, then one has the level of understanding of a calculator, nil.

The third stage ought not be neglected. The solution of the mathematical problem needs to be taken back to the situation to see whether it is any use. This step is frequently neglected by students, who blithely submit negative distances, volumes in linear units, and other "answers" that are not only wrong but also clearly inconceivable. Verifying the mathematical solution is a useful step, but the proof of the pudding is in the dining room not the oven. I think that I am not being idiosyncratic in what I say on this topic In a forthcoming paper, Anna Sfard quotes Shimshon Amitsur as speaking of the value of "the ability to translate real-life problems into mathematical problems, and vice versa" ("Mathematician's view of research in mathematics education—Interview with Shimshon A Amitsur", to appear in A. Sierpinska and J. Kilpatrick (eds.), *What is research in mathematics education and what are its results*. Dordrecht: Kluwer).

There is more to the solving of problems than mathematics. A psychotherapist was written up in a recent newspaper for suggesting that his patients could be helped by seeing their personal problems in dramatic form in movies. When the NCTM urges the study of problem solving on teachers of mathematics, I assume that they mean problems amenable to mathematical solution. Within the limits of our customary instructional situations (which often involve a solitary student reading a text), word problems are the best we can do. If you study them with the care you have already demonstrated and an acceptance of their value, you may be able to suggest improvements in how they are constructed. (I may say that I do not understand the Stella/sweater problem as you quote it.) You might like to give some thought to how, within our customary constraints, we could do something more realistic, more interesting, more effective. But first you need to see the point; I hope that I have aimed you in that direction

Yours truly,

Robert Thomas

Dear Dr Thomas,

I disagree with your characterization of the purpose of word problems in several fundamental ways: first, in terms of your conservatism in accepting the way things are as the way they must always be in mathematics education, second, in terms of your unproblematic acceptance of concepts of separable mathematical and "real" worlds and of story as a transparent bridge between the two.

In your letter, you signal an acceptance of the "customary constraints" of traditional mathematics education "It would be lovely, had we but world enough and time, to get our non-mathematical situations in real life", you write, "but the exigencies of classroom instruction—not to mention examination—make us get along with stories that serve to convey somewhat artificially the aspects of real life in which we can find the mathematics to solve [. . .] Within the limits of our customary instructional situations (which often involve a solitary student reading a text) word problems are the best we can do." This sense of "the best we can do under the circumstances" is one that I want to challenge, and I am certainly not alone in this. Why do you feel that we must conceive of mathematics education as comprising "a solitary student reading a text"? Why the emphasis on examination and on the familiar constraints of traditional classroom instruction in mathematics? It is only by recognizing, pushing and playing with these self-imposed boundaries that we will improve mathematics education.

You write about your intentions as a teacher in setting word problems for your students; for example, you say that the execution of an algorithm is not the point of setting such problems, that there is not only one right way to do problems, and that the interesting part of mathematics which exercises students' creativity is the setting up of the problem, which exercises mathematical insight. I am interested in the intentions of teachers and textbook writers in setting word problems for students, in students' uptake of those intentions, and in the differences between the intentions and uptake. Your intentions, as I read them, include a devaluing of "boring, mechanical" arithmetical calculation in favour of the setting up of algebraic relationships cued by a story; and the idea of "exercise" in relation to insight and creativity (perhaps calling to mind the ideas of faculty psychology). I wonder whether students read the intentions you wish them to. I am particularly interested in the uptake or response of young children new to school mathematics who are encountering word problems for the first time, after many years of immersion in worlds of story. Is it clear to these children that the stories in word problems are meant merely as tenous links to a pre-existing world of mathematics? The calculations which you find tedious and mechanical are likely not so transparent to young children who are trying to make sense of them in their initial school experiences. Many studies show the tremendous creativity such beginners bring to the work of making sense of calculation.

In your letter you put forward what is a fairly traditional view of word problems in mathematics education—that the story in story problems is a (sometimes tenuous) link between mathematics and real life. The concepts that you use unproblematically in this description ("story", "mathematics", "real life", "link") are the very terms that I take issue with The basis of our disagreement comes down to a question of world view, including views of what may be assumed and what must be raised as a problem I think it is important that we problematize terms like "story", "mathematics" and "real life".

First, what is meant by "story" in these problems? They would certainly be considered poor examples of stories in almost every other context, and do not engage us (nor intend to engage us) in fictional worlds with rich, complex characters, imagery, plot, etc. I question how word problem stories relate to our other worlds of story experience—to novels and family histories, fairy tales and riddles, proverbs and parables. I think that an examination of these stories as stories will give us interesting and useful new ideas about ways of using word problems in education. For example, if we treated word problems as if they were parables (a closely-related literary genre), we might spend a week paying attention to the resonances of a particular problem, the ways it interacts with our lived experience and our notions of mathematics, and the gaps and paradoxes in the problem that irritate us and refuse closure. If we treated word problems like riddles, another related form, we might expect them to play with language, with double meanings and puns, and we might notice that they call up complex chains of reference, allusions to the sacred and profane.

You refer to story as a link between mathematics and real life, a link which enables students to find the mathematics in non-mathematical situations."Real life" as an externalized, knowable, objectively verifiable entity is a central problematic in contemporary (post-modern, post-structuralist) philosophy and scholarship, and cannot be taken for granted as a shared assumption The concept of mathematics as a self-contained world apart which owes nothing to our embodied lives is being questioned by a number of contemporary mathematicians. The idea that mathematics is "there to be found", rather than projected upon, non-mathematical situations assumes first a mathematics and a real life which are strictly separate, and second a primacy and universality to mathematics which allows that mathematics can and will be found as an imbedded structure in real-life situations There is little concept of human agency here, neither of the rootedness of mathematical abstraction in lived human experience nor of the psychic effort involved in simplifying the complexity of lived experience to the point where it can be modelled mathematically.

Story viewed as a link or bridge between the disparate worlds of real life experience and mathematics is also a troublesome notion. I am more of the view that it is through language and story that we create our always culturally-mediated worlds — that we "story forth" entities before we can meet them. Typical examples of this idea from semantics involve ourperception of colour or understanding of kinship relationships, where particular languages and cultures define discontinuities in the colour wheel (say, the split between what is counted as "blue" and "green") or the significance of particular kin relationships (say, the importance of maternal uncles) by the words and stories that define those terms. Certainly in mathematics there is a strong sense in which objects are created by being named and talked about. It is this sense of invocation, of story as speech act, that particularly fascinates me in story problems. What is being invoked, called forth or pointed to in such stories?

In your letter, you've said you're trying to aim me in a particular direction; that is, you would like me to suggest improvements in the construction of word problems within the customary constraints of the traditional mathematics classroom. This was certainly the direction taken by the dozens of math educators in the 1970's and 1980's who published studies on the readability, clarity, and age and grade level categorization of word problems within existing structures of mathematics teaching and testing. My work is on an ontological rather than a strategic level; I am asking "what are word problems?" (as a literary, linguistic, mathematical, educational and historical genre) rather than "how can we get students to do them better?" My aim is to take a walk around the word problem genre as an object and to view that object from many different disciplinary points of view. Since our students are all novices to some extent in mathematics, I think that they too make sense of word problems in a multi-disciplinary context (and not just in the context of mathematical intentions) I think that this multi-disciplinary analysis will lend clarity to some of students' intuited difficulties, and will help explain why story problems are seen as "the hated word problems"

Sincerely,

Susan Gerofsky