WHO CAN KNOW MATHEMATICS?

MARGARET WALSHAW

Mathematics, it is commonly agreed, should be accessible to all. Democratic societies subscribe to the view that for individual and societal progress, *all* students should have right of access to mathematical knowledge and that teachers should develop students' capacity to learn mathematics. As a result, each round of published international or national comparative datasets invites intense scrutiny of student proficiency outcomes. Closer inspection of the data highlights student experiences that are inequitable in the sense that for specific groups of students, learning mathematics presents an impossible challenge (Anthony & Walshaw, 2007). The issue of inequities surrounding learning mathematics is extremely complex. Conventional accounts that provide a means to explain equitable learning experiences, it is argued, do not get to the complexity of the issue. An equitable mathematical experience will not be achievable for specific groups of students if we cannot begin to unravel this complexity.

Concerns over students' low achievements relative to international benchmarks have put huge political pressure on education systems in a number of western nations. Stakeholders tend to blame sitting governments at both state and national levels for not doing enough to demonstrate good or internationally-comparable student performance. In my own country, New Zealand, the current response to systemic underachievement consists of a suite of proposals including the passing of a policy on national standards, more public-private partnerships, increased class sizes, and the linking of teachers' salaries to their perceived competence.

There are important synergies between the educational climate in New Zealand and that in other countries, where policy's most recent engagement with the issue of student learning has arisen, in part, through reflections on the changing nature of our mathematics classrooms, which increasingly cater to diverse groups of learners. Increased socioeconomic disparity, as well as cultural and linguistic diversity, have put engagement with marginalized and underprivileged learners sharply into focus. The solution that is often offered to address underachievement pivots around the conflation of equity with equality, in which unequal approaches, unequal access, and unequal opportunities are deemed to fully explain why specific groups of students do not succeed with mathematics. Equality, in these explanations, is privileged over advocacy grounded in preferential treatment for specific groups, based on the understanding that equal outcomes, approaches and access, taken together, yield a comprehensive picture of equitable mathematical access for students, irrespective of any social categories.

This kind of approach has been seriously undermined by a number of researchers who emphasise the relationality of mathematical knowing (*e.g.*, Appelbaum, 2008; Brown, 2011; Roth & Radford, 2011). For these researchers, in trying to paint a picture of equitable arrangements in mathematics education, issues of structures, as well as interactions between contexts and people, cannot fail to intervene. From this point of view, the teacher's practice and the student's learning are always situated within a web of wider influences; social and political factors that impact on learning are hugely significant. Hence, an approach that conceives of mathematics as constructed and as situated within institutions, historical moments, and social, cultural and discursive spaces might signal how mathematics might be made accessible to all learners and, as a result, how persistent inequities in students' mathematics education might be addressed.

This article, then, is an exploration of contemporary thinking about learning mathematics, and within that, a discussion of social justice within mathematics education. It represents the culmination of conversations with many others, my reading of the literature, and a number of recent productive research collaborations. It establishes a direction for what we might do to effect change based on the potential of a radical democratic project for learners of mathematics. At the core of the discussion lies an interest in reconfiguring learning as a political and moral project—an interest that is couched within a disavowal of the possibility of utopian transformation.

Connections and disconnections

Mathematics education has taken great strides over recent years to move "toward a broader understanding of what it means to know mathematically" (Maheux & Roth, 2011, p. 41). These strides have resulted in a shift from cognitive psychology towards a strong reliance on socio-cultural theory (Jablonka, Wagner & Walshaw, 2013). Vygotskian and neo-Vygotskian frameworks that build on the understanding of the prior necessity of social interactions for cognitive behaviour are currently highly influential, but in much of this work the take-up of Vygotskian ideas is eclectic. Many theorists claim an allegiance to the social but nevertheless draw on terms and concepts frequently associated with constructivist positions. In these perspectives, the construction of knowledge still remains the preserve of the individual stable mind, albeit influenced by social and cultural practices. Even as the importance of the social is acknowledged for knowledge construction, the social functions as a constructor of knowing.

In sociocultural formulations that maintain that the social functions as a constitutor rather than a constructor of knowing, teachers and students are both active participants in the learning process. The proposal comes hard on the heels of a renewed respect for "the other" within mathematics education (see *e.g.,* Ernest, Greer & Sriraman, 2009). In these various formulations, learning is embedded within a cultural and social context; it is a collaborative process rather than a function of the individual or the social setting; it is an apprenticeship that occurs through guided participation in social activities with a community that supports and extends understanding; and/or it is mediated by language and other cultural tools. Identities, in these formulations, develop from knowledge which "is distributed across people and their tools and technologies, dispersed at various sites, and stored in links among people, their minds and bodies, and specific affinity groups" (Gee, 2004, p. 33).

Teaching that works to counter the effects of social or material disadvantage is about building on student interests in a collaborative way; co-participating as a learner in a community of learners; engaging in dialogue between and with the students; and developing relationships between the teacher and student in less traditional ways. Crucially, then, in these formulations, equitable practice is more about interactions between the teacher and students than it is about transmitting and consuming knowledge. To that end, learning is characterized by an enhanced, integrated relationship between teachers' intentions and actions, on the one hand, and learners' disposition towards mathematics learning and development, on the other.

More recently, in an attempt to sharpen the modalities that shape equitable practice, Roth and Radford (2011) have proposed a cultural-historical perspective in which learning is created in the spaces and activities that the classroom community share within a web of economic, social, and cultural differences. Hence, knowledge creation cannot be separated from the axes of social and material advantage or deprivation that operate to define students. Thus, it follows that the identity of a student as a mathematical learner comes into being *in relation to* the negotiations that she undertakes, both in the past and the present, with other individuals and communities. Crucially, a change within a particular classroom context may also result in a change in a student's long-term assessment of him or herself as a mathematics learner. That is to say, an activity within the mathematics classroom has a direct bearing on the kinds of mathematical identities that students might take up and the kinds of proficiencies to which they might aspire. What students say and do within the discourses made available has the effect of contributing to the development of a mathematical identity at a given time and place.

If an identity as a mathematical learner is relational and is able to tell us about the nature of an equitable mathematical experience, then it needs to be expressed as something dynamic rather than as a static process or as a property of people, as formulated in the conventional take-up of sociocultural work. In underscoring the relational nature of Vygotsky's ontology, Roth and Radford (2011) merge the cognitive, the social, the historical and the affective. They do not stop at an analysis of the social context nor at an analysis of the individual. Their reading emphasises that culture and history are embodied drivers of thinking and being.

Cultural-historical approaches amplify the contingent and, in doing so, embrace a position that moves beyond humanism's fixed and unitary subject. Such approaches frame identity differently and so inspire a re-shaping of the humanist imaginary. Radford's (2012) reading of emancipation troubles the clarity of the natural order, lifting the notion of emancipation out of the narrower interpretation expressed within sociocultural theory, and moves us towards appraising the enlightenment modern project as "a chimeric and unfulfillable dream" (p. 101). His view of the modern-day social justice project is aligned with the view of scholars such as Foucault. For Radford, rethinking identity and the subject away from the modality of self-sufficiency invites a very divergent mobilization of the emancipation project and the role that project plays in everyday classroom practice for individual learners.

In Radford's view, at the heart of the issue lies the relationship between freedom and truth and, with it, the relationship between the individual and the social. By way of example, Radford discusses the student-centred approach, advocated in many official curriculum documents as the pivot of mainstream education policy and practice. The difficulty arises when the ideas generated by students in the classroom are reconciled by the teacher with the conventional mathematical ideas as outlined in curriculum documents. As Radford notes, students' ideas are invited yet they can never be autonomous since they are "unavoidably engulfed in discourses and epistemes (*i.e.,* systems of thinking) that are not [the students'] own" (p. 104).

Foucault (*e.g.,* 1972) has shown how bodies of knowledge, like school mathematics, are premised on a certain set of truth claims. As such they are caught up in "regimes of truth", and what comes to count as school mathematics does not pre-exist certain normalizing and regulating practices. Knowledge about school mathematics is an *effect* of particular rules of formation. While these rules are often unknown to the actors involved, they circumscribe the possibility of thought concerning what exactly school mathematics is. They also set boundaries on what is taken as mathematical truth. The unmasking of school mathematics as intimately tied to the social organization of power then becomes crucial to our understanding of the emancipatory project focused on the question: "who can know mathematics?" As Radford has asked, "How can the modern subject be the locus of meaning, feeling and intentionality if it has to talk, feel and intend through thoughts and words that are not its own?" (p. 106).

Radford's question draws attention to an issue that many of us have pondered upon from time to time, yet have not been able to address from within the traditional horizon of mathematics education thinking. When we can no longer propose learners as knowing, willing and judging subjects, able to act in an autonomous fashion, the possibility of emancipation is ruled out *a priori*. Instead, we become aware of the contingency of our very being. Trapped within Nietzsche's (1967) dilemma, we are confronted with a difficult situation: that if no Truth exists then there can be no possibility of an ethics and a politics possessed of genuine emancipatory values. When old orthodoxies are unsettled, like this, we are presented with a new set of challenges for the social justice project.

An ethics of the self

Foucault's final work provides us with the possibility of finding a new impetus for "the undefined work of freedom" (Foucault, 1984a, p. 46). In *The Care of the Self* (1984b) and *The Use of Pleasure* (1984c), Foucault offers a theory which encompasses a more complex and differentiated analysis of relations; one that is able to acknowledge the potential of creativity and agency within social constraints. In proposing a modern ethics of the self, infused with emancipatory potential, Foucault aimed at promoting "new forms of subjectivity through the refusal of [a] kind of individuality which has been imposed on us for several centuries" (Foucault, in Dreyfus and Rabinow, 1982, p. 217). This work offers a set of vocabularies for articulating practice which is at odds with the language offered within, for example, the Freirean and Deweyan master narratives of progressive change.

Foucault's argument is that a progressive politics might best be served not through adherence to externally imposed moral obligations, but rather upon an ethic of who we are to be, and what, therefore, it is possible for us to become. As he explains, what matters is how the individual turns herself into a subject. The proposed critical ontology of the self is generated from the idea of *critique as its enabling condition* and it is this idea which forms the basis of his modern ethics of the self. In the classroom and in everyday life, it is through *technologies of the self* that the learner conceives of herself as a thinking, speaking and acting mathematical learner and where she fashions her mathematical identity. These particular practices and techniques influence the ways in which her subjectivity is constituted and the ways in which her mathematical experiences and identity will be shaped. Her thoughts and actions are also governed by them.

It is through the notion of *governmentality* that Foucault is able to offer an interpretation of individual experiences in which domination and resistance are no longer conceived of as ontologically different, but as opposing effects of the same power relations. Institutions and social groups do not own power; rather, power exists in relationships that are discursively produced. Since discourse is both an instrument and an effect of power, discourse can be the medium through which an opposing strategy is prevented as well as the medium through which an oppositional strategy might be put in place. What derives from the notion of governmentality is the idea that learners (and teachers) are active agents with the capacity to fashion their own existences. That is, whilst governmentality targets the learner as the means with which to maintain social control, at the same time it provides her with the very techniques with which to resist this government of individualisation. The category of the "self" suggests an understanding of the learner as an active and never-completed process of enculturation across a vast number of subject positions, some to a much greater degree than others, but over all of which the learner may exert some degree of autonomy.

An ethics of the self requires a distinction between socially imposed "ethics" (which determine which acts are permitted or forbidden and which acts are ascribed positive or negative valence in a constellation of possible behaviours), and internally constructed "morals". In a similar vein to Roth and Radford (in Roth, Radford & LaCroix, 2012) who argue for

activities as a means to praxis and transformation, Foucault argues that in order to understand how individuals constitute themselves as moral subjects of their own actions, we need to examine the practices that "constitute, define, organize, instrumentalize the strategies which individuals in their liberty can have in regard to each other" (Foucault, 1988, p. 19). Such an examination would reveal the different ways in which one's self is formed as an ethical subject.

In such an examination, moreover, the concept of autonomy is reworked to one that is linked to a questioning of what appears as natural and inevitable about one's identity. In its redefinition, autonomy assumes an analytic function through which to explore the ways in which learners act in the mathematics classroom and the ways in which they give meaning to their experiences, activities and identities. That is, this examination looks closely at technologies (or practices) of the self: those rules of conduct the learner sets herself intentionally and voluntarily in order to create herself in daily school life as a "work of art". In Foucauldian understanding, learners, like teachers, curriculum planners, researchers, and so forth, despite their stable appearance, are all merely productions of practices through which they are subjected. The identities these actors have of themselves are made in and through the activities, desires, interests, and investments of others. Hence, the truth about oneself is not something given, not something in our nature, but something we have to discover for ourselves.

There are significant points of convergence between the notion of the self/subject (mathematics learner) as conceived by Roth and Radford (in Roth, Radford, & LaCroix, 2012) and that put forward of the subject by Foucault (*e.g.,* 1970). All promote a historically variable account of subject-constitution. For all of them, the subject is never fully constituted. When they talk about identity, they talk about identity as fluid in nature, forever in process. Theorising identity in this way allows them to differentiate their work from the Cartesian effort to conjure a foundational status for the subject that is claimed to pertain to a rational "man". It allows them to escape the cause-effect logic which is at work in the construction of an essential or fixed identity. It also allows us to move beyond that logic in descriptions of the pedagogical relation involving teachers and underachievers.

In this reading, new coordinates for the identity of a mathematical learner may be mapped out. In Foucauldian understanding, the idea is that the subject is, on the one hand, an agent and, on the other hand, has a connotation of being subjected to. The subject is internally contradictory since it has both the status of a position of agency and the status of being acted upon. That is to say, the subject is embodied with a double valence: it is an ensemble, and never in one place only. A reading of the subject, like this, invested as it is in dynamism, requires the shift in language that Roth and Radford argue for in order to convey the point that the verb "to be" is "always in transformation" (p. 9). Thus a learner's aesthetic self-fashioning is not oriented towards the recovery of an essential inner mathematical identity but towards an exploration of the scope for potential and ways of existing in the world. Blake, Smeyers, Smith and Standish (1998) elaborate:

It is because she has no essence that the subject enjoys [...] a freedom of fragmentation: a freedom that arises in the constellation of differences that constitute a lineage of loose alliances, relations of resistance and mastery, and configurations of fluid interests. The freedom of fragmentation remains real in response to the constant transformation of problems. It puts in question the firmest of principles and established practices. The result is an ethic of responsibility for the truths one speaks, for the political strategies which these inform, and for those ways of relating to ourselves that make us either conformists or dissidents. (p. 62)

The construction of an identity as a mathematical learner is an ongoing process, ever-changing as the student works at reconciling her sense of self as a learner with the normative activity established within her mathematics classroom. Choosing one's mathematical identity is a process that is not wholly conscious but nevertheless accessible to consciousness. It involves the interpretation of a classroom reality which is weighted with existing sanctions and prescriptions. The important point is that through an ethical self-analysis the learner examines everyday classroom practice, problematizing and unravelling the tangled complexities of the pedagogical relations, all the while maintaining an awareness of the possibility of resistance. Thus, on the one hand, a mathematical identity is the locus of cultural interpretations, that is, it is always already caught up and defined within a classroom context. On the other hand, it is also the site at which the individual learner is required to receive and actively interpret that set of interpretations.

In Foucault's words, these interpretations guide the self-fashionings by which they "seek to transform themselves, to change themselves in their singular being, and to make their life into an oeuvre that carries certain aesthetic values and meets certain stylistic criteria" (Foucault, 1984b, pp. 10-11). The subject is always open to reinscription. While the student is complicit, subsumed by more global structures, the reality is that the student *is never reduced to them* (Walshaw, 1999). Transforming the self is a "work of art". As Foucault (1984a) says: "Modern man [...] is not the man who goes off to discover himself [...] he is the man who tries to invent himself. Thus modernity [...] compels him to face the task of producing himself" (p. 42). Through Foucault's concept of ethical self-formation, mathematics classroom learners constitute themselves continually as a work of art, and as the subject of a critical practice of freedom, ever mindful of what is not able to be surrendered: that is, ever mindful of the limits of practice.

Conclusion

Foucault's theorising provides the conceptual tools for a project of freedom, of going beyond the "limits" that descend from the learner's own particular historical situation and circumstances. His work gives new direction to the undefined work of freedom to be achieved by working at the limits that have been imposed on us. But the strength of Foucault's work, as opposed to the display of its individual gems, is that it creates a politically constructive moment by offering the tools for invoking ethical deliberation. Specifically, it embraces the potential for creativity and agency within social constraints. This is important, because, as Foucault sees it, what we might become stands as the political, ethical, social, and philosophical problem of today.

In undercutting the foundations upon which the notions of empowerment and emancipation lie, Foucault's work sets itself up against grand narratives of social progress and the united calls to social justice as a macro-political struggle. Based in practice, the ethical analysis he proposes is able to address the realities of inequitable experiences. For Foucault, our freedom is not located in our so-called transcendental nature; rather it is found in our capacities to contest and change those practices that constitute who we are. Based on the principle of self-critique, the proposal is for an ethics of the self that is experimental, endless, and relinquishes any hope of attaining a complete and definitive knowledge of what may constitute our historical limits.

This proposal, it is argued, has far-reaching implications for equity projects in schools focused on lifting the mathematics achievement of specific groups of students. The political possibilities centre on the individual learner as both the site for a range of possible forms of subjectivity, and subjected at any particular moment of thought or speech, to the regime of meaning of a particular discourse, but which enables the learner to act. The learner is neither the origin of the particular classroom arrangement in which she finds herself; but neither is she a passive member of it, or of the wider education system. The learner has the capacity to modify the relations and arrangements in which she finds herself. If the learner shuns the responsibility of authentic self-creation she comes to be entirely fabricated by others. She cannot simply, of course, wilfully fashion an entirely new self but she can, as Waugh (1992) has argued, use aesthetic strategies to reformulate available resources. Learners do not have a hidden essential mathematical identity waiting to be discovered. Learners are, rather, an artefact, an aggregation of available forms from which they must choose to shape into a coherent identity.

From the perspective of intervention work in the field of mathematics education, a whole new space for critical reflection on the scope and limits of freedom becomes available. What becomes possible is a response to the question relating to how the growth of mathematical capabilities might be disconnected from the intensification of mathematical understandings that are "handed down". On offer is the possibility of the learner moving beyond her current self, and what she might be doing or thinking. The project involves a consideration of how the learner comes to an understanding of the elasticity of her current individual freedom by constantly exploring the limits of her subjectivity. It is to concede that the learner, in general, and the disaffected and disengaged learner, in particular, does this, in the first instance, by questioning the boundaries of the taken-for-granted understandings of learners in school mathematics and revealing how these established forms of identity are necessarily contingent and historically specific. At this point the possibility of transgressing the so-called limits is made available and this, too, is the point where the potential for new forms of subjective experience and new forms of mathematical identity are established.

In summary, on offer is a praxis that attends to failures and refusals. In the slippage from taken-for-granted truths about underachievement amongst specific groups of students, Foucault's ethics of the self asks: how are learners constituted as moral subjects of their own actions? What his work makes possible is a politics which embraces a recognition of the multiple and contradictory aspects of both our individual and our collective beings.

References

Anthony, G., & Walshaw, M. (2007) *Effective Pedagogy in Pāngarau-Mathematics: Best Evidence Synthesis Iteration [BES].* Wellington, New Zealand: Ministry of Education.

Appelbaum, P. (2008) *Embracing Mathematics: On Becoming a Teacher and Changing with Mathematics.* New York, NY: Routledge.

Blake, N., Smeyers, P., Smith, R. & Standish, P. (1998) *Thinking Again: Education After Postmodernism.* Westport, CT: Bergin & Garvey.

Brown, T. (2011) *Mathematics Education and Subjectivity: Cultures and Cultural Renewal.* Dordrecht, The Netherlands: Springer.

Dreyfus, H. & Rabinow, P. (1982) *Michel Foucault: Beyond Hermeneutics and Structuralism.* Sussex, UK: The Harvester Press.

Ernest, P., Greer, B. & Sriraman, B. (2009) *Critical Issues in Mathematics Education.* Greenwich, CT: Information Age.

Foucault, M. (1970) *The Order of Things: An Archaeology of the Human Sciences* (trans. Sheridan, A.). New York, NY: Vintage.

Foucault, M. (1972) *The Archaeology of Knowledge and the Discourse of Language* (trans. Sheridan Smith, A. M.). London, UK: Pantheon.

Foucault, M. (1984a) *The Foucault Reader: An Introduction to Foucault's Thought* (ed. Rabinow, P.) Harmondsworth, UK: Penguin.

Foucault, M. (1984b) *The Care of the Self* (trans. Hurley, R.). Harmondsworth, UK: Penguin.

Foucault, M. (1984c) *The Use of Pleasure* (trans. Hurley, R.). Harmondsworth, UK: Penguin.

Foucault, M. (1988) Truth, power, self: an interview with Michel Foucault. In Martin, L., Gutman, H. & Hutton, P. (Eds.) *Technologies of the Self: A Seminar with Michel Foucault.* Amherst, MA: The University of Massachusetts Press.

Gee, J. P. (2004) Discourse analysis: what makes it critical? In Rogers, R. (Ed.) *An Introduction to Critical Discourse Analysis in Education,* pp. 19-50. Mahwah, NJ: Lawrence Erlbaum.

Jablonka, E., Wagner, D. & Walshaw, M. (2013) Theories for studying social, political and cultural dimensions of mathematics education. In Clements M. A., Bishop, A., Keitel, C., Kilpatrick, J. & Leung, F. (Eds.) *Third International Handbook of Mathematics Education,* pp. 41-68. Rotterdam, The Netherlands: Springer.

Maheux, J. F. & Roth, W. M. (2011) Relationality and mathematical knowledge. *For the Learning of Mathematics* **31**(3), 36-41.

Nietzsche, F. (1967) *The Will to Power* (trans. Kaufmann, W.). New York, NY: Random House.

Radford, L. (2012) Education and the illusions of emancipation. *Educational Studies in Mathematics* **80**(1/2), 101-118.

Roth, W. M. & Radford, L. (2011) *A Cultural Historical Perspective on Teaching and Learning.* Rotterdam, The Netherlands: Sense.

Roth, W. M, Radford, L. & LaCroix, L. (2012) Working with cultural-historical activity theory. *Forum: Qualitative Social Research* **13**(2), art. 23.

Walshaw, M. (1999) An unlikely alliance: mathematics education, post-structuralism, and potential affirmation. *Mathematics Teacher Education and Development* **1**, 1-12.

Waugh, P. (1992) *Postmodernism: A Reader.* London, UK: Edward Arnold.

Euclid's geometry seems to have been proposed as a cosmological theory (cf Popper, 1952, pp. 187-9). Both it's "postulates" and "axioms" (or "common notions") were proposed as bold, provocative propositions, challenging Parmenides and Zeno, whose doctrines entailed not only the falsity, but even the logical falsity, the inconceivability, of these "postulates". Only later were the "postulates" taken to be indubitably true and the bold anti-Parmenidean "actions" (such as "the whole is greater than the part") taken to be so trivial that they were omitted in later proof-analysis and turned into "hidden lemmas". This process started with Aristotle: he branded Zeno a quarrelsome crank, and his arguments "sophistry". This story was recently unfolded in exciting detail by Árpád Szabó (1960, pp. 65-84). Szabó showed that in Euclid's time the word "axiom"—like "postulate"—meant a proposition in the critical dialogue (dialectic) put forward to be tested for consequences *without* being admitted as true by the discussion-partner. It is the irony of history that its meaning was turned upside down.

[…]

The analogy between political ideologies and scientific theories is then more far-reaching than is commonly realised: political ideologies which first may be debated (and then perhaps accepted only under pressure) may turn into unquestioned background knowledge even in a single generation: the critics are forgotten (and perhaps executed) until a revolution vindicates their objections.

Lakatos, I. (1976) *Proofs and Refutations: The Logic of Mathematical Discovery*, p. 49. Cambridge, UK: Cambridge University Press.