Four Views of ICME-5

Rapport sur le congrès ICME V, Adélaide 1984

Nous voulons tout d'abord souligner la grande efficacité de l'organisation, sous la responsabilité de M.F. Newman et M. Carss, qui a permis non seulement une intense circulation

Nombreux sont les enseignants et les formateurs qui ont semblé découvrir à cette occasion l'existence d'une véritable *recherche expérimentale* sur les phénomènes spécifiques de l'apprentissage et de l'enseignement des mathématiques.

Il est vrai que beaucoup de déclarations relèvent encore de la profession de foi, ou constituent des prises de position plus idéologiques que scientifiques. Par exemple en géométrie, l'idée qu'il suffit de donner aux élèves des activités ouvertes de manipulation pour qu'ils s'approprient des notions géométriques, sans que soit posé le problème de la structuration de ces activités en fonction des acquisitions visées ou le problème de l'identification des savoirs construits; en d'autres termes l'institutionnalisation de cette connaissance par l'enseignant.

Non seulement la présence de la recherche a augmenté en quantité dans le programme et dans les activités d'ICME, mais elle a changé de nature. Ainsi dans le cas de l'enseignement élémentaire, on a pu noter que les travaux évoluaient d'une approche de type statistique ou psychométrique vers une approche plus phénoménologique dans laquelle il s'agit moins de compter les erreurs des élèves que d'analyser les procédures qui les accompagnent, dans un effort *d'explication et de compréhension* de leurs fondements conceptuels.

Cette présence de la recherche nous semble devoir être liée au développement important du groupe international Psychology of Mathematics Education (PME) crée lors de la IIIème réunion d'ICME (Karlsruhe, 1976). Les membres de PME se réunissent annuellement pour échanger des résultants et débattre de questions théoriques ou méthodologiques à propos de recherches expérimentales portant sur l'acquisition de concepts mathématiques. Au sein même de ce groupe, et les présentations faites à ICME V en attestent, on peut constater une évolution d'approches statistiques très "quantitatives" vers des approches cliniques plus qualitatives mais aussi plus préoccupées d'explication des processus cognitifs mis en évidence.

Plus récemment une nouvelle évolution s'est amorcée: entrer dans la classe pour étudier dans cette situation particulière les phénomènes liés à l'apprentissage des mathématiques. La situation dans laquelle se trouve l'élève, notamment la situation sociale, a une incidence importante sur la nature et la structure de ses acquisitions et de ses conduites cognitives. Cette évolution est l'objet d'un débat au sein de PME, mais il semble à beaucoup qu'elle est nécessaire dès lors que l'objectif est de contribuer à une meilleure connaissance et à l'amélioration de l'enseignement des mathématiques.

Ce développement de l'intérêt pour la recherche, les confrontations qu'il a suscitées, montrent que les activités de recherche recouvrent une grande hétérogénéité de pratiques, voire de traditions. Dire qu'il y a une tendance marquée vers les travaux empiriques et cliniques ne doit pas laisser penser à un mouvement unanime. Par ailleurs dans le même temps émergent de façon, parfois aigüe, les problèmes des relations entre théorie et pratique. Nous les aborderons plus loin

Ce qui réunit les chercheurs actuellement est la nécessité reconnue par chacun de constituer un ou plusieurs corps théoriques permettant de donner du sens aux travaux de recherche jusqu'ici trop souvent isolés et donnant des fondements solides aux débats scientifiques qui s'engagent. Les résultats d'une recherche ne peuvent être discutés sans référence aux cadres théoriques et problématiques qui l'ont guidée; un bon usage de tests statistiques ne garantit pas la pertinence des questions abordées ni leurs conclusions. La corrélation n'est qu'un indice, un fait, sa signification est à construire, cela n'est pas possible sans fondements théoriques.

Ce débat fondamentaliste sur la recherche sur l'enseignement est complexe et la discussion a semblé parfois glisser vers la caractérisation des chercheurs plus que vers la caractérisation de l'objet de leur recherche. Il y a là un écueil auquel il faudra prendre garde. Cette caractérisation est un des points cruciaux qu'il faudra éclaircir:

> comment définer l'objet d'étude de recherches *spécifiques* de l'enseignement des mathématiques (ce que nous nommons en France: Recherche en Didactique des Mathématiques).

Cette question reprise et développée notamment par Brousseau et Steiner lors de la post-conférence Theory of Mathematics Education (TME), se trouvera sûrement au centre des rencontres à venir sur ce thème. Elle soustend plusieurs autres questions soulevées lors des communications ou des débats comme par exemple: l'importation de concepts théoriques (de la psychologie ou de la pédagogie, etc.) versus la constitution de concepts originaux (home grounded concepts), la reproductibilité des phénomènes didactiques, les difficultés et spécificités méthodologiques.

Théorie, pratique et formation

Les relations entre recherche et pratique ont préoccupé, de façon importante, de nombreux participants à ICME V; peut-être faut-il voir là la conséquence de deux phénomènes concommitants:

—d'une part, un groupe de personnes, les chercheurs sur l'enseignement des mathématiques, est en train de se constituer socialement dans de nombreux pays et au plan international. Que peuvent en attendre les enseignants: de nouveaux censeurs, ou au contraire des alliés face aux difficultés rencontrées dans leur pratique?

—d'autre part, de nombreux enseignants et formateurs commencent à percevoir ce qu'une recherche scientifique pourrait apporter à l'amélioration de leur métier, certains en sont les premiers acteurs. Vont-ils abandonner complètement le contrôle d'un mouvement dont ils ont été les initiateurs?

Ce caractère d'abord politique des rapports entre recherche sur l'enseignement et pratique permet de comprendre pourquoi l'on assiste souvent à des dialogues de sourds. Cette difficulté doit être dépassée pour aborder sur le fond le problème des rapports entre recherche et pratique. Des propositions intéressantes ont été faites par les chercheurs allemands (Wittmann, Steinbring) qui suggèrent l'existence de lieux de pratique intermédiaires autour desquelles se retrouveraient chercheurs et praticiens. Cette solution est celle que l'on retrouve dans les Instituts de Recherche sur l'Enseignement des Mathématiques (IREM) en France, à propos de la formation continue des enseignants

Peut-être les relations seraient-elles facilitées si les débats portaient moins sur les personnes que sur leurs activités. Il n'y a pas de difficultés à être enseignant-chercheur, ce qui arrive dans de nombreux systèmes universitaires, là où se trouve l'incompatibilité c'est dans l'exercice simultané des deux activités. Comme le fait remarquer Brousseau, le critère de démarcation est déontologique: l'efficacité de la pratique et la rigueur théorique peuvent avoir des exigences contradictoires; elles ont des responsabilités au regard de la société, et vis à vis de l'élève qui ne sont pas de même nature.

Ce genre de difficultés a été mise au jour dans l'activité de l'un des ateliers du groupe "Problem Solving." Enseignants et chercheurs, à propos de l'exercice en commun d'analyse—ou d'une tentative—d'une séquence vidéo, ont finalement vu apparaître que leurs questionnements, leurs exigences, leurs motivations ne se rencontraient pas. Pourquoi analyser, quoi analyser, comment analyser l'activité de résolution de problème de deux élèves, autant de questions qui renvoyaient à la position professionnelle de chacun. L'explicitation de cet obstacle était un préalable à l'activité collective de l'atelier, cela a finalement, et très positivement, été le résultat essentiel de son activité.

On a souvent entendu réclamer l'amélioration de la communication entre chercheurs et enseignants. Cela ne dépend pas seulement de l'amélioration des structures et des supports de cette communication. L'accès aux résultats de la recherche ne va pas de soi, d'abord parce que les communications entre chercheurs n'ont pas les mêmes exigences ou le même contexte que la communication enseignant-chercheurs. Un *travail spécifique* sur les résultats de la recherche est un préalable indispensable à leur communication, pour permettre une réelle efficacité pratique.

La formation initiale des enseignants a un rôle important à jouer dans l'amélioration des communications entre recherche et pratique. Déjà on a pu relever l'importance qui est donnée au contact direct du futur enseignant avec l'élève par le moyen d'observations ou de pratiques d'interviews cliniques.

La formation des enseignants du second degré (11-18 ans) semble s'écarter un peu de la seule formation aux contenus mathématiques pour prendre en compte l'élève. Cette évolution est une conséquence directe du développement des recherches sur l'apprentissage des mathématiques qui ont montré que la complexité psychologique d'un concept mathématique n'est pas identifiable à sa complexité mathématique. La formation à l'enseignement lui-même, dans ce qu'il a de spécifique des mathématiques pose plus de problèmes; mais il en est de même dans le domaine de la recherche où les travaux dans les classes sont peu nombreux. Cependant quelques exemples ont été donnés, en France notamment.

Un point de discussion est celui de la finalité de cette formation. S'agit-il de fournir des méthodes pour enseigner ou des connaissances sur l'enseignement (savoirs et savoir-faire)?

Informatique et enseignement des mathématiques

L'informatique a tenu une grande place parmi les préoccupations de plusieurs groupes au congrès. De nombreuses démonstrations sur micro-ordinateurs de logiciels pour l'enseignement ont été faites, notamment au "French corner." On remarque une nette évolution de ces logiciels: alors qu'au début il s'agissait essentiellement de "questions-réponses," avec analyse des réponses de l'utilisateur, on trouve maintenant des logiciels beaucoup plus ouverts, qui permettent une utilisation dynamique et laissent au maître et à l'élève un rôle actif.

Une grande part du "technology theme group" a été consacrée à l'informatique. Les échanges et les discussions ont permis de se rendre compte que dans la plupart des pays "développés," la situation est assez semblable: l'équipement des établissements scolaires en matériel informatique augmente rapidement, et on peut penser que dans cinq ans, presque tous les établissements seront équipés. Mais il y a un écart entre le matériel, mais pas d'utilisateurs, ou bien il y a des utilisateurs potentiels, mais pas assez de matériel! Cela pose naturellement le problème de la formation des enseignants. En beaucoup d'endroits des séances d'initiation à l'informatique ont été mises en place. Mais, et c'est ce qui frappe le plus, les développements se font de façon non coordonnée, dans toutes les directions, sans une solide réflexion sous-jacente. Il en résulte un afflux de didacticiels en tous genres. Il est urgent qu'une réflexion s'organise au niveau international.

Autre point commun à beaucoup de pays: les divergences entre mathématiciens au sujet de l'informatique. Bien que la situation évolue très rapidement, l'informatique demeure pour de nombreux mathématiciens, une mode, un gadget, ou en tout cas une science suspecte.

Cependant, d'importantes convergences ont pu être mises en évidence dans les groupes de travail, et l'informatique apparaît comme un outil à la fois pour les mathématiques et pour l'enseignement des mathématiques Il ne s'agit pas seulement d'un outil technique, mais aussi d'un outil pour la pensée, le raisonnement, la résolution de problèmes. Cet outil pourrait conduire à des "styles de pensées" fort différents de ceux que nous connaissons actuellement.

L'informatique influe sur les mathématiques ellesmêmes. L'informatique pour les mathématiques dépasse largement le stade de la programmation: l'algorithmes dans les mathématiques.

L'informatique permet d'avoir des mathématiques une vision plus expérimentale: l'observation, la visualisation, la simulation, la conjecture, la vérification numérique, sont au coeur de l'activité mathématique. Cette évolution des mathématiques se situe au niveau de la recherche et à celui de l'enseignement. Déjà, on se demande comment devront évoluer les programmes de mathématiques, sous l'influence de l'informatique. Quant au niveau pédagogique, chacun connaît des expériences sur l'utilisation de l'ordinateur comme outil pédagogique. De nombreaux "styles pédagogiques" apparaissent, et il importe de développer une approche multi-usages, tirant profit des diverses façons d'utiliser un ordinateur pour l'enseignement.

Sur l'ensemble de la question de l'influence des ordinateurs et de l'informatique sur les mathématiques et leur enseignement, la Commission Internationale pour l'Enseignement des Mathématiques (C.I.E.M.) a d'ailleurs mis en place une réflexion large et ouverte. Un texte, posant les questions essentielles, a été diffusé* et un colloque en 1985 devra produire un document de réference.

Mais il faut souligner que la réflexion sur l'informatique repose beaucoup plus sur des perceptions et des expériences locales, souvent pleines d'a priori, que sure des études didactiques ou épistémologiques sérieuses. Les travaux et les expérimentations ont porté la plupart du temps sur l'ordinateur et sur les contenus de l'enseignement, plus que sur l'enfant et son comportement. Lorsqu'on parle de la nouvelle attitude de l'élève face à l'ordinateur, face aux mathématiques par le truchement de la machine, lorsqu'on parle du rapport élève-machine, on s'appuie plus sur l'idéologie que sur l'expérimentation. L'ordinateur introduit une position nouvelle de l'enseignant par rapport à l'élève et par rapport au savoir. Mais s'agit-il là d'un phénomène fondamental ou d'un phénomène conjoncturel, qui s'estompera dès que l'informatique aura cessé d'être une nouveauté?

De nombreuses recherches restent à mener autour de l'informatique: recherches sur l'enseignement et l'acquisition des concepts informatiques (Didactique de l'Informatique), mais aussi recherches sur la façon dont l'informatique influe sur l'enseignement et l'acquisition de concepts mathématiques. En développant simultanément ces recherches, des essais, des expérimentations et des innovations dans de nombreuses directions, et en les articulant convenablement avec la formation initiale et continue des enseignants, peut-être arrivera-t-on à plus de cohérence et de solidité, dans ce domaine où les idées et les énergies ne manquent pas.

Y verrons-nous plus clair à Budapest, en 1988?

N. BALACHEFF, C. COMITI, C. LABORDE
Equipe de Recherche en Didactique des Mathématiques et de l'Informatique
Institut IMAG
Université de Grenoble 1
B.P. 68
38402 St Martin d'Hères
FRANCE

B. CORNU
Institut Fourier
Université de Grenoble 1
B.P. 74
38402 St Martin d'Hères
FRANCE

ICME-5 Revisited

Now here's one for all you trivia lovers! ICME-5 was held in Adelaide, Australia August 24-30, 1984. Where and when were the first four held?

Here's another question, and this one might be more challenging. What would induce some 2000 teachers, educational researchers, and mathematicians—mostly residents of the northern hemisphere—to spend thousands of dollars, travel enormous distances from every corner of the world, and suffer the ravages of jet lag in order to spend a week in Australia during the southern winter? I have asked myself this question several times since I got back from Adelaide—indeed, I asked myself the same question after the last ICME—but I still don't have an answer that I'm completely happy with.

I suppose that an ICME conference has a number of fairly obvious attractions for mathematics educators. In a sense ICME is *the* conference for us, at least from my

L'enseignement mathématique, 1984, Tome 30, p. 159-172 perspective here on the western edge of North America. We have the local and annual meetings of the NCTM, of course, but they are really aimed more directly at classroom teachers than at people like us. The AERA annual meeting is closer, but its audience and program are so wide that it's not really the best meeting for mathematics educators either. Here in Canada we have an annual, week-long meeting of mathematicians and mathematics educators under the auspices of the Canadian Mathematics Educators Study Group—CMESG to its friends— and that's much closer to the mark. But an ICME conference should be like having a hundred CMESG meetings simultaneously, and that's what makes it so special. An ICME conference provides the only opportunity we have to meet with colleagues from all over the world and to compare notes with them on the problems, issues, and questions we are all facing. Moreover, the formal program of the conference includes sessions conducted by some of the leading names in our field.

Among the highlights of ICME-5 for me were the plenary sessions, particularly the papers delivered by Jeremy Kilpatrick of the University of Georgia and by Renfrey Potts, a mathematician from the University of Adelaide. Jeremy gave an excellent speech, entitled "Reflection and Recursion," in which he discussed the computer as a metaphor for the processes of teaching and learning mathematics, and the importance of self-awareness to those processes. Professor Potts gave one of the most stimulating and expertly staged presentations it has ever been my good fortune to hear at such a conference. He urged us to consider the importance that discrete mathematics has assumed in mathematics, and to revise the school curriculum to include topics from discrete mathematics.

Most of the rest of the conference program was made up of what were called Action Groups, Theme Groups, and Topic and Study Groups. These were set up to deal with a wide variety of subjects of current interest in the field. Topics included mathematics education for various age groups of students, the role of technology, applications of mathematics, problem solving, the importance of theory and research, evaluation, and psychology, to mention just a few. Participants were expected to join an Action and/or a Theme Group and to meet with that group each day. The proceedings of the conference, when they are published, will contain summaries of the deliberations of each of these groups as well as reprints of the papers which were commissioned for some of them and presented at the conference itself.

I was dissatisfied with a number of things about the conference. First of all, I felt that there were simply too many sessions. This led to poor attendance at many sessions, either because there were too many things for people to choose from at any one time, or because some sessions were scheduled too late in the day. I also felt that there was entirely too much English at the conference: the number of sessions which included non-English-speaking speakers was very small indeed. Having to commit oneself to an Action or Theme Group for the duration of the conference was a drawback too, especially if you decided along the way that you had chosen the wrong one.

It is always interesting to find at conferences like these how universal our problems and concerns are. For example, the shortage of qualified mathematics teachers seems to be a problem just about everywhere, not just in North America. Unfortunately, it seems to remain just that, a problem: no good solutions appear to have been identified as yet. It is also evident that the role of applications and problem solving in the mathematics curriculum is a major concern in most countries. No major breakthroughs on these, or other major points, were made during the conference, at least to my knowledge, but the fact that so many scholars are paying attention to some of these major issues is heartening nevertheless.

Some other things I liked about the conference had to do with the extra-curricular activities. First of all, Adelaide is a very attractive and well laid out city, surrounded by the Adelaide hills. We enjoyed a pre-conference trip by rented car to the nearby Barossa Valley, a premier Australian wine-growing region complete with picturesque vineyards and congenial tasting rooms. We also enjoyed the friendliness and hospitality extended to us by our Australian hosts at various social and cultural events, although I think I heard enough about the Australia Cup race (formerly known as the Americas Cup) to last me a lifetime.

The next ICME is scheduled for Budapest in 1988, and I would like to make a few recommendations to the organizers about some things that might be done to make the conference more fruitful for those attending. First, try to locate the conference at a convention centre or some similar facility where all of the sessions can be held in fairly close geographical proximity to one another. At the last two conferences, sessions have been held in different buildings scattered across large university campuses. Among other things, this results in some things being very hard to find. Second, and this is related to my first point, try to be realistic about the sizes of rooms needed. I attended several sessions in rooms which could hold over a hundred people but there was only a handful of people in attendance. This makes for an uncomfortable situation for all concerned, audience as well as speakers. Third, restrict the number of sessions that are held at any one time and keep the number of late afternoon/evening sessions to a minimum. Finally, and this is the most problematic issue, do everything possible to ensure the quality of the conference presentations by insisting that all papers be submitted in advance to organizers of sessions or groups of sessions, and that all such submissions be rigorously refereed.

My overall rating of ICME-5 is positive; I would give it 7.5 on the universal 10-point scale. I haven't decided yet whether I'll go to Budapest for the next one or not, but I probably will. To miss an ICME is to miss being "where the action is," and to miss a wonderful opportunity to renew acquaintances with valued colleagues from around the world.

DAVID F. ROBITAILLE
Department of Mathematics and Science Education
University of British Columbia
Vancouver, B.C.
Canada V6T 1W5

Letter from Adelaide

> "This congress had wide pages and narrow margins" (with apologies to Grice)

It is one thing to review a book. It is quite another, however, to attempt to review a congress At least with a book, you can presume the same set of words have passed before your and any other reader's eyes. No two people are likely to have attended the same conference and "the conference" is no longer there for someone to attend who did not. A book continually lies in wait and so a review can attract new readers. Reading a congress review may have to be the primary experience, as with reading theatre reviews of plays you know you will never see.

To a considerable extent, this will be a deliberately personal account in an attempt to avoid a bland, impersonal summary. One of my assumptions concerning the likely readership of this piece is that a sizeable portion of it will have either attended ICME-5 or known and already talked with someone who did Consequently, I shall not spend any time either scene-setting or on the structural organisation of the meeting itself. That said, I would like to note that one of the distinctive features of the Adelaide congress was the inclusion of the so-called Action and Theme groups. The Berkeley meeting in 1980 was relatively uniform in structure, with a wide choice of presented papers throughout the day. In Adelaide, the structure was across rather than within days. As a consequence, by being asked to decide on Theme and Action topics beforehand, a sizeable portion of each day was spoken for prior to the manifested reality

I attended the theoretical mathematics section of the Tertiary Academic Institutions Action Group, despite the regular graveyard slot of 8:30 - 10:00 a.m. The initial focal concern of our group of about 25 people was the notion of rigour In the next few paragraphs, I should like to try to convey some thoughts which came out of the discussions our group had. I have included a detailed discussion of the working of one group partly as a number of points of interest emerged worth of record in their own right. My other reason is to indicate one instance of what is possible from a group of motivated strangers, and a standing start, in four meetins of an hour each on consecutive days.

We started by examining some of the connotations of the word itself, with suggestions including:

> *hard/firm/harsh* (the rigours of boarding school Carries with it a punishment ethic and value system. It is not good for you unless it hurts. A moralistic term.)
>
> *in depth* (level of detail, carrying with it a value of more is better)
>
> *formal* (a game, or a more serious activity Who makes the rules and why?)
>
> *solid/worthy/reliable* (will it take your weight? Something to put your trust in. Recall William Clifford, "Remember, then, that scientific thought is the guide of action; that the truth which it arrives at is not that which we can ideally contemplate, but that we may act upon without fear.").

Rigour is an odd sort of word in that it is clear neither in its grammatical function nor its referrent. We talk as if rigour were a thing.

What is the purpose of rigour? Only people have purposes, which recalls Hammer's aphorism that "the most neglected existence theorem in mathematics is the existence of people." Thom claimed (during ICME-2) that he would be happy to sacrifice rigour for increased understanding, and suggests that mathematics has never been seriously derailed as the result of an error Some in the group were insistent that clarity is a property of the seeing (the underlying mental perception), not of the saying (the articulation). It is therefore a fallacy to attribute rigour to an expression, whether spoken or written.

The scope of the group's discussions spread to questions of proof and intuition, and the role that certain proof styles can play in defeating or destroying intuition The formal machinery of analysis was examined in this light using as example an epsilon-delta proof of:

$$\lim_{x \to 1} (2x + 5) = 7$$

The perspective of testing the definition of limit against this known reality (or perhaps what we would want to be the case) was explored as an alternative to proving the result in the sense of providing security of knowledge where before there was doubt This perspective provides a variant answer to the question of whether it is worth proving, because it is usually seen either as obvious and/or not worth knowing.

In the process of formalising intuitions in order to extend the range of agreement, which seems to me to be one of the aims of mathematics, there has to be a movement back and forth between the formal and the intuitive. In mathematics, there has been a considerable move from intensive to extensive definitions, where the former capture the meaning and purpose better, but where the latter prove more computationally tractable One problem of accepting extensive defintions without careful exploration and examination is that they frequently allow in "monsters" Students frequently find their intuitions and their theorems being defeated by something they were not even aware of as being in the game. This was described in terms of "It may be in the rules, but it isn't cricket". The spirit in which the game is played is also important.

Finally one possible strategy was proposed to help in this regard, namely Brecht's *Verfremdungseffekt,* the distancing of the observer from the everyday and commonplace, making the familiar strange in order that it may be examined. Part of this has to do with holding before oneself the possibility that a result or concept might have been otherwise. This approach attempts to justify the desire for proving the apparently obvious by making it less obvious One example discussed was the intermediate value theorem. Proof by picture is hard to contend with, as it is hard to draw a plausible potential counterexample. There is also a problem of what the student is making of proffered counterexamples. The result seems obvious and is therefore perceived as not worth proving (i.e. the investment of effort) or even not worth knowing. Using

$$f: Q \to Q, f(x) = x^2 - 2,$$

for instance, might indicate some of the subtlety of the result, and points up that the theorem says as much about the nature of the real line as that of continuous functions on it

This approach can be described as showing that theorems tend to be *closed* in a metaphorically topological sense, that is, if you perturb the assumptions even slightly, counterexamples appear. Part of the difficulty is that students tend to see nothing but theorems in lecture courses and therefore do not come to value them; they appear to be commonplace.

Philip Davis, in his sub-plenary lecture on proof, addressed some similar questions in the philosophy of mathematics. He claimed that the only question that has been looked at by philosophers in the last hundred years is "What makes mathematics true?" He contrasted private with public theories of mathematics, alluding to the social dimension of mathematics attended to in the latter, and claimed that all the current "-isms" (e.g., formal-, logic-, intuition-, ...) are private theories which require only one ideal mathematical knower. Only in public theories, where mathematics is seen as social practice, does an account of teaching become relevant and can the notion of discovery be addressed. A similar community and tradition-oriented account of mathematics can be found in Philip Kitcher's recent book, *The Nature of Mathematical Knowledge*, in which he set himself the task of producing an account of mathematics which a working mathematician would recognise. This novel orientation reverses Thom's insight that "In fact, whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics," and suggests that the teaching of mathematics and its traditions should be of considerable interest to the philosopher.

Adelaide provided an opportunity for Hans-Georg Steiner to develop his plans for a group working on the theory of mathematics education, and a two-day post conference was held immediately following ICME-5. He had earlier offered a suggested aim for mathematics education, namely that of providing some stability, to act as a damping force against cyclic pressures which seem to be increasingly prevalent in political and social decisions about the teaching of mathematics. In order to be able to make a rational reaction to these pressures, he insisted on the need for, not an immutable body of theory and experiment, but a stable platform from which to observe, perform experiments and return in order to evaluate and interpret the results.

Bob Davis ran a couple of informal sessions during the congress, ostensibly on the place of understanding in mathematics, which brought to light some of these socio-political pressures. Whether or not it was his intention, one of the considerable forces at work seemed to involve enlisting the support of delegates to combat a perceived swing in the United States back towards rote methods of instruction. This provided a dissonance of expected tone which I felt worthy of considerable attention. We were being strongly invited by certain members of the audience to attend to a political dimension in mathematics education, namely any force we may collectively possess as a pressure group of international opinion. Other voices expressed concern regarding which community, if any, we could claim to speak for other than the fifty or so assembled individuals. In the remainder of this report I intend to consider some of these quite general, quasi-political issues concerning the various mathematics education communities, rather than report the activities of a specific group. I shall illustrate these by means of events arising from ICME-5.

Due in part to the broad but vague nature of the mathematics education enterprise, there is a wide disagreement concerning the goals of such a congress. Should it be seen as predominantly a meeting of researchers in an academic discipline, such as the comparable meeting of the International Congress of Mathematicians? Such conferences in mathematics announce new results, illustrate new techniques and allow integrative survey lectures of areas of current concern. Certainly Jeremy Kilpatrick's plenary lecture on the increasing incursion of the concepts of reflection and recursion from mathematics into mathematics education fitted that description. But in mathematics, there is a far greater homogeneity of audience which makes both the task and the required level of dissemination clearer

More than a third of those attending the congress were teachers of school mathematics, and another considerable group in attendance were teacher educators. ICME congresses provide a large-scale opportunity for teachers to meet and interact with researchers, though I do not want to reinforce a sense of dichotomy between these groups. Teacher educators were able to meet together and discuss varying practice and developments in their area. Another role which the congress performs can be likened to that of a book fair, that is providing a venue for books, materials, videos and software to be displayed and examined. Can ICME also be seen as a giant in-service programme for practising teachers? What possible aims should it aspire to and are any of these conflicting? By trying to be all things to all who have any connection with the teaching of mathematics at some level, the congress can be diffuse and sprawling

One of the general uncertainties about mathematics education is its fuzzy discipline barriers, in other words, who can reasonably be said to belong to it? ICME shares this same confusion concerning what are and should be its aims, together with who should decide them. These concerns among others I believe arise out of the transitional period we are currently in whereby mathematics education is emerging as an academic discipline in its own right. It already possesses some of the contemporary markers, such as specialist journals, although it lacks some of the coherence, either theoretical or methodological, which many other more established disciplines an offer. It even lacks a broad consensus concerning what the important questions are (see Freudenthal's address in the Berkeley proceedings or the last couple of issues of this journal).

There are also institutional pressures at work One instance of this is whether mathematics education is situated in mathematics departments as an adjunct discipline, or in education departments as a specialist area, or as a separate entity in its own right. At a larger level they involve the relation of ICME congresses to ICMI, which is after all controlled by the IMU. This imposed hegemony was underlined by the final presidential address on the topic of Hausdorff dimension and fractals What was made of this presentation, irrespective of its virtues or otherwise at a mathematics level, by almost anyone in the audience was beyond me. It did serve the political function of underlining the fact that the president of ICMI is a mathematician appointed by the IMU

These discussions, which hinge on questions of control, status and access, reflect the above uncertainty at a global level concerning the role and intended audience for such international congresses. As a result of individual initiatives, in particular by Marjorie Carss, new channels of access to ICME-5 were created, as well as employing the more conventional national institutional structure. My opinion is that Adelaide was much better for this. I hope that the organising committee of the 1988 Congress in Hungary will not rely solely on the academic, institutionalised route, which tends to be overrepresented with established mathematicians. Another indication of the emerging discipline of mathematics education is that there are alternative routes other than that of defected mathematician, by which academics in mathematics education are coming into being

From a personal point of view I greatly enjoyed and gained from the meeting The social hours were most beneficial, particularly in enabling extended conversations to occur, as well as finding congenial dinner companions. Spare time for precisely such encounters needs to be available, yet due to the pressure on the timetable, nineteen different occurrences would immediately fill any such vacant spot if it were programmed. Although it is a commonplace remark, I do not even primarily attend such conferences to listen to formal presentations. They provide the framework to deviate from without which ICME would be the milling chaos which was evident at the Monday Barossa Valley outing. If Hungary is to be a comparable occasion in 1988, more than a formal structure is required.

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P S. My final thought concerns the question of cost For financial reasons the organisers wanted as many Australians to attend as possible. From the other side, a very rough estimate of the participants costs (irrespective to some extent of whether they were met personally, institutionally or nationally) came to some two million pounds, leaving aside the question of salaries of the delegates during the period of their visit. Thinking somewhat crassly purely in investment terms, the question has to be raised "Is it worth it?" That is, might not a greater global improvement in mathematics education be brought about by a more direct application of such resources? I am aware that this argument is slightly specious in that the same money may not be available for other things, but I nonetheless think the question is worth asking, in particular, what is being bought with the money. I am not intending to question all international conferences, but feel that the nature of ICME congresses, together with the nature of the subject, allows it to be raised.

A comparable question can be raised about computers and their relation to mathematics education. The Congress ended with a non-debate on the role of computers in mathematics education ("Miracle or Menace") between Philip Davis and Hugh Burkhardt. One crucial point related to the above phenomenon was made by Burkhardt (billed as the enthusiast) when he noted that to cast doubt on a current euphoria is frequently seen as reactionary, harmful and ultimately seditious.

While not courting the charge of Luddite, a defensible reaction (and one of the many points not made by Philip Davis) is surely one of concern at the incursion of highly sophisticated and expensive computing equipment into schools at a time when, in Britain at least, there are serious financial shortages being imposed in the educational system. Computers are frequently smuggled in under claims of improving the teaching of mathematics, yet the Cockcroft report noted with concern that it was primarily mathematics teachers who were being used to teach the burgeoning computing and computer studies courses in schools. This shift is acting to the detriment of school mathematics teaching. If, as with the Congress, the intent were really to improve the teaching of mathematics *specifically,* surely comparable or greater gains would be achievable by spending those enormous sums more directly (while being aware that in Britain the Government subsidies are also covert support for Britain's computing industry). My reaction to these sorts of arguments is that they are redolent of a justification for the enormous cost of the American space programme in terms of spinoffs such as Teflon. I have no doubts whatsoever that computers warrant considerable attention in schools. My feeling is that it is deleterious and harmful to both causes to confuse the relationship between them.

Faculty of Mathematics
The Open University
Walton Hall
Milton Keynes, MK7 6AA
England

ICME Past...ICME Still to Come

The International Congresses on Mathematical Education might well be regarded as the "other Olympics"—the mathematical Olympics. Take away the vying for gold (that belongs to the mathematical olympiads?) and the resemblance is close. There is the long period of preparation of the ICME venue (ICME-5 in Adelaide was eight years in planning); there are the opening and closing ceremonies, the core and peripheral events, the personal best performances of participants—some new, some tried "olympians"; there is international colour and camaraderie; there are "incidents"; and there is the particular flavour given to each of the "games" by the host nation. The passing of the fifth ICME heralds the coming of the sixth in Hungary in 1988 and provides an occasion for reflection. One can reflect on the most recent set of performances or one can reflect comparatively on successive ICMEs. To do the latter provides an interesting review of the changing issues and emphases in mathematics education over the past 15 or more years.

In choosing to review ICMEs rather than ICME-5 I am faced with certain dilemmas. How can I do so adequately when I was not present at ICME-1 in Lyon, France, 1968? How can I comment on ICME-5 when responsibilities as a member of the National Program Committee considerably diminished my participation in the Program itself? How can one comment satisfactorily on one ICME, let alone four when the complexities of the program are such that it is possible to sample in six days only a smattering of the total offering? Indeed, I regard the greatest challenge in attending any ICME is to select a personal program that will leave one neither satiated nor frustrated nor disappointed but entirely satisfied. In reviewing ICMEs past, I have chosen to record what made an impression on me as a participant, what seemed to be major issues or themes that recurred throughout a Congress, what were some achievements and what seemed to me to work less well.

ICME-2, Exeter, England, 1972. I regard this as the ICME of thematic unity. Could it have been Piaget's message to the Congress that seemed to make the psychology of mathematics education permeate so much of the proceedings? Although I did not attend any of the sessions of the Psychology group, I found myself constantly involved in discussions of issues that had been raised there. Of course, this Congress was the genesis of the International Group for the Psychology of Mathematical Education which had its eighth conference in Sydney, Australia in August 1984. Curriculum was a dominant theme at Exeter; not as much a review of developments under the "new mathematics" as a display of curriculum achievements. Also prominent were considerations of what was good classroom practice, the British Program Committee under Elizabeth Williams giving particular emphasis to the notion that what was new in the "new mathematics" was newness in approaches to teaching as well as (if not more importantly than) newness in content. Computers, too, were making their appearance amongst the technology sections.

ICME-3, Karlsruhe, West Germany, 1976. This appeared to me to be the ICME of unity of purpose. There was a seeming recognition that among the participants there existed a diversity of backgrounds, practices and problems and there was also a need to find solutions and statements that took account of these. Karlsruhe seemed to be the truly international ICME: there was a good representation from a variety of language groups at most sessions, there was a feeling of working together in most groups and outside formal sessions, there was free and eager exchange and communication. This was also the ICME where "back to the basics" was a burning issue. In a sense to encounter this was perplexing for participants from developing countries who had experienced a much later movement for curriculum reform to take account of the "modern" mathematics. Now, so soon, they were hearing the apparent call for a reversal of trends. Other matters which captured attention were the women and mathematics group and Seymour Papert's presentation of Logo. For me, the most positive and productive aspect of Karlsruhe was the functioning of the Working Groups. My own experience was in the Elementary Teacher Education group under the enlightened guidance of the later Arthur Morley. The atmosphere was one of purposeful activity, exchanging ideas and sharing the task with others from many countries of preparing a common report. Happily the contacts made on that occasion have been long-lasting and fruitful.

ICME-4, Berkeley, U.S.A., 1980. What promised on paper to be a highly coordinated and participatory program proved, in my experience, to be considerably less so in practice. There was much less the feeling of personal involvement. The one issue that seemed to me to find some measure of common concern was that of national standards of mathematical performance. Was it that the back-to-the-basics movement had by this time acquired a political backlash and the international mathematics education community was feeling under some threat? Probably to the contrary, as the group which seemed to be the focus of most attention both inside and outside the formal program (though again I did not attend any of its sessions) was the research group. Increased research interest and activity would seem a proper response to outside pressures. Perhaps it was the spread of the congress over such a large campus that made the Berkeley experience seem so compartmentalised and less rewarding—for me, at least.

ICME-5, Australia, 1984. Among the chief aims of the National Program Committee for this ICME were that: (i) a balance should be sought between information giving and participation; and (ii) that the needs of classroom teachers should be taken into account. Both aims were achieved with mixed success. The degrees of balance depended on the perceptions of the chief coordinators of

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Finally, we allow that the stages analyzed above are not definitive, but they reflect a first attempt to represent the evolution of student's programming procedures.

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the various groups, as did the degree of success in catering for teachers' interests. There was indeed, a high proportion of teachers among the participants and their responses seemed generally favourable. The Congress provided them with the opportunity to hear and to talk with people of international repute; they were able to exchange ideas and perceive shared problems with confrères from many parts of the world; and it was an occasion which enabled them to measure the degree to which their own thinking matched levels elsewhere. We are still too close to ICME-5 but it could be that from the events in Adelaide a great deal of productive, co-operative achievement might flow. The potential for this will depend on the ability of the groups' chief coordinators to sustain the impetus. It is too easy for established routines and immediate local pressures to take precedence over new, long range and remote initiatives. Topics which tended to recur in several sections during ICME-5 were: the role of problem-solving and modelling; the need to translate research findings into effective classroom practice; the high priority that should be given to pre-service and in-service teacher education; the important place of language in mathematics; and the need for mathematics learning to be a socially interactive process. A useful and much appreciated feature of the Adelaide program was the provision of periodic summaries of proceedings of certain groups. This was one way of resolving the difficulty of conflicts in interests and timetabling

Finally, one of the pervading influences of ICME-5 was the Cockcroft Report. This seems to have risen like a beacon, providing direction and signalling support for many positive issues that have emerged in mathematics education over the past 25 years: active learning, social interaction, mathematics as a mental process, flexibility in one's repertoire of teaching skills to match the varying needs of learners and so on.

Out of all this, what does ICME achieve and what is its future? Mathematics education has witnessed its own knowledge explosion over the past 25 years. Consider the three-volume research report that was fed to the Cockcroft Committee. ICME has a continuing and important role to play in bringing people in related fields together for the sharing of information and ideas. The future strength of ICME will depend, it seems to me, however, on its ability to preserve the balance between information-giving and the involvement of participants. One way to achieve this is through the continuity which Working Groups might provide in the form of reports on the present state of affairs, new issues, problems, solutions and practices and the like. Such working groups should be allowed to rise and fall as the need demands. It would seem important also to maintain the Action Groups, from early childhood to tertiary levels, and to preserve another balance: that between mathematics and mathematics education. Finally, the difficulty of finding satisfactory solutions to the associated problems of publicising research and improving practice would seem to mandate in favour of making every ICME equally accessible and attractive to the mathematics educator/researcher and the mathematics teacher at every level.

JOHN S. CONROY
Faculty of Education
Macquarie University
Sydney, N.S.W. 2113
Australia