LEVERAGING THE PERCEPTUAL AMBIGUITY OF PROOF SCRIPTS TO WITNESS STUDENTS' IDENTITIES

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It is not easy to define identity. One can hold multiple identities. For instance, a student can be a mathematics major, a student at University X, a junior (*i.e.,* third-year student), a transfer student, and a tutor, while also being an older brother, a soccer team's goalie, an East Los Angelian, a Californian, a Chicano, a US-citizen, a surfer, a new employee at a local coffee shop, a high school valedictorian, a boyfriend and a grandson all at the same time. Each term or phrase provides a set of meanings, whose relevance is determined not only by the situation but also by the others present in the situation and the ways those present position themselves and are positioned. Indeed, in some situations these terms and phrases are merely identifications or affiliations, while in others they shape who one sees oneself to be and how one is seen by others at a given time and place. The facets of oneself that shape one's identity are temporal and situated. For these reasons, researchers interested in identity and its role in students' learning of mathematics have moved towards socio-cultural perspectives which take a relational approach to identity in which "identities are (re)constructed in spaces and moments" (Gutiérrez, 2013) and negotiated with others in context:

> I define *identity* as a dynamic view of self, negotiated in a specific social context and informed by past history, events, personal narratives, experiences, routines, and ways of participating. An identity is who one is in a given community and, as such, is both individually and collectively defined. Although an identity is related to a role or ways of participating (*e.g.,* the role of a *problem poser* in a mathematics classroom), I define *identity* in a broader sense. An identity also encompasses ways of being and talking; narratives; and affective components such as feelings, attitudes, and beliefs (Bruner, 1994; Gee, 2005; Wenger, 1998)—aspects not necessarily included in the term *role*. (Bishop, 2012, p. 38)

Why study identity?

Adiredja and Andrews-Larson (2017) note that the construct of identity, while highly prevalent in the K-12 literature in the United States, especially that focused on equity, has received less attention among researchers studying undergraduate mathematics education. This is surprising for three reasons.

First, as Bishop (2012) notes, national organizations (*e.g.,* NCTM, 2000) have called for K-12 teachers to attend to and develop their practices, due to their impact on the affective components of students' identities (*i.e.,* students' dispositions, affect, mindset, desire to persist, *etc.*) and consequently on students' learning outcomes. As affective components do not stop impacting student learning outcomes when students reach university mathematics classrooms, there is a need to further examine the identities students develop.

Second, as Boaler (2002b) argues, there are theoretical and empirical reasons for viewing learning as constituted through interactions between students' knowledge, identities, and practices (see Figure 1). For instance, Boaler notes, "practices such as working through textbook exercises, in one school, or discussing and using mathematical ideas, in the other, were not merely vehicles for the development of more or less knowledge, they shaped the forms of knowledge produced" (p. 43). Indeed, teaching practices and curricular materials not only promote particular learning environments, but also call on students to engage with mathematics in particular ways with particular tools. Thus, while engaging in the work of learning, students' practices, ways of knowing, being and seeing both mathematics and themselves necessarily shift. This is a point also made by Wenger who argues, "learning transforms who we are and what we can do, it is an experience of identity" (Wenger, 1998, p. 215). It stands to reason then that research on the teaching and learning of undergraduate mathematics stands to benefit from studies that move beyond models of practice and knowledge by exploring the inter-relationships between knowledge, practice and identity.

Third, mathematics majors (who identify themselves as mathematical in a permanent and lasting way) are often called on to reconceptualize what it means to be 'mathematical' during their undergraduate mathematics education. For instance, for some students being mathematical may mean having a good memory and being able to employ computational algorithms accurately and efficiently, without necessarily knowing why or how those algorithms work. And, acting mathematical might entail being compliant, as opposed to inquisitive or independent. Yet, upon entering a proof-centered university course these same students may be called on to be mathematical by demonstrating a propensity

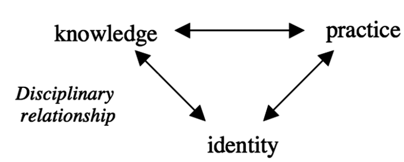

Figure 1. Boaler's (2002b) model of classroom learning.

towards deductive critique, a penchant for abstraction, and a willingness to engage in particular disciplinary discourses which are both argumentative and highly-assertive. It follows that being mathematical may not entail developing a singular mathematical identity but rather flexibly enacting multiple mathematical identities which range from the submissive to the assertive and from the obedient to the authoritative. Given these (potentially conflicting) demands, there is cause to explore undergraduate mathematics students' identities.

On identities and discourse

Bishop's (2012) definition of identity, quoted above, highlights two important facets. First, identities are not haphazard but rather are "informed by past history, events, personal narratives, experiences, routines, and ways of participating" (p. 38). Second, identities are interactively constituted within environments through interactions with others. This second facet raises the question, how are identities interactively constituted? The perspective taken is that one of the primary means for interactive constitution is *discourse*. This perspective is informed by the works of Gee (2001, 2005) who argued that identities are created through discourses, Sfard and Prusak (2005) who defined identity in terms of internalized communications and narratives, and by Bishop (2012) who posited, "discourse plays a critical role in enacting identities." It is also informed by Setati (2005) whose studies of multilingual classrooms in post-apartheid South Africa demonstrated the ways class and power are enacted during mathematics classroom discourses. Specifically, in this article it is argued that identities are *constructed* (Heyd-Metzuyanim, 2017) through discourses as interlocutors position themselves and others in relation to their current social context, classroom environment (its artifacts, content and practices), institutional setting, and history. Thus, as argued Bishop (2012), a viable means for exploring identity is *discourse*: "the spoken and written words, semiotic systems, representations, and gestures of participants as they use language to communicate, interact, and act" (p.44).

The proof script methodology

Researchers have conducted a host of studies on students' reading strategies, comprehension levels and difficulties with mathematical proofs. These studies have either employed proof comprehension assessments (e.g., Mejia-Ramos, Fuller, Weber, Rhoads & Samkoff, 2012), clinical interviews (*e.g.,* Weber, 2015), or novel methodologies, such as proof scripts (*e.g.,* R. Zazkis & D. Zazkis, 2014). The latter entails having students produce a written dialog where the interlocutors discuss a given proof, elaborating on key points, and highlighting difficulties identified by students—*i.e.,* what Koichu and Zazkis (2013) refer to as students' *problematics*. The dialogs are then analyzed to create models of students' understandings of content and practices, their perceptions of key ideas, and their ways of reasoning about *problematics*.

This methodology emerged in response to current issues related to the study of students' ways of knowing mathematical proofs. Indeed, as noted by Koichu and Zazkis, past research examined students' difficulties from the researchers' point of view. Thus, methods were needed that afforded the identification of students' problematics and avoided *expert's blind spots* (Nathan, Koedinger & Alibali, 2001). Not surprisingly, it has proved fruitful; especially with regard to identifying difficulties not seen by experts (see Koichu & Zazkis, 2013). Furthermore, the field was in need of methodologies that not only shed light on students' ways of reasoning, but also attended to the discursive nature of proof. In particular, those working with the proof script methodology have built on Sfard's (2005, 2007) theorizations of thinking, to argue that thinking is inherently communicative.

> Although thinking appears to be an inherently individual activity, there is no reason to assume that its origins are different from those of any other uniquely human capacity: Like all the others, this special form of human doing is most likely to have developed from a patterned collective activity. At close look, the best candidate for the collective activity that morphed into thinking through the process of individualization is interpersonal communication. It seems, therefore, that human thinking can be regarded (defined, in fact) as the individualized form of the activity of communicating, that is, as communication with oneself. (Sfard, 2007, p. 571)

Hence, the methodology emerged as a means to employ discursive approaches to studying students' understandings of mathematical proofs, which were reflective of particular theoretical models of students' thinking.

Leveraging perceptual ambiguity

Given researchers' theorizations of identity, especially those that posit identities as interactively constituted by and negotiated through discourse, it is surprising that students' identities have not been explored using the proof script methodology. One issue may have been that characterizing students' mathematical understandings requires a specific way of viewing students' discursive interactions, whereas seeking to witness students' identities requires a wholly distinct way of seeing. This takes us to the notion of perceptual ambiguity.

In 1915, the cartoonist W.E. Hill published the drawing in Figure 2. When viewing the drawing one of two images will appear, either a young lady with her head turned or an elderly woman looking down pensively. Both images are

Figure 2. W.E. Hill's cartoon.

present. Yet, there is one drawing. Both images reside in the same set of lines. Yet, one can only see one image at a time. This is why the drawing has what psychologists call *perceptual ambiguity.* Perceptual ambiguity refers to a characteristic of an image for which one's grouping of certain contours, images, or ideas supports one's perception of a figure, object, or meaning while one's grouping of other contours within the same image promotes a different singular perception which cannot simultaneously be perceived. Such drawings were of interest to psychologists for they demonstrated vision is an active rather than passive process; what is seen is constructed by the viewer actively.

In this article, the construct of perceptual ambiguity is of interest for it aptly describes a potential affordance of proof scripts: namely, that proof scripts can be leveraged to witness students' identities. To demonstrate this affordance, I share a two-part analysis of a pair of proof script excerpts which were written in response to the task in Figure 3. In the first part of the analysis, I demonstrate how proof scripts can serve as a mechanism for examining students' understandings. In the second, I demonstrate how they can be leveraged to witness students' enacted mathematical identities.

Examining students' understandings

To demonstrate how proof scripts can function as a mechanism for examining students' understandings, I consider two proof script excerpts drawn from the same *Introduction to Proof* (ItP) university course and part of a larger sample of 43 scripts. Upon examining the task, the reader may feel there is not much to say with respect to the given proof. However, the ItP students had little difficulty elaborating on this, arguably brief, argument. Indeed, the majority of students produced dialogs in which either Gamma or the student elaborated on why four cases are called for. Furthermore, many explained not only the generalized Law of Trichotomy but also the role of axioms and/or definitions in mathematics. To see this, consider Student A's proof script excerpt (Figure 4).

This script excerpt provides evidence of several key understandings: (1) the status of the Law of Trichotomy within the theory of real numbers; (2) the meaning of the term axiom within the discipline of mathematics; and, (3) a perhaps tentative understanding of contradictions. Consider for example, the remarks in lines 3 and 4, where Student A indicates an awareness of the Law of Trichotomy's status as an axiom. These remarks are important, for as many who have taught introductory courses know, students at this stage of development often have difficulties distinguishing between axioms, definition, and theorems even though these distinctions are important to the practice of proving. Likewise, in line 4, one can see not only is Student A aware of the status of the Law of Trichotomy but also is able to articulate what it means to say a statement is an axiom. And, while such remarks may seem trivial to those with an advanced understanding of mathematics, they are far from trivial for novices in ItP courses, who may struggle to understand what does and does not need to be proven. Lastly, looking to lines 1 and 2, one can glean preliminary evidence of yet another important understanding. When Gamma asks why the first three cases cannot "be true," Student A responds "because of the law of trichotomy." In other words, Gamma is reminded, essentially, of the existence of an axiom and then reminded in line 4 that axioms are statements "regarded as being the truth." From this exchange, one is afforded a glimpse of an emergent way of reasoning in which the student recognizes that when an inconsistency has occurred within a mathematical theory, it must be resolved by deference to that which is taken to be true (*i.e.,* given a choice between a result and an axiom one chooses the axiom and label the result 'false' or 'impossible'). And, while it is clear that more evidence is required to suggest such understandings were anything other than emergent, it is still the case that Student A's response, especially that in line 2, is of interest. Remarks such as these suggest *theoretical thinking*: thinking "concerned with internal

Assignment:

Part 1. Start by reading the proof and identifying what you believe are the "*problematic points*" for a learner when attempting to understand the theorem and its proof. A *problematic point* is anything you think is incorrect, is confusing, or is correct but warrants further discussion. List these "*problematic points*" in a bulleted list.

Part 2. Write a dialogue between you and Gamma in which you and Gamma discuss the theorem and its proof. The dialog should address the *problematic points* you identified (and listed in your bulleted list) through questions posed either by you or Gamma.

***Theorem*:** For any real numbers x and y, if $x \leq y$ and $y \leq x$ then $x = y$

***Proof*:**

1) Assume x and y are real numbers such that $x \leq y$ and $y \leq x$.
2) Then ($x < y$ or $x = y$) and ($y < x$ or $y = x$).
3) We will consider four cases
   Case 1. $x < y$ and $y < x$.
   Case 2. $x < y$ and $y = x$.
   Case 3. $x = y$ and $y < x$.
   Case 4. $x = y$ and $y = x$.
4) In Cases 1 through 3 our assumptions contradict the Law of Trichotomy.
5) We are left with Case 4.
6) Case 4. $x = y$ and $y = x$.
7) Therefore, $x = y$.
8) The result follows. ♦

Figure 3. Proof scripting task.

Excerpt A

1 Gamma: Why can't the first three cases be true?
2 Me: Because of the law of trichotomy.
3 Gamma: What's the law of trichotomy?
4 Me: The Law of Trichotomy is an axiom we use. An axiom is a statement that is regarded as being the truth or accepted as true. So this axiom states that only one of the three following cases may happen: either $x < y$, $y < x$, or $x = y$. Applying this knowledge we can see why the first three cases don't work.

Figure 4. Student A's proof script excerpt.

coherence of conceptual systems" (Sierpinska, 2007, p. 54), which Sierpinska argued is necessary for students to develop a sensitivity towards contradictions.

Beyond students' understandings of the components of mathematical theories, the proof scripts also often indicated students' emergent understandings of a generalized proof structure and their desire for a more detailed proof. In particular, several scripts included the proofs of Cases 1 and 2, with the necessary axioms and definitions, and an explanation as to why Case 3 was not needed, if a proof of Case 2 was given. These observations are exemplified by Student B's proof script excerpt. The spelling and punctuation are as the student wrote it [1].

As in the previous excerpt, Excerpt B provides evidence of several emergent understandings which are important to the practice of proving. Consider for example, Student B's remarks in line 9 [2]. The remarks indicate an understanding of the mathematical (logical) definition of the symbol ≤ and the fact that the definition takes the form of a disjunctive statement; *i.e.,* a form that justifies the partitioning of the proof into cases. And, again, it is important to note that such understandings are important for novices to develop. Indeed, those who teach ItP courses will readily attest to the fact that novices may use "proof by cases" in the strangest of places. Beyond an emergent understanding of the conditions under which proofs by cases are appropriate, another important understanding seems evident in Excerpt B. Turning our attention to line 11, one can see that the student has an observable, possibly preliminary, understanding of a practice that is important to proofs at this level; namely, that sub-proofs which are identical in structure are not replicated within a proof, instead replication is avoided with the phrase 'without loss of generality'. Running the risk of sounding repetitive, I will note here, as I have above, that novices (ItP students) struggle to understand when it is appropriate to argue 'without loss of generality' and often use the phrase inappropriately. Thus, it is noteworthy that Student B is able

Excerpt B

1 Me: What's the word Mockingbird?
2 Gamma: No much ese, just working on this pinche proof!
3 Me: Chale, that stuff aint easy homes.
4 Gamma: Que no, check it out and see if I got this mierda right?
5 Me: It looks good Homes except in linea 2 and 3 you got no detail ese. You need to explain cases 1–3!
6 Gamma: Don't yell at me ese!
7 Me: Stop acting like a chavala!
8 Gamma: Whatever homes!
9 Me: Anyways, lleva, with Line 2 you didn't state Axiom 6 which lets you split the inequalities foo.
10 Gamma: Chingada madre! I forgot Axiom 10 in Line 3 for the cases 1, 2, 3.
11 Me: And for Line 4 you forgot Definition 6. Another thing for Case 2, if you prove it by contradiction using Axiom 10, because $x \neq y$, Case 3 is found without loss of generality because of Case 2.

Figure 5. Student B's proof script excerpt.

to specifically describe when and where the phrase should be used in the elaborated proof.

Leveraging proof scripts

Before answering the question of how proof scripts can be leveraged to witness students' enacted mathematical identities, three clarifications are needed. These concern the term 'witness', the phrase 'enacted mathematical identity', and the question of which foci are required to witness students' enacted mathematical identities. The term 'witness' when not used in a religious sense, harkens an image of a bystander who is present at an event and able to attend to evidence. If one looks to students' discursive interactions within dialogs one does not merely observe but rather is present at an event and able to attend to evidence. Moreover, witnesses are *present at* but not *present in* events. Indeed, when examining a students' proof script, one does not become part of the discursive interaction but rather is a type of bystander; someone who has knowledge of the surroundings and participants, but is not in the sphere of action. It is for these reasons that the term 'witness' is used in relation to exploring students' identities in the context of students' proof scripts; the term witness conveys the status of one who is present at, but not present in, a sphere of action and who is able to attend to evidence.

Beyond clarifying how the term witness is used, it is also important to discuss what is meant by "enacted mathematical identities." According to Bishop (2012), an *enacted identity* is an identity "existing in the present moment, is not temporally independent; it looks backward in that it reflects aspects of previously enacted identities and it looks forward in that it anticipates and influences future enactments" (p. 42). And, a student's *mathematical identity* is "the ideas, often tacit, one has about who he or she is with respect to the subject of mathematics and its corresponding activities […] a person's ways of talking, acting, and being and the ways in which others position one with respect to mathematics" (Bishop, 2012, p. 39) Thus, an *enacted mathematical identity* is an identity presented in a particular moment, which both reflects past and anticipates future enactments, while at the same time revealing how one positions oneself and others with respect to mathematics. Here it is important to note, some may argue students' scripted selves should be regarded as *possible selves* (see Markus & Nurius, 1986) [3]. Yet, mathematical identities enacted in scripts are not merely possible selves, but rather are selves brought forth into a public arena (*e.g.,* one potentially involving a teacher, other students, and researchers), through constructed interlocutors. Thus, a scripted self is not only an enacted identity, but is also reflective of one's *agentic identity*: one's "conceptualized notions of self which (are) related to their capacity to act and influence within pertinent figured worlds" (Ward-Penny, Johnston-Wilder, and Lee, 2011).

The last point of clarification concerns the question of which foci are required to witness students' enacted mathematical identities. The approach taken in the present work was to examine the dialogs with respect to the following questions: (1) How are the scripts' interlocutors positioned with respect to each other; (2) How are the scripts' interlocutors positioned with respect to the knowledge and practices at play; and (3)Which type of discourses are invoked in the script; in particular, are the discourses procedural, conceptual, contextual, or another form of discourse (see Gee, 2005; Setati, 2005; Thompson *et al.,* 1994)?

To demonstrate how these questions were employed in the analysis, I revisit the excerpts and illustrate how, when one stops attending to what mathematical understandings are evident in students' discursive acts and attends to the relative positioning of the students, their knowledge and practices, a new—not yet perceived—image can emerge.

Returning to Excerpt A but shifting our attention to the questions listed above, one sees that Student A is not positioned as an unknowledgeable or uncertain peer, as was the case in many of the sampled scripts, but rather Student A constructs himself as the knowledgeable other—he emerges through the discursive interactions as the authority in the dialog. Gamma's only role is to ask questions and listen to the answers. Gamma can ask for clarification but is not a source of knowledge. Furthermore, it is not merely the case that Student A emerges as the authority who is sharing information but also that Student A is freely able to enact a position of power, for Student A positions himself as one who can decidedly determine that which is true. Indeed, while many scripts involved dialogs where students deferred to other authorities (*e.g.,* "in our notes it says" or "the instructor said"), in the dialog Student A is the authority.

In contrast, the relative positioning of Student B and Gamma is quite different. Certainly, Student B is knowledgeable but Student B is not the only one who contributes to the elaboration of the proof. For instance, the reader will notice that while Student B tells Gamma that more details are needed (see line 5 of Excerpt B), Student B does not provide those details in full. Instead, Student B positions Gamma in ways that allow for Gamma to make contributions (see lines 9–10). Moreover, the banter that occurs throughout the proof is not hierarchical. Instead, Student B constructs a discourse in which the verbal jockeying is playful and conveys a camaraderie between the students. The discourse is not aimed at a positioning of one student over the other. Instead, it conveys a shared endeavor focused on explanation and mutual advancement. This point is conveyed in particular in lines 1–5, where Gamma's remark "just working on this pinche proof" is met by "Chale, that stuff ain't easy homes" and other encouraging remarks such as, "It looks good Homes except in linea 2 and 3." Indeed, what one witnesses through this dynamic interplay of ideas is not an emerging power structure by rather Student B's development and use of a *voice of solidarity:* a voice that conveys from one to the other, I understand your struggles and am willing to help you with them (see Setati, 2005) [4].

Lastly, in relation to Excerpt B, it is important that one be aware that the dialect employed in the dialog is not the taken-as-shared dialect of undergraduate mathematics texts (and I would argue mathematics classrooms). Instead, Student B has created a discourse in which the constructed interlocutors, through their interaction, skillfully pinpoint key gaps in the proof and the axioms and definitions necessary to fill those gaps. At the same time s/he maintains the dialect common *at that time* to students *in that area*— essentially translating sophisticated mathematical ideas into an urban dialect that employs terms outside of formal English and Spanish (*e.g.,* ese means "that" in Spanish but is slang for "man" or "dude" in parts of Mexico and California). Hence, Student B interactively positions himself as one bridging the great divide between the institutional home of the discipline, where mathematical practices are both recognized and valorized, and a community often structurally excluded from the discipline. Indeed, in Excerpt B one sees an instance where through a dynamic interplay between the student's knowledge, identity, and practices the student engages in the enculturation of mathematics, rather than solely a student's mathematical enculturation.

Discussion

Recognizing that, as argued by others (Bishop, 2012), identities are not only an important educational outcome, but also critically inter-related to our knowledge and practice (Boaler, 2002b; Gutiérrez, 2013), I have sought to demonstrate how the perceptual ambiguity of proof scripts can be leveraged to witness students' enacted mathematical identities. Perceptual ambiguity in this context refers to an affordance of a static artifact, when that artifact can convey particular meanings through one's intentional focus on specific attributes, yet convey a distinct set of meanings should one's intentional focus shift to other, present but not yet attended to, attributes. The artifact—in this case a student's proof script—being static does not change, but rather our goal oriented activities do when actively "seeing" the artifact.

Looking to the two-part analysis, one sees that when attending to the students' problematics and key points, one can work to discern the students' emergent understandings. In so doing, one detects a particular (temporal and situated) perception of the student. Yet, if one shifts ones attention to how the students position themselves and others in relation to the knowledge and practices at play, one can witness students' enacted mathematical identities; *i.e.,* one can begin to limn an identity presented in a particular moment, "a person's ways of talking, acting, and being and the ways in which others position one with respect to mathematics" (Bishop, 2012, p. 39). And in so doing, one discerns a different perception of the student. Moreover, one cannot observe both perceptions at the same time. Examining students' understandings requires that one brings mathematics to bear on the question of what is known by whom and in which ways. In contrast, witnessing students' identities demands attending to the students' discursive acts so as to understand how students dynamically position themselves, not only with respect to others but also to the knowledge and practices employed in a particular context. It means seeing how the constructed interlocutors are oriented to each other, the knowledge and practices, as opposed to questioning which mathematical meanings and practices are required for a discursive act.

Concluding Remarks

The aim of this article is to demonstrate how the perceptual ambiguity of proof scripts can be productively leveraged to study not only students' emergent understandings, but also to witness students' enacted mathematical identities. In this sense, the article's main contribution is methodological.

With this said, it is important to note that there is another potential contribution. To see this, one must recognize that if taken up Wenger's claim that learning is an experience of identity (Wenger, 1998) necessarily calls on those who design and co-construct learning environments to take responsibility not only for what is learned, but also for the identities that are constructed. It stands to reason then that as instructors we must recognize we cannot ignore that some in our classrooms are emerging as authoritative individuals who gravitate towards power discourses, while others do not. Moreover, we must recognize we are not only creating opportunities to engage with mathematics, but also opportunities to develop into mathematical selves and that, for some, finding one's place within the taken-as-shared culture (and its ways of speaking) may mean engaging in discourses that feel distinct from one's own and require either the student or the mathematics be enculturated. Thus, recognizing that proof scripts can be leveraged to witness students' identities may mean engaging in a process that leads to an irrevocable awareness of the ways in which our institutional settings are impacting students' ways of talking, acting and being and their positioning of themselves and others with respect to mathematics. It may mean recognizing that we are not interacting with students' identities, but rather creating institutional spaces within which agentic mathematical identities are constituted.

Notes

[1] Student B wrote at the beginning of the script "Warning: Foul language." While I can translate the dialog (I am a native Los Angelian), I have chosen not to. It is my belief that even if the appropriate terms were used, the student's meanings would be changed and misrepresented.

[2] Student B meant Definition 6 rather than Axiom 6, as is evident from the student's subsequent remarks (see line 11).

[3] I would like to thank Dov Zazkis for raising this point.

[4] I would like to thank Luis Leyva for pointing out that the dialog conveyed an instance in which students engage in a discourse of solidarity; a point I have connected to the work of Setati.

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The discovery of the unconscious implies that there can be no such thing as absolute knowledge, or total knowledge—the unconscious always remains unknown. Ignorance thus becomes a component of knowledge. [...] This suggests that teaching should be more about confronting resistance to knowledge than about lack of knowledge.

Rosamund Sutherland (1947–2019) in *Consciousness of the Unknown*, **13**(1), p. 43

Editor's Note: Reflections on Ros's life and work will appear in the next issue.