Communications

A Response to Nunokawa's 'Surprises in Mathematics Lessons'

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In a recent article in FLM, Nunokawa (2001) integrates a collection of insights from the work of Japanese researchers with the aim of illuminating a particular aspect attributable to mathematics lessons that acts as a hook for students' interest in mathematical ideas. He refers to this notion as 'surprise' Interestingly, Nunokawa's sense of surprise is intimately connected to a particular standpoint on the nature of mathematics: that is, he expressly suggests that mathematics is some*thing* that exists outside of, and separate from, individuals. As such, Nunokawa announces a particular orientation to the notion of surprise This short piece is a response to his sense and use of the notion of surprise.

I agree with Nunokawa's declaration that surprise may be a "critical factor" in order for learning to happen. Alas, Nunokawa's focus on mathematics as a "social-cultural-historical reality" - a frame for mathematics that he borrows from Hersh - is where Nunokawa and I part company. To be clear, it is not that I disagree with Nunokawa's position on mathematics and knowledge; but I do disagree with his notion of surprise. It is, rather, that cognition might be more usefully viewed as a set of nested, emergent forms of knowing, pushing mathematics across a collection of different bodies, *e.g.* biological bodies, social bodies and bodies of knowledge, the body politic and ecological bodies. [1] In this manner, surprise can also be understood as a nested, emergent phenomenon. As such, surprise is an event of *emergence* [2] that is brought forth in different forms depending upon the level of emergence.

Prepare to be surprised

Nunokawa suggests that the deliberate structuring of a surprise in the shape of particular mathematical problems can bring about a student's interest in mathematics. In other words, we might think that surprises can be planned where we might surprise others. The thing is that we are not planning a surprise party. While one might create some of the necessary conditions for a surprise party, the person for whom the surprise is for needs to be expecting something other than an unexpected gathering waiting for him or her The *lived* experience of surprise, however, is not something that can be planned One might be able to create some of the conditions for surprise, but it depends upon the structure of some other system.

Surprising events happen. They are happenings in the sense that they are chance events that come out of the interaction of a number of different parts acting in concert. Surprise, therefore, is an emergent phenomenon that arises in different forms at different levels For example, it may be a shared experience at the level of a society as in the World Trade Center disaster on September 11th, 2001 It may be a cultural phenomenon, as when the ancient Greeks 'discovered' the phenomenon of irrationality. Similarly, surprise manifests itself at the psychical level of ideas where discrepancies between our own expectations of and for a stable world meet up with our actual perceived experiences with the world in which we (and our emergent ideas) are already a part.

Paradoxically, *one must be prepared to be surprised.* In other words, we must have a variety of experiences that prepare us for the unexpected If there were no experiences to compare, then we could not be surprised Phenomenologically, we say that we are surprised when we experience an unexpected event or happening. This is to say that something was expected, but did not happen for us I will explain my judicious use of the preposition *for* rather than *to* below

A divided world

Similar to the views of the mathematician John Casti (1992), I also believe that surprises happen when there is a discrepancy between the expectations of a closed, abstract system, *i.e.* what we know, in interaction with the elements of another open system, *i.e* the world around us. In other words, we are *prompted* to be surprised, but not in some causal sense. Thus, surprise does not happen *to* us, but *for* us In this sense, it is a property of an emergent form. Moreover, on another level or for a different system or phenomenon, one might be able to describe the same notion of surprise It will, however, present itself differently because of the different interactions within and between other different layers or nested, emergent forms.

Nunokawa supposes that lessons can be deliberately structured so that they *possess* an element of surprise. In fact, he writes about it as if they could be in*corpo*rated into the lesson - embodied or united with a body, in this case a body of knowledge (e.g curriculum guides or the knowledge of a teacher who is planning for some surprising moment) However, I contend that surprise is something that *comes from* or *out of*, as in an *event* (Latin: *ex- + venire*), the interaction of different bodies. Surprises are event-full moments or happenings.

Nunokawa's model immediately points to particular bodies in relation to one another: the mathematical world and the real world. Moreover, each body - the mathematical world and the real world - is marked off with a box, a distinguishing and distinguished boundary which metaphorically suggests a container schema. The logic of the container schema easily offers a binary choice of being in or out of the container Thus, something is either of the mathematical world or it is not

Although Nunokawa does not suggest that this is true of mathematics, his *model* and *our* logic of the container schema and language all suggest this Additionally, Nunokawa's model shows the relations between these two worlds: however, the relations suggest particular uni-directional interactions. Starting with some initial information about and in the real world gives rise to two possible interactions:

(1) a uni-directional interaction or manipulation of the world that creates some new information in the world;

(2) a uni-directional interaction that generates some mathematical formulation.

Two other interactions are also present in this model. Within the mathematical world, a uni-directional interaction or operation in mathematics creates some new information in mathematics. The final interaction is a uni-directional interaction from newly constructed information in the mathematical world to new information in the real world. With this in mind, Nunokawa pursues the theme of surprise as a phenomenon that arises in the gaps: that is, he pursues four different types of surprises *caused* by the gaps between the mathematical world and the real world, marked by the uni-directional interactions in the model he uses.

A world of surprising interactions

In every case that Nunokawa presents as an illustrative example of surprising events, he writes of an implied gap between states and a realization of a gap between states that gives rise to the possibility of a "kind of surprise in the students" (p. 44). Moreover, he claims – and can claim only after the fact – that a surprise acts as a "*trigger* for further learning" (p. 44, my *emphasis*). In one story involving an iteration of a mathematical operation applied to an initially-given random number and its subsequent result, he claims this creates a "rather neat" result. What is interesting is Nunokawa's remark in the following line, where he says that "it is expected that this gap will produce student surprise". I wonder if Nunokawa would be surprised if this did not happen: that is, if the *student* were not surprised, since the described problem appears always to work and does not present any unexpected results other than the anticipated one.

I really do wonder if the students involved in these tasks which Nunokawa describes really were surprised (in the phenomenological sense). Perhaps he would say that the pursuit of a mathematical understanding of the problems they faced was an indicator of being surprised. However, phenomenologically, this cannot be so. Nevertheless, he claims that they were surprised because the students were prompted to pursue a mathematical reason for the surprising result.

The feeling of surprise

Nunokawa provides us with a number of student remarks about the various lessons he presents in his article. The student comments sometimes explicitly announce an expression of surprise. An *expression* of surprise does not constitute, however, nor would necessarily correspond to, a *feeling* of surprise. Nunokawa's article does not quite express the possible phenomenological, lived-experience of surprise very well.

Clearly, students did make remarks which one might take for an experience of surprise. Consider some of the following:

> "Ah, that's a triangle."
>
> "I didn't think that I could see such a wide area of the city from that building."
>
> "I was very surprised, because I noticed that we could see further than I had expected."
>
> "At first, I doubted whether mathematics could deal with issues in sports."

Certainly, there are expressions of wonder, doubt, but also of the unexpected. Wonder is definitely a human experience which is coupled to surprise (Fisher, 1998). It is questionable that doubt could be a mitigating factor which generates a surprise for an individual. To say, for example, that "I doubt that is the case" only to find out a little later that it actually *is* the case, would not be enough to generate a *feeling* of surprise. In this case, there is no sense of being *overcome* by something.

Surprise has this way of sneaking up on us. Interestingly, the etymology of the word *surprise* has something insightful to say about this notion of "sneaking up on". As van Manen (1990) writes:

> Being attentive to the etymological origins of words sometimes put us in touch with the original form of life where the term still had living ties to the lived experiences from which they originally sprang. (p. 58)

Thus, the word *surprise* is closely related to such notions as "a sudden unexpected attack", "to come upon unexpectedly", "to take unawares", "the feeling or emotion excited by something unexpected, or for which one is unprepared" or something "akin to astonishment and wonder, caused by an unexpected occurrence or circumstance". There is a sense of being "overtaken" (French: *sur- + prendre*). [3]

With this in mind, one must be prepared to be surprised – fortified against some attack – so that one might be taken over. One must be caught off-guard or even trip over something unexpectedly to be surprised. There may be a sense of something coming from behind or out of one's visual blind spots. Would you be surprised that human beings have such a physical blind spot at the back of our eyes? And, yet, we cannot see that which we cannot see. In this manner, there is always the possibility that we may be surprising ourselves! In this sense, we are constructing a world of surprise, but it is more towards trying to keep pace with a changing world in an adaptive sense rather than some architectural sense of constructing such a world.

Planning for surprises in mathematics lessons

Nunokawa's article is about teachers planning lessons for students so that they might "feel surprises easily" (p. 48). Although his paper does suggest that teachers might be able to plan for such events, ultimately the experience of surprise does not happen in this manner. Yes, we might prepare ourselves for surprising moments; however, this does happen somewhat differently.

Phenomenologically, a variety of different experiences must be had for an individual to be surprised. Moreover, a certain level of diversity is still not enough. To be surprised, there must also be a certain level of redundancy of

experience: that is, a particular experience must be common enough that it first fades into the background, so that it becomes overly-familiar for us. It would be, then, that one's guard would be down when an unexpected attack is made, *i.e* something completely unfamiliar has happened. At a neurological level, Austin (1999) describes this phenomenon of encountering the unexpected as the brain telling us that it does not know what to do or make of a certain situation. There is a pause and then we realize "all of a sudden" that we have "discovered" something new, as if it were there all of the time.

Perhaps it is not surprising to see how Nunokawa's sense of surprise manifests itself. By assuming that the world exists "out there", separate from us, it then becomes possible to think about planning a lesson which includes a surprise embedded in the task beforehand. Certainly, the right conditions must be present in order to be surprised. For Nunokawa, this condition is the creation of one or more planned gaps. However, this raises the question: a gap between what?

The world is full of surprises. Every moment is a surprising moment. Now. Now. And, now. Did you notice anything surprising? To use a *cliché*, I wouldn't be at all surprised if you said "no". But why are we not surprised (in every moment)? To repeat, surprises happen *for* us - not *to* us. To be surprised means that we must be attentive to our current, lived, embodied experiences in the world. In other words, we must be open to being surprised so that we might be prompted to be surprised. But the world cannot necessarily cause us to be surprised without our partial collusion - particularly when we are inattentive to it and the world fades into a background of over-familiarity.

To end, this is not to say that Nunokawa's analysis of the mathematics lessons he examines is wrong or not useful. It is more that the question of what creates a surprise for us comes from individuals' own experiences and so cannot be answered by saying that surprises can be created in isolation from ourselves. Certainly, Nunokawa's analysis of surprise as emerging from a gap is in line with my and Casti's sense of a discrepancy between two states. But Nunokawa and I depart on one important matter: the phenomenon of surprise is an *emergent* form arising from some locally-determined interactions between two systems - one closed (our own knowledge of the world) and the other open (the world) - which creates enough of a discrepancy between the expected and the unexpected to produce the lived experience itself. In any event, it might be better to simply leave open the idea that any of these students that Nunokawa writes about actually experienced surprise.

Notes

[1] This is an organizational and interpretive strategy borrowed from the work of Brent Davis, which is presented and discussed at length in *Engaging Minds* (Davis *et al.*, 2000).

[2] Kelly (1994) argues that when the word *emergence* is scrutinized, it disappears and becomes "meaningless". Nevertheless, he writes that complexity scientists would suggest the meaning of emergence is like: "that organization which is generated out of parts acting in concert" (p. 458). Here, I would stretch Kelly's notion of organization to include phenomena that manifest novel, unanticipated structures, patterns, properties or processes.

[3] These meanings for and connotations of 'surprise' are taken from Webster's Ninth New Collegiate Dictionary (Miriam Webster, 1983) and the Oxford English Dictionary (Simpson *et al.*, 1993).

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Revista EMA: an Unexpected Challenge

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Contributing to the consolidation of the group of mathematics teachers in Columbia as a professional community has been one of the aims that has guided the work of *Una Empresa Docente* [1] over the last ten years. Such a purpose refers to the reconstruction and enrichment of certain aspects of the culture in which the teacher's work is embedded: in particular, those that relate to the systematic achievement of classroom experiences [2] and the diffusion of accounts of such experiences and their results. In the present short piece, we present and discuss some of the difficulties we have faced while encouraging the publication, in the *Revista EMA* journal, of the classroom experiences of teachers. Facing them while maintaining the established objectives of the publication creates, in turn, questions with respect to the role that the journal editors could and should play.

Introduction

For three years, *Una Empresa Docente* produced an information bulletin about projects, reflections, courses and events carried out in mathematical education by our workgroup and other mathematics teachers in the country. When the *Revista EMA* journal was created in 1995, it put forward as its main objective to contribute to the consolidation of the group of mathematics teachers of the country as an academic community, by means of three definite purposes.

- To supply all of its members with access to the information that is relevant for the profession they share. This includes knowing about the work of