ON THE SOCIAL NATURE OF MATHEMATICAL REASONING

WOLFF-MICHAEL ROTH

It has become a truism to state that mathematical knowledge or "meaning" and the norms and practices of (school) mathematical communities are negotiated and socially constructed before individuals construct (internalize) them for themselves. This social-constructivist discourse is exemplified in the following quotation, which states, with reference to Vygotsky, that:

> students' interpretations of what takes place on the intermental plane will be internalized and become a part of their intramental resources for knowledge construction. These concepts are used here to distinguish between ideas and thinking that are shared during a discussion (intermental) and the thinking and ideas that […] are not shared (intramental). This distinction emphasizes the importance of the content and characteristics of classroom discussions as an influence on the learning process. Only the intermental part of the discussion becomes a source of concepts, arguments, and goals that the students can use to construct knowledge. (McNair, 2000, p. 198)

In this discourse, clear distinctions are made between the intermental and intramental, that is, between the shared, socially constructed and the individually constructed. The classroom is considered as a "context" that "influences" the "learning process," and, therewith, the content of learning. The product of intramental construction inherently is not shared. This discourse is found in many discursive and communicative studies of mathematical learning, such as when making the common claim that "goal-oriented activity […] can be internalized to produce personal meanings" (Ng, 2016, p. 313). A distinction is made between what is available in the public sphere between people and what exists within the person following a process of internalization. In this approach, primacy is given to classroom talk not just as the origin of knowledge but also as the origin of norms, as in, for example, the statement that "the social norms […] were established during the concluding data analysis discussions" (Cobb & Tzou, 2009, p. 152).

I defend an alternative discourse, in which mathematical reasoning is never individual, but social through and through. This is so not because humans construct mathematical reasoning in social relations, but because mathematical reasoning and any mathematical norm is and exists as social relation and is, therefore, part of the visible interactional order of society, rather than in the heads of individuals. This discourse is grounded in the one Vygotsky (1989) used when, for example, he stated that "the relation between higher psychological function [….] was at one time a real [*real'nim*] relation between people" (p. 56, original emphasis, underline added, translation corrected). For the late Vygotsky, there is no intramental different from intermental, because "the alien is identical with one's own and one's own exists as an experienced reality of Other" (Mikhailov, 2001, p. 26). The discourse I defend takes into account a critique of the constructionist approach in science studies where "the social character of domain-specific skills and reasoning may only be an incidental feature" (Livingston, 2008, p. 212); and it is consistent with the position that for students to negotiate anything like social or sociomathematical norms they already have to know these norms (Radford & Roth, 2011).

In this article, I make a case for a notion of the social that is much deeper than that in current theoretical approaches, where the social is but a context. I argue for a primacy of the social in the sense of Vygotsky, for whom the genetic origin of all higher functions and personality are social relations—*i.e.*, "sociogenesis is the key to higher behavior" (Vygotsky, 1989, p. 63). Writing about the primacy of the social is another way of saying that the social is "essential to what practitioners do, that it's unavoidable and, therein, that it's omnipresent throughout a domain of practice" (Livingston, 2008, p. 210). This social, exists in the form of, and is manifested by, an interactional order: the sequentially ordered turn-taking patterns of classroom talk. This visible order subsequently finds its expression in the actions of students in their exchanges with other students. It is not the classroom talk that brings about the interactional order (*i.e.*, social and socio-mathematical norms) but, conversely, the interactional order determines what is said and done, and how what is said and done is to be taken. Thus, at the point at which students apparently act according to a norm, what they are really doing is maintaining the conditions for acting in certain ways while monitoring others to act accordingly.

In the following, I begin by presenting a brief case study and then develop this position. I use an asterisk to mark that the specifically mathematical nature of the events and reasoning remains to be determined: at the outset, the children, members to the setting, do not and cannot know what is specifically mathematical and what is not.

Mathematical* reasoning first *is* a relation between people: a lesson fragment

In this fragment from a second-grade classroom (7–8 years), the norm under consideration is the tie between a mathematical action and a verbal account giving a reason for the

For the Learning of Mathematics 36, 2 (July, 2016)
FLM Publishing Association, Fredericton, New Brunswick, Canada

action. In the fragment, the mathematical action is sorting (categorizing) based on a mathematical (rather than some other) property. This norm first exists as social relation, that is, as an interactional order. This order is the result of joint work; and if it is joint work, we must question the distinction between intermental and intramental. The fragment derives from the first lesson in three-dimensional geometry for the class. The main task on that day was a game of sorting mystery objects that the children pulled from a black plastic bag and that they were asked to place on (a) a mat of an existing group or (b) a mat of its own. The teacher, Mrs. Winter, started the game by placing the object she had pulled (a cube) on a colored mat and then laying another mat next to it. Then one child after another took a turn, including Gina.

Fragment 1

➔ 01 ((*Gina places her object, as in the image from the video, below.*))

02 (0.8) ((*Gina retreats to her seat.*))

➔ 03 W: now can you tell us what you're thinking?

04 (3.5) ((*Gina scratches her ear and brings her hand to her chin as culture associates with thinking.*))

05 W: there must be something different ((*Mrs. Winter gesticulates towards objects on the floor*)) because you gave it its new, its own category; can you tell us what you thought was different between the two ((*Mrs. Winter points to the cube and cylinder*))

06 (0.8)

➔ 07 G: they're different shapes? ((*Changes gaze and body from being oriented towards objects to face of Mrs. Winter.*))

This exchange is typical for this mathematics* lesson, during which only the second to last of the 22 students taking turns immediately offered up reasons for discarding a number of groups before placing his own object. Some of the remaining 21 students replied immediately after Mrs. Winter asked them to state their thinking, and in many other cases, more extensive exchanges were required, some much longer than that with Gina, before the student eventually stated a reason accepted *de facto* in the course of the game. That is, when the invitation to provide the basis for the classification was not forthcoming, additional *joint* work was accomplished (such as, in the fragment, what follows after turn 05) to bring about a possible statement that would count as having been an underlying thought. The statement in turn 05 makes it possible to produce an after-the-fact statement of a form of reason that accounts for the different placement of Gina's object. This statement is specifically invited to be about a difference that was attributed to a thought that had occurred before—as evident in the use of past tense: "tell us what you *thought* that was different." Whatever can be told, therefore, can be attributed to reason; and, conversely, the form of reason under consideration inherently is considered to be tellable. Because Gina had placed her object on a different mat, there "must be something *different* between the *two*," and that difference is such that it can be told. In this fragment, the sequence of turns also *was* a relation: student sorts–teacher solicits reason–student provides reason. It is a version of a fundamental structure of practical (mathematical*) reasoning that will come to be observed widely throughout this classroom during the following week.

The *social* nature of mathematical* reasoning

In this section, in analyzing the preceding fragment, I demonstrate that the nature of mathematical* reasoning is much more fundamentally social than suggested in social-constructivist studies and in the scholarship on negotiation of social and socio-mathematical norms. First, I show that what the classroom members (teachers and students) do already is *social* (irreducibly joint) work as distinct from individual work that is somehow combined. Second, the *social* nature of the socio-mathematical* norms first exists in and as interactional order (social relations). These relations, that is, the interactional order, also produce and make visible any teacher "power" that may be observed (in any evaluative turn that may follow) rather than the teacher's power determining the social order; power is the result of the interactional order rather than explaining it (see Foucault 1975).

The social nature of joint work: speaking to be understood

Language-use is fundamentally social. Joint work {speaking | actively receiving} not only happens in relation but constitutes the relation. This can be seen in the following retranscription of the fragment designed to take into account the active reception of speech. The retranscription makes apparent the sociological dimension of speech: communication is social (joint) work as each intelligible word belongs to speaker and recipient; and it makes apparent the diastatic but irreducible nature of responding, consisting of the unity of actively receiving and replying (Vološinov, 1930). Every word, therefore, has both sociological and psychological dimensions that are orthogonal to each other, and therefore cannot be reduced to each other (Figure 1). In saying, "can you tell us what you thought was different between the two," Mrs. Winter does not just make public something utterly individual. She uses language, which is not her own; instead, she uses an "alien" language that has come to her from the other and, in her speaking, returns to the other, here Gina, her classmates, and the visiting research team. Moreover, the statement is designed *for* Gina. That speaking, designed for the other, received by the other, already takes into account the nature and intelligibility of a word for this other. The general intelligibility of these words is presupposed in the act of speaking. In replying appropriately, Gina also makes available that she has actively attended to Mrs. Winter. Her statement, "they're different shapes" is directed towards and has taken up the "can you tell us what you thought was different between the two." The intelligible word *is social* or it is not at all; the social nature of the word certainly is neither constructed nor negotiated. But there is more to the social.

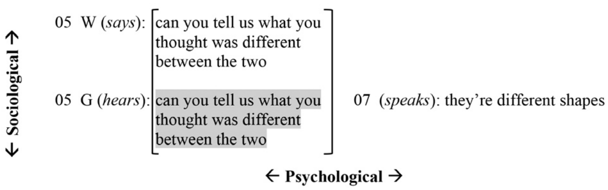

Figure 1. A transcription that also features hearing makes explicit sociological and psychological dimensions of talk.

Gina provides what we recognize to be an *answer* to the *question* posed by Mrs. Winter: in saying, "They're different shapes?" (turn 07), Gina has accepted the invitation to reply. But that turn *in itself* does not constitute an answer. If we, witnesses and observers, only were to hear Gina, we would not know that the statement is a reply. Given the rising intonation, we might even surmise that we are hearing a question. The nature of turn 07 as an answer cannot be found in linguistics or in logic, that is, it cannot be found in the semantic or syntactic nature of the expression (Vološinov, 1930). Instead, the nature of the turn qua *answer* is a sociological (social) fact. It is the consequence of the sequential organization of the turn taking that is and makes the relation between people. The nature of the statement as answer arises from the positioning of the turn with respect to another turn that can be heard as a question. That is, the very nature of the turn as an answer is premised on the social and its "norms," to which people orient in their doings, which they produce to be heard as such, and to which others can be expected to attend as well. In fact, if an answer to a hearable question is not forthcoming, this "non-occurrence is an event, *i.e.* it is not only non-occurring [….] it is absent, or 'officially' or 'notably' absent" (Schegloff, 1972, p. 76). When Mrs. Winter says, "there must be something different" (turn 05), she also states that there is something that has to be accounted for but which has not yet occurred. As such, the absence of an answer following turn 03 is legitimately used to make inferences, which, in school situations, may be that a student does not know, does not understand the question, *etc*. When there is no reply even though the pause is getting very long, Mrs. Winter speaks again (continues to speak) (turn 05).

The social origin of socio-mathematical* reasoning

In this subsection, I demonstrate in which way mathematical* reasoning, in being communicated and thereby implying its intelligibility, is inherently social rather than individual and personal. The statement, "there must be something different because you gave it its new, its own category" not only presupposes and implies a norm but also, in the request to "tell us your thinking," holds Gina accountable to it. If a new category is formed, then there must be something different; and this difference has to be accounted for in terms of a reason. That is, what has been done is treated as not conforming to a norm that is yet to be enacted. The teacher turn formulates what will eventually be emerging: the socio-mathematical* norm of tying a mathematical* action to its account (reason). This norm is presupposed to be intelligible; and it is observable as such in the sequential organization of the ordered turn taking (*i.e.*, in the interactional order). Mathematical* reasoning is social because it is witnessable and observable as that real turn-taking relation first. At some later point in this curriculum, this same mathematical* reasoning was observed in the actions of individual students. However, although produced by individuals, it was no less social (Vygotsky, 1989). To show the social nature of the work of mathematical* reasoning, I begin with the structure of practical actions.

The structure of the practical action in the fragment, categorizing by shape, may be denoted, following Garfinkel and Sacks (1986), in this way: "doing [sorting by (geometrical) shape]." The "doing" refers to the actual work, and the parenthesized statement "sorting by geometrical shape" is a gloss members (*e.g.*, the two teachers in the classroom) use to characterize this work. Mrs. Winter provided precisely such a gloss when she started this part of the lesson by saying, "We are doing sorting activities, we are not going to do them by color or by shape"; and the actual work was accomplished by means of ordered and ordering turn sequences. It was completed in and as *joint* work (*i.e.*, in the irreducible {query | reply} pair). The sorting actually is a composite of (a) the act of attributing an object to a group that in this classroom has or does not yet have a name and (b) the act of stating a reason for the attribution. *Specifically* mathematical *is the characteristic of the classification, "by shape," and the tying of the placement to its specifically* mathematical *account (reason).* In other subject areas, different classificatory characteristics would have brought out the specificity of those fields. But the children will be able to conceive of the specifically *mathematical* in these activities once they are already competent in the practice, allowing them to distinguish them from practices in other areas. In the fine arts, for example, students might not be asked at all to provide reasons for grouping objects, such as on a canvas or in an exhibit.

The fragment exemplifies how the children, in their first geometry* lesson, did not tie classificatory actions to accounts, exemplified here in Gina's placement of the object on a separate mat and then retreating towards her seat without having provided a reason that "explains her thinking." But reasoning is mathematical* only when it can be tied to a specific (verbal) account. That tying of action and account was one of the "socio-mathematical* norms" observed later in the unit but that did not exist early in the unit. How then did that norm emerge? In that it was an interactional order (social relation) first; it *is* the social relation anchored on two turn pairs, one that Mrs. Winter completes and the other one that she initiates:

I	G:	((*Gina places her object, as in Fragment 1.*))	
I	W:	now can you tell us what you're thinking?	II
	G:	they're different shapes?	II

In this fundamental structure, there are two turn pairs (I, II) hinging on one turn. That middle turn (a) completes Pair I by evaluating what has preceded to be lacking something, precisely a statement of the thinking; and it initiates another pair (II) that invites Gina to state her thinking. Gina (eventually) accepts the invitation by stating that the two objects have different shapes.

The three-turn exchange sequence manifests and reproduces a social relation. And it is *as* that relation that the socio-mathematical* norm exists. Gina can witness it; and all the others can witness it. The socio-mathematical* norm is not somehow hidden *in* the relation: the relation *is* the socio-mathematical* norm. That is, the norm that binds a placement and its reason together first (*i.e.,* during the first lesson) exists as an interactional order, as a social relation; the same work that produces the relation also produces the tie. Together, Mrs. Winter and Gina do this work. Their social relation, the interactional order, is at the origin of the norm, not some negotiation. It is as the sequence of classification, solicitation of thinking, providing reason—a sequential order of distributed work in which the student takes the first and third, the teacher the middle slot—that we first observe the "socio-*mathematical** norm" of classifying (sorting) three-dimensional objects. There is an interactional order at work that is witnessable by those present: classificatory action followed by provision of reason. It first exists in the sequential turn-taking order of student (S) and teacher (T) in the form S-T-S and later is observed in this classroom in a single turn (S). That is, there is a reduction of an interactional order S-T-S → S that parallels the process of pairing from classifying | requesting accounting | accounting to classifying | accounting.

There is a second way in which the S-T-S sequence is important, for it is here—in the evaluative function that that middle turn falling to the teacher takes—that children can find whether what they have done does or does not conform to some (sociocultural) norm. We see this at work when Gina, immediately following the fragment shown, uses size in restating her answer ("the square one's a bit shorter, and that one is a tiny bit taller"). She does so even though Mrs. Winter repeatedly has said that they were not sorting according to color and size. Gina has an opportunity to find out that this action does not conform to the rule to be learned when Mrs. Winter, in the next turn, sanctions the account ("what are two things we said we weren't going to sort by?"). On the other hand, when a sorting | reasoning pair passes, children can discover the norm in the form of the two-turn relation that has just passed. This not only holds for what the norm can be said to state explicitly but also to any contingency that the rule/norm may be said to imply. This phenomenon is known as the "etcetera clause" (Garfinkel, 1972, p. 28). What a rule/norm (and the implicit agreement) does or does not state can therefore be *discovered* after the fact. Thus, it is not that students *first* constructed the norm and then behaved accordingly. Instead, the children *discover* some norm (rule) when their immediately preceding actions subsequently are marked in terms of conformity or violation of the socio-mathematical* norm.

In the foregoing, actions come first and the norm is discovered as a result of the interactional order. More generally, any explicitly stated rule/norm (*e.g.*, "no sorting by color or size") only makes sense if the domain within which it operates is already known. Children learn the grammar of their mother tongue only *after* they already know to speak generally and only after they speak (mostly) grammatically correctly. Instructions—such as (a) sorting but not by color and size or (b) sorting and providing reasons—make sense once the students already know how to act in the ways that the instructions describe. That is, rules and norms do not *determine* (cause) their practical actions as seen in the lesson fragment. Instead, instructions constitute descriptions that can be used after some action has occurred to determine the degree to which the preceding action can be said to have adhered to the norm. And it is through the interactional order—the teacher saying that an action is to be or is not to be according to some rule—that a child's action becomes cultural (Vygotsky, 1989). When the reasoning (practical action) is in accordance with the norm (rule), then the current activity continues. It is when there is divergence between norm and the foregoing reasoning that the child is held publicly accountable [1].

As the fragment shows, not just any account suffices in the context of the mathematics* lesson. The children participate in a form of activity where the interactional order excludes certain descriptions (properties) of objects—those pertaining to color and size—as the basis of accounting for categorization. The rejection of an account (i.e., "we're not sorting by size") does not mean that the classification itself was incorrect. It is the pairing {object ↔ verbal account} and the nature of the second part that is at issue in the context of this lesson. In the verbal account, which has to correspond to the result of the (classificatory) action, only some statements are accepted and thereby marked as acceptable—those that unbeknownst to the children, initially conform to the historically evolved socio-mathematical* norms—whereas others, though legitimate in other contexts are not legitimate (*e.g.,* classifying by color or size). The socio-mathematical* norm provides for the orderliness of the classroom as an aspect of the mathematical* world: both in how it is produced and how it is recognized. Thus, when sorting (categorizing) by mathematical* shape is linked to a mathematical* account, then what has been done is *observably* and *recognizably* mathematical*. Members see that something mathematical* is occurring, and those acting do so to produce the visibility of what is mathematical* in the doing.

The primacy of the social in mathematics (classrooms)

When children come to a mathematics classroom, they do not *construct* knowledge and rules bottom up; instead, the "knowledge and rules of social interaction have a whole cultural history behind them and therefore pre-exist the interaction that takes place in the classroom" (Radford, 2008, p. 224). Classroom events are not only ordered and ordering, but its members make that order visible (*e.g.*, sorting + stating reasons). Saying that children learn in the classroom, a social setting, in itself does not render the forms of reasoning social. The context constitutes the social in a weak sense (Livingston, 2008). Instead, reasoning is social in the strong sense because it first exists as the observable interactional order. The interactional order—*e.g*., the sequential organization of turn taking—determines the sense of what is said and done in the saying, as much as the extent to which what has been said corresponds to the truth (Sacks, 1992). This is opposite to the ordinary ways in which talk and relation are analyzed and theorized, which give priority to classroom talk and take the interactional (social) order as secondary. It is not the classroom talk that determines the interactional order—the "social norms" and "socio-mathematical* norms"—but the other way around, the interactional order determines (a) what is said and done in talk and (b) how what is said is to be heard and understood. Because the new ways of relating, the "classroom social norms" and "socio-mathematical* norms" exist *as* the relation (in addition to occurring *in* the relation), the social nature of the norms *exists in* the orderly nature of the turn-taking routine, the primary phenomenon. The primacy of the social arises from that first relation in ontogeny where a child participates in some practice.

When mathematical practice is investigated, mathematicians are found to act in ways that makes order visible, which then allows others to recognize them as doing legitimate mathematics (Livingston, 2008) [2]. Mathematicians are always on the lookout for the specifically *mathematical* in the action of their peers. For example, "everywhere where proving is going on, provers are managing and supervising the ordinary, routine ways in which proving is done" (Livingston, 2015, p. 216). They do so not merely in general but locally, (a) in the here-and-now of every proof while it is unfolding without time out and (b) as the technical details of the particular proof-in-the-making. This maintenance of the visible and observable order is part of the background expectancy that characterizes mathematicians. What they do and the accounts they produce, the paired nature of doing and accounting (reasoning) that we observe *as* social relation in the classroom above, is typically mathematical in the community of mathematicians. With their actions, mathematicians are reasoning in ways that maintain the conditions for reasoning in those ways so that they can be seen as acting in recognizably mathematical ways.

In mathematics, there are pairs of actions that go together so that when one happens, the other does as well. Proving moves and proving accounts go together (Livingston, 1986). For example, placing an object in a category set and then accounting for the placement in verbal terms would constitute such a pair. If one is done, the other one is done as well. We may describe this tie between an action and an account as a norm. But such a norm is not determinate, which is to say, it neither *causes* the account once the action has been completed nor does the account cause the action, in those cases where the account has been provided before. Mathematicians orient so as to provide for both, actions and associated accounts, and, thereby, they orient to doing mathematics in a recognizable way.

When children are learning mathematics*, the observable order characteristic of mathematicians is not initially in place. As Radford (2008) points out, however, classrooms are not spaces "where knowledge and rules of interaction are negotiated anew" (p. 224). How then do the children come to act in ways that at their very core are typical of (school) mathematical culture? The example presented here shows that children did not "construct norms," for they would already have to be knowledgeable about the domain that will be governed by the norm. Instead, what we observe is how that norm, the tying of classificatory actions and verbal accounts, exists *as* social relation first. That is, the children participate in exchanges where those socio-mathematical* norms will come to be recognized as constituting the visible order.

A crucial aspect of these exchanges is the teacher, who, in taking a specific place in the interactional order, allows existing sociocultural orders to be observable. Here, this order pertains to the type of justifications that are allowed and to the tying of sorting acts and associated reasons. That interactional work includes an aspect of qualifying (sanctioning) what occurs; and Vygotsky (1989) suggests that it is precisely in this way that any higher psychological function comes about. Importantly, the higher psychological function always remains social. The interactional order provides opportunities for children's cultural learning and development, which has been described as "the growth of a world and is directed to the production of objective features of the persons' environment that 'any competent member can *see*'" (Garfinkel, 1972, p. 30, emphasis added). In the fragment, the children are enabled to see mathematical* reasoning precisely because it exists in the form of an ordered relation of actions and accounts mapped on the interactional order (*i.e.,* S-T-S). This relation attributes a crucial positioning of the teacher in the second turn slot of the interactional sequence. It is here that their contributions to the joint work are "progressively enforced and enforceable compliance of the developing member to the attitude of daily life as a competent societal member's way of 'looking at things'" (Garfinkel, 1972, p. 30).

Coda

Studies of the social order of things do not conjecture the social as the underlying truth, such as in the "social construction" of knowledge or norms, but instead re/discover the social in the *witnessable* order of everyday life. The discourse on social norms in mathematics classrooms and on socio-mathematical norms presents the members of a class as the constructors of the social. The social is derived and secondary. This is consistent with the critique of "negotiation" in mathematics classrooms, which presupposes that the students already have something to negotiate (Radford, 2008). To re/discover the social in mathematical reasoning and doing means to find and show how the social comes first and enables mathematical reasoning and doing. Vygotsky (1989) names this the *sociogenetic* method. To find out the origin of the *social* in mathematical reasoning and knowing, we need to investigate the witnessable interactional order. What is social about the work that results in any negotiation and the socio-mathematical norms that result from it? How do members orient to recognizably producing and monitoring the observability of order, an order that is the origin of the phenomena resulting from that same producing and monitoring?

Acknowledgments

I am grateful to Eric Livingston and Christian Greiffenhagen for their comments, feedback on, and questions to be addressed in an earlier draft of the manuscript.

Notes

[1] Latour (2013) makes the same case when he exhibits "a slight defect in construction," using the concept of *instauration* among others to correct the incorrect conceptualizations of (a) the source of action, (b) the direction of action, and (c) the qualification of the action as good or bad (*i.e.*, norm).
[2] For this reason, too, the order is discoverable on the part of members and researchers alike.

References

Cobb, P. & Tzou, C. (2009) Supporting students' learning about data generation. In Roth, W.-M. (Ed.) *Mathematical Representation at the Interface of Body and Culture,* pp. 135–170. Charlotte, NC: Information Age Publishing.

Foucault, M. (1975) *Surveiller et Punir: Naissance de la Prison.* Paris, France: Gallimard.

Garfinkel, H. (1972) Studies of the routine grounds of everyday activities. In Sudnow, D. (Ed.) *Studies in Social Interaction,* pp. 1–30. New York, NY: The Free Press.

Garfinkel, H. & Sacks, H. (1986) On formal structures of practical action. In Garfinkel, H. (Ed.) *Ethnomethodological Studies of Work,* pp. 160–193. London, UK: Routledge & Kegan Paul.

Latour, B. (2013) *An Inquiry into Modes of Existence: An Anthropology of the Moderns.* Cambridge, MA: Harvard University Press.

Livingston, E. (1986) *The Ethnomethodological Foundations of Mathematics.* London, UK: Routledge & Kegan Paul.

Livingston, E. (2008) *Ethnographies of Reason.* London, UK: Routledge.

Livingston, E. (2015) The disciplinarity of mathematical practice. *Journal of Humanistic Mathematics* **5**(1), 198–222.

McNair, R. E. (2000) Working in the mathematics frame: maximizing the potential to learn from students' mathematics classroom discussions. *Educational Studies in Mathematics* **42**(2), 197–209.

Mikhailov, F. T. (2001) The "Other Within" for the psychologist. *Journal of Russian and East European Psychology* **39**(1), 6–31.

Ng, O-L. (2016) The interplay between language, gestures, dragging, and diagrams in bilingual learners' mathematical communications. *Educational Studies in Mathematics* **91**(3), 307–326.

Radford, L. (2008) The ethics of being and knowing: towards a cultural theory of learning. In Radford, L., Schubring, G. & Seeger, F. (Eds.) *Semiotics in Mathematics Education: Epistemology, History, Classroom, and Culture,* pp. 215–234. Rotterdam, The Netherlands: Sense Publishers.

Radford, L. & Roth, W.-M. (2011) Beyond Kantian individualism: an activity perspective on classroom interaction. *Educational Studies in Mathematics* **77**, 227–245.

Sacks, H. (1992) *Lectures on Conversation 1964-1972.* Malden, MA: Blackwell.

Schegloff, E. A. (1972) Notes on a conversational practice: formulating place. In Sudnow, D. (Ed.) *Studies in Social Interaction,* pp. 75–119. New York, NY: The Free Press.

Vološinov, V. N. (1930) *Marksizm i folosofija jazyka: osnovye problemy sociologi eskogo metoda b nauke o jazyke.* [Marxism and philosophy of language: application of the sociological method in linguistics]. Leningrad, USSR: Priboj.

Vygotsky, L. S. (1989) Concrete human psychology. *Soviet Psychology* **27**(2), 53–77.

385. Ask yourself: Is it conceivable that someone learn to calculate in his head without ever calculating aloud or on paper?—"Learning it" presumably means: Being brought to the point of being able to do it. Only the question arises, what will count as a criterion for being able to do it?—But is it also possible for some tribe to be acquainted only with calculation in the head, and with no other kind? Here one has to ask oneself: "What will that look like?"—And so one will have to depict it as a limiting case. And the question will then arise whether we still want to apply the concept of calculating in the head here—or whether in such circumstances it has lost its purpose, because the phenomena now gravitate towards another paradigm.

Wittgenstein, L. (1953/2010) *Philosophical Investigations* (4th edition). Chichester, UK: John Wiley.