THE USE OF ORIGINAL SOURCES AND ITS POTENTIAL RELATION TO THE RECRUITMENT PROBLEM

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When students select or deselect mathematical sciences as part of their higher education, they may in fact be basing their choices on incorrect assumptions. David Pengelley has referred to this problématique as "reality vs. fantasy" [1]. The phrase has to do with university or upper-secondary school students not having an accurate idea of what mathematics is about when practised as a scientific discipline by, for example, pure and applied mathematicians in universities. From my own previous studies (*e.g.,* Jankvist, 2011b), I have found that upper-secondary school students' answers to questions of what professional mathematicians do typically range from having no clue at all, to believing that they perform some kind of "clean-up job" that consists of finding "errors" in already existing formulas and proofs, or more efficient ways of calculating already known quantities, *etc.* Often such views have to do with the students' beliefs about mathematics as something static and rigid; a belief of course not unrelated to the typical presentation of mathematical topics in textbooks. Few students seem to believe it possible that mathematicians can come up with *new* mathematics. Students therefore know neither what they accept to study if they choose to engage with the mathematical sciences, nor what they reject if they do not. The claim I make in this article is that the study of original mathematical sources can provide students with a truer image of mathematics as a scientific discipline, both pure and applied. History in general and original sources in particular show mathematics-in-the-making, as opposed to mathematics-as-an-end-product (Siu & Siu, 1979), as found in typical textbook presentations. This allows students to think not only about mathematics itself, but also about the aspects which surround the discipline and foster it, which in turn seems to give them a more favorable and, not least, a more accurate view of the subject.

Using history and original sources of mathematics in the teaching and learning of mathematics has been an integral part of mathematics education research for around four decades, since the creation of the ICMI affiliated study group on *History and Pedagogy of Mathematics* (HPM) in 1972/1976. During this time, HPM's focus has shifted from advocacy, through more or less personal reports on using history in teaching, to empirical research on the effects and efficacy of integrating history in classrooms and study programs [2]. Another shift within HPM research has to do with the use of original sources. While many of the earlier papers address the use of history through secondary sources, retelling of history, or even anecdotes, newer studies address students' reading of original sources and the various ways of making such a demanding endeavour meaningful (Jahnke *et al.,* 2002).

The reasons for resorting to original sources may have to do with the use of *history as a tool* or the use of *history as a goal* (Jankvist, 2014a). In the first case, the focus is on the teaching and learning of the "inner" issues of the discipline of mathematics (*in-issues*), such as abstract mathematical concepts, ideas and notions, theorems, proofs, conjectures, *etc.* or mathematical competencies and skills (recent examples are Barnett *et al.,* 2014; Clark, 2012; Kjeldsen & Blomhøj, 2012). In the second case, the focus is on meta-perspective issues (*meta-issues*) such as developing students' historical awareness of the evolution of mathematics and its interplay with culture and society, or with other disciplines *etc.,* through a use of original sources or excerpts (*e.g.,* Furinghetti, 1993; Jankvist, 2011a; Kjeldsen, 2012). The use of original sources may, however, help to address more general educational problems, such as: *retention* of students once they have entered the mathematical sciences at university level; *transition* of students between educational levels, *e.g.* between upper-secondary level and undergraduate level, or undergraduate level and graduate level; and, not least, the problem of the *recruitment* of students to the mathematical sciences in the first place.

From a "logical" point of view, it may be deduced that when the use of original sources can develop students' awareness and image of mathematics as a discipline, it should also have an effect on their potential recruitment to the mathematical sciences. This claim remains to be addressed empirically. This article is a first small step in this direction. More precisely, I refer to empirical data from a study in which upper-secondary school students worked with original sources as part of their mathematics education. The reason for addressing the recruitment problem specifically is it that this problem may be regarded as the most "fundamental" of the three mentioned above, in the sense that recruitment is a necessary condition for both transition and retention [3].

HAPh-modules: a reading of original sources

From February 2010 to May 2012, I followed a Danish upper-secondary school class of 27 advanced mathematics and physics students who, through two teaching modules, were exposed to extensive readings of original sources. An overall purpose of these modules was to introduce the students to aspects of *history, application, and philosophy* of mathematics (abbreviated as HAPh) and to do this simultaneously in one module relying on original sources (see Jankvist, 2013). In the first HAPh-module, implemented in April-May 2010, the students read Danish translations of the following texts [4]:

- Leonhard Euler (1736) *Solutio problematis ad geometriam situs pertinentis;*
- Edsger W. Dijkstra (1959) *A Note on Two Problems in Connexion with Graphs;*
- David Hilbert (1900) *Mathematische Probleme: Vortrag, gehalten auf dem internationalen Mathematiker-Kongreß zu Paris 1900* (the introduction).

The overall theme of this module was mathematical problems, as addressed by Hilbert in his introduction to the 1900 lecture. To make Hilbert's quite general observations a bit more concrete, the students first read the two other texts, each of which addresses a mathematical problem. Euler's paper from 1736 addresses the Königsberg bridge problem: how to take a stroll through Königsberg crossing each of its 7 bridges once and only once. Today, this paper is considered the beginning of mathematical graph theory. With the dawn of the computer era two centuries later, graph theory (and discrete mathematics in general) found new applications. Dijkstra's algorithm from 1959 solves the problem of finding the shortest path in a connected and weighted graph. This algorithm is used in almost every internet application that has to do with shortest distance, fastest distance or lowest cost. Dijkstra also discusses a method for finding minimum spanning trees, a problem relevant for the building of computers at the time, and since then used in telephone wiring and other applications.

The second HAPh-module was implemented in September-October 2012. The students read Danish translations of the following three texts:

- George Boole (1854) *An Investigation of the Laws of Thought on which are founded the Mathematical Theories of Logic and Probabilities* (chapters II and III);
- Claude E. Shannon (1938) *A Symbolic Analysis of Relay and Switching Circuits* (first parts);
- Richard W. Hamming (1980) *The Unreasonable Effectiveness of Mathematics.*

The title of Hamming's paper made up the theme for this module. Hamming discusses "the unreasonable effectiveness of mathematics" from the viewpoint of engineering (and computer science), asking why it may be that such comparatively simple mathematics suffices to predict so much. To provide the students with a concrete example, they were first introduced to Boole's idea of a two-value algebra and the context in which he conceived it in 1854, namely that of trying to describe language (and thought) from a logical point of view. Next, the students studied a later application of Boolean algebra by Shannon from 1938. By relying on a set of postulates from the now further developed Boolean algebra ($0 \cdot 0 = 0$; $1 + 1 = 1$; $1 + 0 = 0 + 1 = 1$; $0 \cdot 1 = 1 \cdot 0 = 0$; $0 + 0 = 0$; and $1 \cdot 1 = 1$) and by interpreting these in terms of circuits, Shannon was able to deduce a number of theorems which can be used to considerably simplify electrical circuits and their construction.

The implementation of these two modules consisted of groups of students working with the texts, while the teacher circled the classroom answering questions. No actual blackboard teaching took place. The written presentation of the original sources to the students followed the approach of "guided reading" (Barnett *et al.,* 2014). This approach offers a sensible way of dealing with the occasional inaccessibility of original sources. The idea is to supply or interrupt a student's reading of the text by explanatory comments and illustrative tasks along the way, while not in any way altering the original text itself. While such interrupting tasks often deal with the mathematical in-issues, at the end of each module the students were asked to prepare a set of essay-assignments dealing explicitly with meta-issues. Here, the students were to relate the three original texts of the module to each other, discuss various aspects of history, application, and philosophy (see Jankvist, 2013), as well as discuss which text they preferred and why (see Jankvist, 2014b). Each module ran over approximately ten 90-minutes lessons.

Three questions

During the 2-year period of the study, the students contributed to four questionnaires and interview-sessions. In order to get a direct indication of the potential effect of the HAPh-modules and the students' work with original sources in relation to the recruitment issue, I included the three following questions as part of the fourth and final questionnaire in March 2012:

1. Have the two modules provided you with a different view of what mathematics is; how it comes into being; and what it is used for? If yes, explain how and in what sense. If no, then why not?
2. Did the two modules encourage you to study or in any way concern yourself with mathematics (and/or natural science) after upper-secondary school? If yes, how and why? If no, why not?
3. Whether you answered "yes" or "no" to the above question (2), do you then consider the two modules to have provided you with a more enlightened basis on which to either select or deselect mathematics (and/or natural science) to be part of your future education? If yes, how? If no, why not?

The fourth questionnaire was completed by 24 students. Of these, 16 participated in follow-up interviews. The students' different answer combinations are given in Table 1. The reason for distinguishing between students who participated in interviews and students who did not is that sometimes students would alter their answers during the interviews. In particular, some students would change their original answer to question 3 (Q3), since apparently the phrasing in the questionnaire was not entirely clear to all of them. Any such changes are taken into account in the column "Q & I" and thus reflected also in the column "total".

Combination			Q & I	Q only	Total
Q1	Q2	Q3	16 students	8 students	24 students
Yes	Yes	Yes	6	0	6
Yes	No	Yes	7	1	8
No	No	Yes	2	0	2
Yes	Yes	No	0	1	1
Yes	No	No	1	5	6
No	No	No	0	1	1

Table 1. Number of student answers according to occurring combinations [5].

A first observation based on Table 1 is that a total of 21 students (88%) agreed that the HAPh-modules provided them with a different view of what mathematics is, how it has come into being, and what it is used for (Q1). This pattern is in line with previous findings related to the use of original sources (*e.g.,* Jahnke *et al.,* 2002; Barnett *et al.,* 2014). Furthermore, 7 students (29%) agreed that the HAPh-modules encouraged them to study mathematics or natural science as part of their future studies. Of these 7 students, 4 had already decided to pursue a higher education programme related to the mathematical sciences, but the remaining 3 may be characterized as potential "win over" students. Regarding question 3, a total of 16 students (67%) agreed that the HAPh-modules provided them with a better foundation to either select or deselect mathematics and/or natural science as part of their future education. Taking into account that 15 of these students participated in follow-up interviews and that 4 of these students altered their answer during the interview after the meaning of question 3 had been explained to them more clearly [6], it is reasonable to assume that the total number could have been considerably higher had all 24 students been exposed to follow-up interviews. The observation that the majority of these students actually answered "yes" to question 1 (5+1 students) supports this claim further, since the majority of the interviewed students who answered "yes" to question 3 also answered "yes" to question 1. I shall return to this observation later.

A deeper look at students' answers

To deepen and illustrate in which ways the students acquired a more accurate image of the discipline of mathematics, I present and comment on a selection of students' answers from the questionnaire and follow-up interviews [7]. In particular, I point out how to a large extent, it was meta-issues involved in the HAPh-modules that seemed to favorably influence the students' view of the subject regarding their potential recruitment to the mathematical sciences. The selection of quotations is based on two criteria: firstly, of course, they are representative of and illustrate the responses of all the students; and, secondly, the students were able not only to articulate the presence of an effect, but also to some degree what caused it.

Nikita was one of the students who in a follow-up interview, expressed being more open to pursuing studies related to the mathematical sciences than she had been prior to the HAPh-modules:

> I do think that I have got more of a reason to select it, than I had in the beginning [of upper-secondary school], because we've seen several different aspects of it [mathematics] due to these modules. If I had only been working with the textbook and so, my answer would definitely have been "No", I believe, because it's very monotonous and much of the same, whereas with the two modules we've had the opportunity to think differently and view things through different lenses and, yeah, see the interrelations in a more comprehensive way than we usually get things presented. So, personally I've discovered that there is much more to mathematics than what it says in the textbook. [...] I think it surprised me that someone actually has been sitting and working with these things, and then arrived at this. Because before I've only thought about mathematics as something just being there, and us as just having these and these things which we could make use of. I've never given it a thought that someone had sat down and worked on it and arrived at something to be used in certain contexts. I've never thought about it like that, only in the way that it's in the textbooks and that's just the way it's given. (Nikita, March 29th, 2012)

In both questionnaire and interview, Nikita is quite clear in answering "yes" to question 3, almost as if this is implicit in her answer to question 1. Due to Nikita's positive change from questionnaire to interview regarding question 2, she is counted as a "yes-yes-yes" student in table 1. Regarding her encounter with original sources, Nikita had on a previous occasion explained:

> Not only did you have to understand what it was about, you also had like the language of it, and it has been a different way of thinking compared to the mathematics we are usually taught, where we have this formula and it works like this, this, and this. Here you got all the background knowledge, and how he arrived at it, *etc*. For me, I personally think that I get much more interested, when I see it all, than if I'm only told that now we are studying vectors and we must learn how to dot these vectors and then we must be able to calculate a length, right. That's all very good, but what am I to use it for? Whereas, when you know about the background, the development up till today, that I think was exciting. (Nikita, November 3rd, 2011)

The following extract from an interview with a student Tobey illustrates the "yes-yes-no" combination and the change of this into "yes-yes-yes":

Tobey: Yes, they [the original texts] gave me an understanding of how you need to think completely different. [...] It has been quite an instructive experience in that regard; kind of an *aha*-experience once you began thinking about it in relation to all the [questionnaire] questions afterwards.

UTJ: Besides you being surprised due to the two modules, did they have any other impact on you?

Tobey: What they impacted is that I now may consider, well not to study mathematics directly, but to study something where you use mathematics to more than what you use it for in physics, because it is a deeper discipline than what you usually think it to be, with just formulas, plus, and minus. [...] There's more to it. It can be applied to several things, at least in relation to these ... (*phrase incomplete*), it would be cool to look at those problems which have been posed, but which have no solution yet. It would be cool to be involved in finding the solution to just a single one of them. It would be completely awesome.

UTJ: But then you answer ["No" in question 3] here, regardless of you wanting to study math or not, then the modules might have provided you guys with a more, well, made it so that you could either select or deselect on a more enlightened basis?

Tobey: But it has! I mean, after these [questionnaire] questions my answer has definitely changed, because yes, they can do that. (Tobey, March 29th, 2012)

Both Tobey and Nikita are representatives of the previously mentioned potential "win over" students as a consequence of their studies with the original texts in the HAPh-modules. For both of them it seems quite clear that this "encouragement" to possibly pursue mathematics further is due to the effect of the original texts on their view of mathematics. In terms of meta-issues, Nikita gives as reasons the interrelations between different parts of mathematics which the texts reveal, the fact that mathematics has come into being by the hands of human beings, and, not least, the different way of working when studying an original source, including its language, as opposed to the regular textbook. Tobey stresses the dimension of creativity in research mathematics and refers enthusiastically to the posed but yet unsolved problems in mathematics (with an implicit reference to the text by Hilbert)—a meta-issue of mathematics as a discipline that is not usually touched upon in textbooks. He even mentions that it would be "cool" contributing to the solution of such a problem, changing this to "completely awesome".

Katharine, also a "yes-yes-yes" student, said the following when asked immediately after the second HAPh-module about reading original sources:

> I like the original [texts] better. You kind of get inside the head of those people and think, well that's how they ... (*phrase incomplete*), because, if there is another one [an author of a secondary source] trying to interpret it, then I feel that they can't really figure out the original, so they take it to a lower level. Whereas I feel that you are challenged more when reading the original [text]. You get to sense how he [the author] has structured it, how he has thought, and so. [. . .] You felt that you got to know them a little more personally and how they expressed themselves using mathematics, explained [things], and so. Also, it provided you with ideas on how to express yourself mathematically, in your hand-in tasks *etc*. I found that very exciting. (Katharine, November 3rd, 2011)

Katharine is also an example of a "win over" student, since prior to the second HAPh-module she had no intention of pursuing the mathematical sciences. In the interview, Katharine mentions a crucial aspect of studying original sources, namely that there are no intermediaries to distort the communicated message. In terms of meta-issues, you get a glimpse of how the mathematicians actually thought; they talk to you; you feel you get to know them; it simply gets more personal, as Katharine remarks. This experience is, of course, closely related to the creativity of which Tobey speaks and the opposition to regular textbooks mentioned by Nikita.

Regarding the other students who gave positive answers to question 2 (table 1), the thing to notice is that the modules, and the reading of original sources, did not diminish their desire to pursue the mathematical sciences. In the case of Christopher, it may even have enhanced it:

> Well, you can say that what gave me some [insight] was all this philosophy, which lies behind, but also the way in which it has evolved [and] that it has evolved in order to describe a certain thing; for example that Boole used it to describe one thing, and then Shannon saw, okay, I apply it for this other thing and then develop it according to that. This connection; that it is two completely different things they are working with and they then can use the same [mathematics], that this mathematics can be applied in so many different contexts. (Christopher, March 29th, 2012)

Christopher was already set on studying something related to the mathematical sciences, but he stated in his questionnaire answer that "the modules definitely did not reduce this desire". Christopher mentions several meta-issues, such as that mathematics has a philosophical dimension, that its evolution is often directed by extramathematical circumstances and that a piece of mathematics developed for one purpose may later prove useful for another.

A student representing the "yes-no-yes" combination is Salma, who gave the following three questionnaire answers of which the first particularly relates to mathematical meta-issues. Question 1: "Yes. It has shown me how mathematics develops, and at the same time how mathematicians work with mathematics. And that mathematics is its own language." Question 2: "No, I must admit that it hasn't. I do find it [mathematics] quite interesting, but there are things which excite me more." Question 3: "I have never considered studying mathematics. But if I had, then it would have been nice with these modules, since I feel that you will know much better what you agree to study."

Sophia, another representative of the "yes-no-yes" combination, explains her encounter with original sources in mathematics as follows:

> Regarding the modules, even though it has been a little dry from time to time, I do think that it has been nice to get the historical [dimension], to read the original texts, and do it the way they did, the people who developed things. [...] in order to get it at a slower pace, to try and figure out "what the fuck is going on here?" That is, instead of just sitting and doing exercises, which you do in school. To try something completely different, something which might be more similar to what they [the originators] actually did. (Sophia, March 27th, 2012)

The data discussed above suggests that the use of original sources may encourage some students to study the mathematical sciences. The more important observation, however, is related to question 3: two-thirds of the students agree that the modules enabled them to either select or deselect future studies involving mathematics and/or natural science on a more enlightened basis. For the majority of these students this appears to be directly related to the original texts having provided them with a different view of mathematics as a discipline (Jankvist 2014b)—a view which now includes a wider spectrum of meta-issues related to the discipline. That the use of original sources can change students' views of mathematics is not a new finding. What *is* new, however, is the empirical suggestion of a direct connection between a students' positive answer to question 1 and a positive answer to question 3 [8]. Hence, the more important observation is not necessarily that a use of original sources may "win" some students over to the mathematical sciences, but that the students who are won over are done so on a more enlightened basis. Equally important is, of course, that the students who deselect the mathematical sciences also do this on a more enlightened basis, as illustrated by the responses of Salma and Sophia. Moreover, the students who agree to being able to select or deselect on a more enlightened basis, all do so with reference to the meta-issues brought forth by their study of original sources.

To return to the outset of the article, four decades of HPM research demonstrate the many benefits of using history and in particular original sources in the teaching and learning of mathematics. In research about the use of original sources, the focus has mainly been on their use to assist students' understanding of mathematical in-issues such as concepts, theorems, proofs, *etc*. (history as a tool). As is evident from the quotations in this study, however, many of the students' reasons for how original sources changed their view of the discipline of mathematics have to do with the meta-issues of the discipline (history as a goal), including historical awareness of mathematics as a still evolving, human involving and culturally based discipline, and that it does make sense to talk about *new* mathematics. Of course, the HAPh-modules were designed to focus on in-issues as well as meta-issues (Jankvist, 2013). Indeed it seems to me that if we are to pursue the full potential of original sources in relation to the problem of recruitment, and possibly also that of retention, an effort should be made to balance a focus on in-issues with a focus on meta-issues. Knowledge of mathematics as an academic and practised discipline appears equally important as knowledge of mathematics itself in this regard. And original sources can provide both.

Some further remarks

As I have mentioned, recruitment is a necessary condition for both transition and retention; without any recruitment, the other problems would not exist. Given this dependency, it appears clear to me that recruitment on an enlightened basis is part of the initial solution to the other problems, in particular that of retention. If students who enter the mathematical sciences at tertiary level have a more realistic image of the discipline they are about to study, then one would expect a higher degree of retention among such students. And, as illustrated through the student quotations, a use of original sources can indeed contribute to the development of such a realistic image, because, as remarked by Sophia, they did something "more similar to what they [the mathematicians] actually did".

The reason, though, for referring to recruitment on an enlightened basis as only *part* of the solution to the transition and retention problems, is that the use of original sources may also affect these other two problems once they occur at tertiary level (Jankvist, 2014a). For example, well chosen original sources motivating the introduction of mathematical concepts, definitions *etc*., may be much easier to access for students than modern textbooks that present the abstract "aftermath" (*e.g.,* Mosvold *et al.,* 2014). If, say, introductory undergraduate courses could be designed based on such suitable original sources, then students might be better accustomed to higher mathematical thinking and thus experience an easier transition from upper-secondary school. In relation to retention, original sources can provide the context which textbooks often strip out: recall the comment by Nikita on missing a reason for studying vectors. Also, original sources can make mathematics more personal, providing students with the opportunity to relate to and identify themselves with mathematicians, get a glimpse of their thought processes *etc.,* which are exactly the things that Katharine brings up.

The above mentioned lack of context is closely related to another educational problem: that of *interdisciplinary* teaching and learning, which often presents a didactic dilemma. On the one hand, students are told that interdisciplinary work is extremely important (for various different reasons); on the other hand, the students are often only shown somewhat artificial and situationally constructed examples, which make the dimension of interdisciplinarity appear pasted-on (Jankvist, 2014a). Because original sources are contextually based, provide authenticity, and deal with reality, even if it is a historical one, any given interdisciplinary elements within them are likely to illustrate much better to students the importance of interdisciplinarity in research and society [9].

Another potential benefit of using of original sources is indicated in the quotation of Nikita, who talks about the *language* of the original source illustrating a "different way of thinking" compared to the usual mode of presentation in the mathematics classroom. It is no secret that many students find challenging the language of mathematics and the logic of language, in particular with regard to the comprehension of mathematical reasoning and the notion of proof [10]. Perhaps original sources have yet a role to play in this respect, one which is not usually associated with the benefits of having students read and work with original sources as part of their mathematics education. Hence, the potential roles of original sources still to be explored not only encompass the triple aspect of recruitment, transition and retention; meaningful interdisciplinarity and the development of a better appreciation of mathematical language are also on the agenda.

Notes

[1] Pengelley made this remark during a panel on "Empirical research on history in mathematics education: current and future challenges for our field" at HPM2012 in Daejeon, Korea, organized by Uffe Thomas Jankvist along with panellists David Pengelley, Yi-Wen Su, and Masami Isoda.

[2] This shift can be seen in the pages of *For the Learning of Mathematics.* Freudenthal (1981) is an example of advocacy; the special issue FLM 11(2), edited by John Fauvel, includes personal examples of the use of history in teaching mathematics, as well as contributions by Katz (*e.g.,* 1997) and van Maanen (*e.g.,* 1997); empirical studies have been published by Arcavi *et al.* (*e.g.,* 1987) and Furinghetti (1997) among others.

[3] The present work has – as part of the STAR-project – been supported by the *European Social Fund* through grant no. ESFK-09-0024. The development of the two HAPh-modules was supported by the *Danish Agency for Science, Technology and Innovation.* A previous version of the paper was presented in WG12 on History in Mathematics Education at CERME8.

[4] The precise references to the original sources may be found in the teaching modules, which are available as texts no. 486 and no. 487 at: http://milne.ruc.dk/ImfufaTekster/

[5] No answers of the combinations "no-yes-yes" and "no-yes-no" occurred.

[6] As in any other interview situation, there is always the possibility that the interviewee is trying to please the interviewer by answering what (s)he thinks the interviewer wants. The way of trying to avoid this was to invite the students to elaborate on their answer to question 3, and when doing so some students would realize more clearly the meaning of the question and change their answer from "no" to "yes".

[7] All student quotations have been translated from Danish.

[8] If we follow through with this, we may assume that the 6 "Q only" students who answered "yes" to Q1 but "no" to Q3, might have altered their answers had they been explained the question more thoroughly in an interview session. Taking the 2 "Q & I" students who answered "no" to Q1 but "yes" to Q3 as a source of error, we would get 20 out of 24 (83%) instead of 16 out of 24 (67%). But this is of course to some degree speculation.

[9] See Kjeldsen and Blomhøj (2012) for an example of using original sources on physics and mathematics, and Tzanakis and Thomaidis (2000) for discussing the close historical development of these two disciplines.

[10] Together with Mogens Niss, I have recently developed and implemented a "Maths Counsellor" program for Danish upper-secondary mathematics teachers, whose purpose shall be to detect students with learning difficulties in mathematics, diagnose in what these difficulties consist, and intervene accordingly. In relation to the topic of mathematical reasoning and proof, we found that a major stumbling block for students had to do with the actual reading and interpretation of mathematical arguments and proofs, which is of course a prerequisite for ever encountering the matters of mathematical content or logical structure involved.

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