

CRITICAL CONSTRUCTIVISM: INTERPRETING MATHEMATICS EDUCATION FOR SOCIAL JUSTICE

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The literature of mathematics education for social justice is rich in well-elaborated and challenging examples. It is a pleasure to dig into this inspiring educational productivity. However, at times this literature reveals a certain degree of conceptual laziness. Specifically, one seldom finds detailed elaborations of the very notion of social justice.

It is convenient to use this notion in a common-sense formulation. In fact, it appears superfluous to argue in favour of a mathematics education for social justice: who would ever try to argue in favour of a mathematics education for social injustice? However, taking a closer look at the notion of social justice, we soon come to realise tremendous complexity and a diversity of interpretations. The notion of justice has deep philosophic roots, and its possible meanings have been interpreted and reinterpreted ever since antiquity.

In this article, I am going to provide an interpretation of a mathematics education for social justice, which on the one hand relates to philosophic discussions of social justice, and on the other hand relates to classroom practices. I see mathematics education for social justice as providing opportunities for students to *formulate visions and hopes* and in this way supporting them in *articulating conceptions of social justice*.

In order to get to this understanding, I start out with an example, *Bolsa Familia*, which I will refer to during the following discussions. Then I consider *ethical realism* and *ethical anarchism* before formulating the position of *ethical constructivism*. This constructivism provides the departure for my interpretation of mathematics education for social justice.

Bolsa Familia

João Luiz Muzinatti teaches mathematics to students from an upper-middle-class neighbourhood in São Paulo. He organises some of his teaching as project work. One of the projects concerned *Bolsa Familia*, which refers to a programme of family support [1]. This programme was implemented as part of the social programmes of PT (*Partido de Trabalhadores*, The Workers' Party) in order to reduce the extreme poverty in Brazil.

When starting the project, Muzinatti asked his students for their views about *Bolsa Familia*. A common reaction was that this social programme was questionable, and furthermore, that an unequal distribution of wealth in society is fair enough; those who are working more and have studied more should also earn more. This claim was supported by statements such as: poor people in general are lazy; if one really wants to do something and to make progress in life,

it is possible; and *Bolsa Familia* provides a temptation for lazy people to continue doing nothing.

As a next step in the project, Muzinatti presented more information about the social programme, for instance about how much a single family would get in financial support. Such information surprised the students; they realised that one could easily spend this amount of money on a single trip to a supermarket—and the money had to last for a whole month! With such information available, the project became developed.

This example is similar to many examples of mathematics education for social justice. In the literature, one finds examples pointing out the many forms of suppression, exploitation, racism, sexism *etc.* that take place in our society. What differentiates *Bolsa Familia* from most of these examples, if not from all of them, is that while mathematics education for social justice normally engages students that suffer social injustices, Muzinatti engages students in comfortable positions. He works with students who apparently benefit from economic injustices. Later I will return to this observation.

Ethical realism

In the *Republic*, Plato addressed the idea of justice. Plato's philosophic outlook was deeply inspired by mathematics. Mathematics provided him with a context in which empirical validations are irrelevant, as mathematical statements do not concern physical objects, but ideas. The world of ideas was considered by Plato as being real. For instance, the mathematical notion of triangle has a real reference, namely the triangle that exists in the world of ideas. In fact, according to Plato, any mathematical object has a real existence in the world of ideas [2]. Plato's conception of ideal references reaches much further than mathematics. It also applies to a notion like justice. Plato's approach in clarifying any such notion was to try to discover its proper reference in the world of ideas.

By *ethical realism* I understand a position which shares with Platonism the idea that it is possible to identify some proper interpretations of ethical notions. However, in my interpretation ethical realism need not assume the very existence of a world of ideas, but it assumes that it makes sense to try to *discover* what, say, justice *really* means.

While the notion of 'justice' has been addressed since antiquity, the notion of 'social justice' first appeared in the 19th century. This notion was coined by Luigi Taparelli, who highlighted its religious features. From such a perspective, the meaning of social justice could be discovered through

careful studies of the Bible. These ideas were explored further in *The Constitution Under Social Justice (Costituzione secondo la giustizia sociale)* written by Antonio Rosmini-Serbati and first published in 1848. Here we find the notion of social justice related to a range of political ideas circulating at the time, but simultaneously to strong religious assumptions.

Classical Marxism does not explicitly address the notion of social justice, but without great difficulties it is possible to integrate the notion within a Marxist outlook. Thus, the capitalist order of things represents a profound degree of injustice. This injustice, however, is temporary as, in the end, the very logic of capital will destroy capitalism and make space for a new order of social justice. Classical Marxism operates with a version of ethical realism, to the extent that it assumes it is possible to identify social injustices as well as to discover proper directions for establishing social justice.

Ethical anarchism

A radical refusal of any version of ethical realism came from Frederick Nietzsche who exercised an *ethical anarchism*. This anarchism inspired the whole postmodern movement. Here, however, let me concentrate on Nietzsche's own anarchism.

Nietzsche claimed that Platonism represented the malignant disease of philosophy. The Platonist concept of truth is based on a distinction between appearance and reality, claiming that true knowledge concerns reality whatever we have to do with mathematics, ethics, or anything else. But, according to Nietzsche, there is no reality behind appearance: "The 'apparent' world is the only one: the 'real' world has only been *lyingly added*..." (2003, §2). Lyingly added by Platonism. There is no real world, no world of ideas, which truth may refer to. Truth is not about anything. It has no reference. It is an illusion. It is a way of talking. It is just a "mobile army of metaphors" (2010, p. 29).

Such complete elimination of truth opens the way for the most radical form of perspectivism. Concepts do not have proper references; in fact they do not have any references. Instead, they drift around in an open landscape of appearances. There is no Archimedean point outside the stream of life from where one can look at things, judge things, and formulate true statements.

Nietzsche's perspectivism concerns epistemology as well as ethics, and he has only ironic remarks to make with reference to a conception such as democracy (1998). According to Nietzsche, democracy is just a pitiable expression of slave morality, which puts values upside down by celebrating the poor, powerless, suffering, deprived, sick, ugly, *etc.*, while considering the noble, powerful, and beautiful to be evil, cruel and lustful. Similar remarks can be made with respects to other notions from the ethical domain as, for instance, social justice. There is nothing to be celebrated with respect to such a notion, and there is nothing to be discovered. Social justice is just a mobile metaphor.

In terms of modernity and postmodernity

It is possible to associate ethical realism with a modern outlook, and ethical anarchism with a postmodern outlook.

As a 'modern departure' for a mathematics education for social justice, one could try to identify principal injustices in

society. One could explore: colonialism which leads to stigmatising discourses, even after explicit colonial structures have become dismantled; capitalism which maintains economic suppression and exploitation in all spheres of life; racism which seems to multiply in ever new discourses; and sexism which seems able to proliferate through a variety of patterns of suppression. Such a departure is found in many versions of mathematics education for social justice that, as a consequence, can be seen as a realisation of a political agenda. Such an agenda, then, might assume some of the grand narratives of Modernity—to use a notion coined by Jean-François Lyotard (1984)—in particular the narratives inspired by Marxism or related positions. A 'modern departure' would tend to assume some validity of such narratives, and in this sense embark on a version of *ethical realism*.

By not assuming any grand narratives, a 'postmodern departure' for a mathematics education for social justice might bring the discussion closer to *ethical anarchism*. Inspired by the approach of Michel Foucault, Thomas Popkewitz (2004) addresses the power dynamics that can be associated with mathematics education. An overall observation he makes is that the format of the mathematics curriculum serves particular interests and that mathematics education turns into a procedure for making people governable. Acknowledging this, it appears questionable what 'progress' and 'making improvements' might mean with respect to mathematics education. With reference to Popkewitz, working for social justice through mathematics education becomes an expression of romanticism, negating the power of the economic and political structures in which mathematics education is encapsulated.

In the following section, I will try to open the way to a *third* possible departure for interpreting mathematics education for social justice, not assuming any ethical realism nor any ethical anarchism. I want to open the way for a constructivism with respect to ethics, which I will refer to as *ethical constructivism*. I will leave it as an open question to what extent this constructivism relates to a modern or to a post-modern outlook.

Mathematical constructivism

Constructivism is a well-elaborated position in the philosophy of mathematics, and I will elaborate a little on this position before I explore constructivism with respect to social justice and ethical concepts in general.

The Intuitionism of L. E. J. Brouwer does not assume the pre-existence of any mathematical reality. Instead, it claims that mathematical objects are mental *constructions*. As a consequence, a theorem like Goldbach's conjecture, that is still not proved or disproved, is neither true nor false. Only a mathematical constructive process would provide the theorem with a definite truth value. Intuitionism seems ready to follow Nietzsche's attack on Platonism by claiming that the mathematical reality has only been lyingly added.

In *Proofs and Refutations*, Imre Lakatos (1976) investigates the dynamics of mathematical development in terms of proofs and refutations, and in this way he contributes to a further elaboration of constructivism. Lakatos talks about proof-generated concepts, and he illustrates how such concepts emerge through complex processes of interaction.

Proof-generated concepts do not have any *a priori* references to pre-existing mathematical objects, but they make new formulations of theorems and lemmas possible and open for new ways of proving.

Lakatos makes a careful study of Euler's polyhedron theorem, and he shows how the very notion of polyhedron develops through the process of proofs and refutations. The mathematical object 'polyhedron' is not any well-defined Platonic entity of which true properties gradually become discovered. Rather, we are dealing with a mathematical conception under construction—a construct whose properties become established as part of the process. And this is the general idea of mathematical constructivism: mathematical objects become constructed, and through this process their properties become formed. This applies not only to polyhedrons, but to any mathematical objects: vector spaces, groups, real numbers, *etc.*

At the beginning of the 1930s, Ludwig Wittgenstein distanced himself from the philosophic perspective he had expressed in *Tractatus*. During this intensively productive intermezzo, he formulated a thoroughly constructivist view of mathematics, his *Remarks on the foundations of mathematics*, negating any version of Platonism. Wittgenstein gave a series of lectures, and over a long period Allan Turing joined in. Turing confronted Wittgenstein by expressing a version of Platonism as broadly assumed among working mathematicians. This was certainly experienced by Wittgenstein as a fruitful challenge, and once when Turing was not able to join the lecture, Wittgenstein simply cancelled it (Monk, 1990).

Wittgenstein claimed that any mathematical law is an expression of social conventions. This, however, does not mean that 'anything goes'. The notion of social convention does not make us slide deep into relativism. Let me illustrate by comparing mathematics and language. In language, one meets many rules and conventions, and one can make mistakes with respect to grammar and spelling. However, grammatical rules and conventions do not represent any grammatical Platonic reality; rather, they are formed through a long process of social construction. Following Wittgenstein, what can be said about language applies as well to mathematics. Mathematical laws are expressions of grammatical rules for the use of words like 'and', 'or', 'if...then', 'the following', 'in total', *etc.* Such rules are developed during time. Mathematical rules are not rooted in any Platonic reality, but in social conventions [3].

Ethical constructivism

I want to consider constructivism not only with respect to mathematics but with respect to ethics as well. While mathematical constructivism concerns the formation of a concept like polyhedron, ethical constructivism concerns the formation of a concept like social justice [4].

An important idea in ethical constructivism was presented by John Rawls (1999). Let us imagine a meeting for the formulation of principles of social justice. We assume that the formulation of such principles cannot be justified by pre-established references, but has to emerge from the meeting. According to Rawls, this is possible on the proviso the meeting takes place under two conditions: *First*, the people who

join the meeting are going to become members of the society in which the formulated principles are going to be implemented. *Second*, the people are not going to know in advance which position they will come to occupy in society; they will not know if they will become rich or poor, men or women, adult or children, ill or healthy, disabled or not, *etc.* Giving these two conditions, Rawls stipulates that the meeting could result in a conception of social justice.

Rawls' thought experiment illustrates that ethics can be considered as having a discursive basis, assuming that a conception of social justice emerges through interaction and communication [5]. As the construction is supposed to take place under conditions that never can be realised, I refer to the Rawls' meeting as an *idealised meeting* for constructing a conception of social justice. However, could we imagine a process of ethical constructions not located in an idealised surrounding, but in real-life processes?

As a start, let us imagine a real-life meeting taking place in the year 1700. The discussion may concern slaves, and one might reach the conclusion that in a just society, everybody (to be understood as every man) has the right to own the same number of slaves. We can imagine a meeting taking place the year 1800 addressing what social justice would mean with respect to women. That social justice could have anything to do with women's right to vote would not occur in the discussion. Instead, the debate might concern the type of politeness that men should exercise with respect to women; when to open the doors, what words to use, and how deep to bow. We can imagine in the year 1900 the discussion might emphasise that in a just society anybody suffering from homosexuality have the right to an adequate treatment. And one can imagine that in the year 2000 a meeting might address what social justice means with respect to refugees. Here, representatives from the Danish Government might advocate that, in a just society, refugees' jewellery should be confiscated. We can think of any such meeting as *real-life meetings about social justice*.

Considering such real-life meetings, we are reminded of Nietzsche's claim that there is no escape from the particularities of perspectives. We are always submerged in a network of preconceptions, stereotypes, and ideologies. Following Nietzsche, we are going to recognise that social justice is a human, all too human, construction. It might appear that ethical constructivism easily slides into ethical anarchism. However, there is more to be said with respect to real-life meetings for constructing conceptions of social justice.

Classrooms as real-life meetings

Let me now return to the project *Bolsa Familia*, which addressed a specific form of family support. However, the project also addressed much broader issues: Is it fair that people in poor conditions get such a support? What is the cause of their misery? Who could be held responsible? What about children in miserable positions? Could they be held responsible? What to think of the unequal distribution of wealth and resources? All such questions have to do with conceptions of social justice.

In fact, I suggest that one interprets a project like *Bolsa Familia* as having to do not only with *learning about* social

issues but simultaneously with *articulating conceptions* of social justice. I suggest that we think of a project work as organised by Muzinatti as a *real-life meeting for constructing conceptions of social justice*.

I do not interpret the project as first of all having to do with teaching students *about* social issues. Rather, I see it as providing a space for challenging pre-conceptions and implicit conceptions of social justice. The students have not experienced the economic difficulties that *Bolsa Familia* is trying to compensate for. However, by putting this example of family support on the agenda, the students' conception of what is just and fair might be reworked.

As already highlighted, the literature in mathematics education for social justice is rich in examples of projects addressing unemployment, taxation, sexism, racism, immigration, *etc.* Let us consider a possible project about taxation, which we could refer to as *Tax Policy*. Like *Bolsa Familia*, *Tax Policy* could raise many issues concerning social justice: What should be subjected to taxation? What scale should be used for income tax? Should everybody pay tax? What parameters should be taken into account when income tax is calculated? Marilyn Frankenstein has paid particular attention to issues about taxation and pointed out: "I have decided that community education about tax policy will now become the focus of my professional work." (2012, p. 49).

It is possible to elaborate *Tax Policy* as part of a political agenda, and Frankenstein is explicit about this possibility. She points out that "we teach categories of analysis and theories of how society works" (2012, p. 54). However, I suggest that a project such as *Tax Policy* can also be seen as providing opportunities for the students to articulate and reconsider their conceptions of social justice. Thus, discussions about tax policy would make it possible for students to formulate for themselves what a fair redistribution of economic resources could mean, and what social justice could mean in general.

I suggest interpreting projects like *Tax Policy* and *Bolsa Familia* from the perspective of ethical constructivism. They open up a way for constructive processes that lead students to articulate conceptions of social justice.

A constructive process

Lakatos presents the constructive process through which the notion of polyhedron becomes formed as guided by a method of proofs and refutations. I do not have the aspiration of being able to provide a similar method guiding the formation of conceptions of social justice. However, I do not see such a process as being 'free' in the sense that 'anything goes'. The *real-life meetings for constructing conceptions of social justice*, as taking place in the classrooms, can include many elements, and let me just refer to a few of them.

Argumentation and information. Lakatos presents the process through which the notion of polyhedron becomes elaborated as being rich in argumentation. Similar observations can be made with respect to the construction of an ethical notion such as social justice. Thus, the project *Bolsa Familia* draws on information presented in figures and statistics, and it includes a range of argumentation. The students came to know the actual amount of money provided in family support. They had the possibility to compare this

with budgets from their own experiences. They gained new insight into other peoples' life-conditions. As a consequence, they could revise some of their original claims about, for example, poor people being lazy and not wanting to do anything due to the family support they receive. In this way, pre-established conceptions of social justice might be challenged and reformulated in a process rich in argumentation and information.

Dialogue. Lakatos outlined the complex interaction that takes place when mathematical notions develop, and he presented this historical process in the form of a dialogue taking place in a classroom. My point is that such interaction is also taking place when ethical conceptions are developed. I see ethical constructions as taking place in a complex dialogic process. This can be characterised in different ways; one is found in Alrø and Skovsmose (2002) outlining dialogic acts such as, for instance, listening, reformulating, and evaluating. The interaction described by Muzinatti (in progress) with respect to the project *Bolsa Familia* is of a dialogic nature.

Diversity. An important feature of Rawls' idealised meeting was that the people joining the meeting did not know which position they were going to assume in society. Naturally, there are no real-life possibilities for complying with such an assumption. However, the assumption points in an important direction: when social justice is addressed it is important to consider a diversity of situations and life-conditions and to articulate different perspectives. Such diversity can be present in the classroom, either in terms of a diversity among the students or in terms of a diversity of information and perspectives addressed in the classroom. Muzinatti articulated a diversity of perspectives in the upper-middle-class classroom environment, and for me, diversity is an important feature of a real-life meeting for the formulation of conceptions of social justice.

Critical expressions of visions and hopes

Certainly, the students working with *Bolsa Familia* have visions about the future, both with respect to personal possibilities and with respect to society. Visions need not be explicitly formulated—they might be implicit, even obscure—still they can be challenged. Visions can be elaborated and, maybe, transformed in new directions.

I see a close connection between visions for the future and conceptions of social justice. In fact, I find that conceptions of social justice represent condensed articulations of visions for the future.

The project *Tax Policy* could address implications of a change in the format of taxation. Paying particular attention to such possibilities, one can see the project as including an articulation of hopes for the future. Frankenstein, as well as many others working with mathematics education for social justice, is deeply inspired by the work of Paulo Freire who talked about a pedagogy of hope. Following this line of thought, I find that a *mathematics education for social justice can be interpreted as an education, which addresses visions and hopes for the future*.

The notion of *sociological imagination* might be useful for emphasising this idea. This notion was coined by Charles Wright Mills (1959) capturing the idea that there are different ways of looking at sociological observations. One can

take them as given facts, but also as contingencies that could be changed. One can think of sociological imagination as a way of expressing alternatives to what is the case for the moment. Sociological imagination is a way of expressing visions and hopes. For instance, one can address the tax system as a contingency that could be different. More generally, I see conceptions of social justice as *expressions* of sociological imagination.

Let me briefly return to the anarchist departure for a mathematics education for social justice. The reason that I do not feel tempted to explore this line of thought further is that I find it might lead to underestimation of the relevance of sociological imagination and the related notions of visions and hopes. I do not follow the line of thought that might relegate sociological imagination to being an expression of romanticism. Instead, I find that such imagination is crucial for the constructive processes that lead to conceptions of social justice [6].

Let me admit that the notion of critique has been overworked in philosophy and certainly also in my own writings. But let me be excused; I like the notion. I want to talk about *critical constructivism* and not just about constructivism in order to emphasise that any human constructions are tentative and could turn out to include very problematic features.

In order to formulate a critical constructivism with respect to mathematics, one needs to take an important step beyond what both Lakatos and Wittgenstein are suggesting. One needs to be open to the possibility of addressing constructed mathematical concepts through profound critical investigations. Such critique concerns not only the formation of mathematical knowledge, but also the possible applications of such knowledge. Furthermore, one needs to consider not only explicit applications, but as well the range of implicit functions of mathematics knowledge and techniques in society. This is crucial for critical constructivism with respect to mathematics [7].

Critical constructivism also concerns ethics. As previously indicated, by imagining meetings taking place in the years 1700, 1800, 1900 and 2000, one can conceive visions and hopes as formulated from particular sets of preconceptions. One cannot assume the existence of any overall perspective that provides general justifications of visions and hopes. Sociological imagination is tentative and preliminary. It is in need of critique.

Mathematical as well as ethical constructions are surrounded by huge uncertainties. In fact, I relate critique and uncertainty closely [8]. A critical engagement is a profound recognition of the uncertainties that accompany any constructive process, also processes that lead towards conceptions of social justice.

Notes

- [1] For a full description of the project, see Muzinatti (in progress).
 [2] Platonism with respect to mathematics has been broadly assumed and, as emphasised by Hersh (1998), it makes part of the outlook of the working mathematician.
 [3] The conception of social constructivism as a philosophy of mathematics is formulated explicitly by Ernest (1998). His presentation makes particular

references to Lakatos and Wittgenstein.

[4] A preliminary formulation of such ethical constructivism was outlined in Skovsmose (2012).

[5] For a further elaboration of discourse ethics, see, for instance, Habermas (1991).

[6] For a discussions of sociological imagination see also Skovsmose (2011) and Skovsmose and Penteado (2016).

[7] In our upcoming book *Philosophy of Mathematics*, Ole Ravn and I also address features of such a critique.

[8] See Skovsmose (2014)

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Grand Manan Sand Dollars
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